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CONNECTED SPACE IN TOPOLOGY

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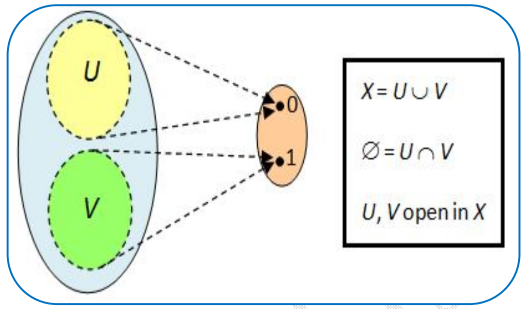

ABSTRACT:

From the intuitive point of view a connected space is a topological space which consists of a single piece. This property is perhaps the simplest which a topological space may have, and yet it is one of the most analysis and geometry. In this paper we introduce different type of connected space in topological space, Examples, theorems are present.

KEYWORDS: *Connected space, disconnected space, totally disconnect, path connected, locally connected spaces.*

INTRODUCTION:

In Topology and related branches of mathematics connected spaces in a topological space that cannot be represented as the union of two disjoint non empty open subsets. Connectedness is one of the principal topological properties that are used to distinguish topological spaces. A subset of a topological space α is a connected set if it is a connected space when viewed as a subspace of X.

SEPARATION:

A subspace Y of a Topological space (x,z) is said to have a Separation if and only if there exists subsets A,B of X, such that (Here A is the closure of A in X)

1) $Y = A \cup B$
2) $\bar{A} \cap B = \emptyset = A \cap \bar{B}$.

CONNECTED SPACE:

Let X be a topological space has separation of X is affair (u,v) of disjoint nonempty open subset of X. Whose union is X, the space X is said to be connected if there does not exist a separation of X.

Note:

Another formulation a space X is connected if the only subsets if X that are both open and closed in X or the empty set and X itself for if A is not empty proper subset of X that is both open and closed in X. Then the set U= A, V= X- A constitute a separation of X.

Disconnected:

Our space X is said to be disconnected if it is not connected, that is if it can be represented in the form X = A∪B, where A and B are disjoint, nonempty and open, and if X is disconnected, a representation of this form is called a disconnected of X.

Examples:
1. The empty set and singleton set are connected.
2. ℝ the space if real numbers with usual Topology is connected.
3. Any Topological vector space over a connected field is connected.
4. The set {0,1} is disconnected with usual Topology.
5. Q is not connected.

Theorem:

If the set C and D form a separation of X and Y is connected subspace of X. Then Y lies entirely within C or D.

Proof:

Since C and D are both open in X. C ∩ Y and D ∩ Y are open in Y. Also there are disjoint and there union is Y. If they were both nonempty they would consitive separation of Y but Y is connected. Therefore one of them is empty, Y lies entirely within C or D.

Theorem:

The image of a connected space under a continuous map is connected.

Proof:

Let f: X→Y is continuous map and X is connected to show Z= f(x) is connected, since the map obtained by f. By the image of space Z is continuous. Consider a continuous surjective map g: X →Z.

Suppose Z= A∪B is separation of Z into disjoint nonempty open set in Z. Then $g^{-1}(A)$ and $g^{-1}(B)$ are disjoint open set in X whose union is X. They are non empty because g is serjective the form separation of Y contradicting the assumption that X is connected.

Problem:

When A is connected, show that $\bar{A}$ is connected

Solution:

A= $\{(0,y); -1 \le y \le 1\}$, B=$\left\{(x,y); y = \sin\frac{1}{x}\ 0 < x \le 1\right\}$ in R^2 with $\frac{1}{x} = 2n\pi + \frac{\pi}{2}$ and $2n\pi + \frac{3\pi}{2}$

(or) x= $\frac{2}{(4n+1)\pi}$ and x= $\frac{2}{(4n+3)\pi}$ x→ 0 as n→ ∞ but $\sin\frac{1}{x}$ oscillates between +1 and -1

Each point of A is an accumulation point of B. Hence A and B are not separated and $\bar{B}$ = A∪B is connected also note that B is connected. Since it is the continuous image of the connected set [0,1] therefore $\bar{B}$ in $\mathbb{R}^2$ is also connected. $\bar{B}$ Is a classical example in topology called a topologists since curve.

TOTALLY DISCONNECTED SPACE:

A space is totally disconnected if its only connected subspaces are one point set.

NOTE:

A Totally disconnected space is a topological space X in which every pair of distinct points can be separated by disconnection of X. This means that for every pair of points x and y in X. such that x ≠ y, there exists a disconnection X = A∪B with x ∈ A and y ∈ B.

Examples:

The following are examples of totally disconnected spaces.

1. Discrete spaces

2. The rational numbers
3. The irrational numbers
4. The cantor set and the cantor space
5. The baire space
6. The sorgenfrey line

Problem:

Show that with the topology τ generated by the open closed interval (a,b] is totally disconnected.

Solution:

Let p, q ∈ ℝ say p < q. Then G = (-∞,p] and H= (p,∞) are open non empty disjoint sets whose union is ℝ. Therefore G∪H is a disconnected of ℝ. G and H are nonempty. Since p ∈ G and q ∈ H. Therefore (ℝ, Z) is totally disconnected.

Theorem:

Let X is a Hausdorff spaces. If X has an open base whose sets are also closed then X is totally disconnected.

Proof:

Let x and y be distinct points in X. Since X is a Hausdorff x has a neighbourhood G which does not contain y. By our assumption, their exist a basic open set B which is also closed such that x ∈ B contained or equal to G. X=B∪B^1 is clearly a disconnection of X which separates x and y. In this theorem if the space X is also compact then the implication can be reversed, and the two conditions are equivalent. Therefore X is totally disconnected.

Example:

Consider R^ω is the box topology we can write R^ω has the union of a set A consisting of all bounded sequence of real number.

Solution:

The set A and B of all unbounded sequence these sets are nonempty and disjoint. There union A∪B = R^ω also each open in the box topology. If a = $a_1, a_2, \ldots \in R^\omega$, the open set

$U = (a_1-1,a_1+1) \times (a_2-1,a_2+1) \times \ldots \in R^\omega$.

Consist entirely a bounded sequence if a is bounded and of unbounded sequence if a is unbounded. Thus even though R is connected. R^ω is not connected in the box topology.

Note:

However R^ω is connected in the product Topology

Path connected:

Given point x, y ∈ X a path in X form x to y is a continuous map f : [a,b]→ X of some closed interval in the real line into X such that f(a) = x; f(b) = y.

A space X is said to be path connected if every pair of points of X can be join by a path connected space X is connected.

Example:

All convex sets in a vector space are connected because one could just use the segment connectily them, which is f(t) = $t\vec{a}+(1-t)\,\vec{b}$.

Example:

Define the unit sphere S^{n-1} in $\mathbb{R}^n$ by the equation $S^{n-1} = \{ x:\|x\|=1\}$

Solution:

If n > 1 it is not connected. When n=1, S^0 = {-1,1} is disconnected. For the map $g : R^n$- {0} → S^{n-1} define by $g(x) = \frac{x}{\|x\|}$ is continuous and surjective now R^n- {0} is path connected and g is continuous. Therefore S^{n-1} is path connected.

Example:

The ordered square is connected but not path connected.

Solution:

B a linear continum the order square is connected. Let p = 0×0 and q = $|x|$ Suppose that there is a path connected f: [a,b] → I_0^2 joining p and q, we derive a contradiction. The image set f([a,b]) must contain every point $x \times y$ of I_0^2. Therefore for each x ∈ I the set $U_{\underline{x}} = f^{-1}[\underline{x} \times (0,1)]$ is an nonempty subset of [a,b] by continuity it is open in (a,b). For any point x × y ∈ (X × (0, 1)) There exists an open $(x \times y - \in, x \times y + \in) \subset (x \times (0,1))$, x × (0,1) is open in I_0^2 and hence U_x is also open in [a,b]. Choose for each x ∈ I, a rational number $q_x \in U_x$. Since the set U_x are disjoint, the map $x \to q_x$ is an injective mapping of I into q.

For $x \neq y \Leftrightarrow U_x \neq U_y \Leftrightarrow q_x \neq q_y$

This contradicts fact that the interval I is uncountable.

Locally connected space:

A space x is said to be locally connected at x if for every neighbourhood u of x there exists a neighbourhood of v.

If x is locally connected at each of x point then x if self it said to be locally connected.

Locally path connected:

A space x is said to be locally path connected at x if for every neighborhood u of x their exists a path connected neighbourhood v of x ⊂ u.

If x is locally path connected at each of x its point then if it is said to be locally path connected.

Example:

1. Intervals on ray R are both connected and locally connected.
2. X= (0,1) ∪ (2,3) ⊂ ℝ with subspace topology is locally connected but not connected.
3. The subspace [-1,0) ∪ (0,1] ⊂ ℝ is locally connected, but not connected.
4. Q is neither compacted nor locally connected.

Problem:

Every discrete space is locally connected.

Solution:

For if p ∈ X then{P} is an open connected set containing P which is contained in every open set containing P. But X is not connected if X contained more than one element is X with discrete topology is totally disconnected.

Theorem:

The space X is locally connected if for every open set V of X, each component of U is open in X.

Proof:

Suppose X is locally connected, let U be open set in X, Let C be component of U. If $X \in C$, we can choose a connected neighbourhood V of X such that $V \subset U$. Since V is connected it must lie entirely in the component C. Therefore, every point that of C has a neighbourhood V contained in C. Therefore C is open in U but V is open in X. Therefore C is open in X.

Conversely,

Suppose that the component of each open set U of X is open in X. Given $x \in$ X and a neighbourhood U of X. Let C be a component of U containing x, now C is connected and its open in x. Therefore X is locally connected.

Note:

A space X is locally path connected iff each path component of X is open in X.

CONCLUSION:

To conclude, the title can be classified different of type of connected space in topological space and problems are presented.

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