REVIEW OF RESEARCH
IMPACT FACTOR : 5.7631(UIF) UGC APPROVED JOURNAL NO. 48514 ISSN: 2249-894X

VOLUME - 7 | ISSUE - 12 | SEPTEMBER - 2018

INTERACTING DARK FLUIDS IN LRS BIANCHI TYPE-V UNIVERSE

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ABSTRACT:

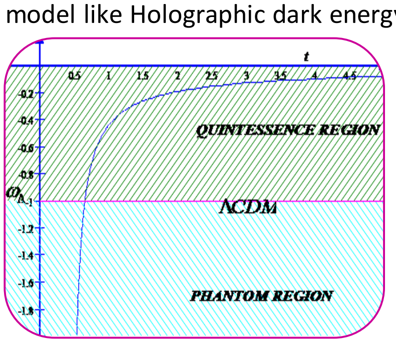
This paper deals with the study of interacting holographic dark energy (DE) and cold dark matter (CDM) in Locally Rotationally Symmetric (LRS)Bianchi type-V. Thesolutions have been obtained for Einstein's field equations. Also,the behaviors of the obtained solutions have been discussed.

KEYWORDS: *LRSBianchi types-V space-time;statefinder parameters;interacting dark fluids.*

1. INTRODUCTION

The universe is expanding in an accelerating manner indicated by recent observations of SNeIa (Type Ia supernovae) [1, 2]. Many observations of CMB (cosmic microwave background) [3, 4]in coherence with LSS (Large Scale Structure) [5, 6] denote that the universe is spatially flat. An exotic component is known as 'Dark energy' (DE) having huge negative pressure dominated the universe. Analysis of cosmological observations under the study of Wilkinson Microwave Anisotropy Probe (WMAP)[7-10] state that $2/3$ rd ofthe total energy of the universe is occupied by dark energy whereas remaining $1/3$ rd is taken up by dark matter (DM) and baryonic matter(ordinary matter). The term 'Dark Energy' is specially used for the unknown form of energy which is not detected directly and do not cluster as other ordinary matter does. This observation can be explained by assuming that at large scale the Einstein gravity model of general relativity (GR) break down and a more general action describes the gravitational field. The simplest component of DE which is supported by current observational data is the cosmological constant (Λ) having equation of state $\omega = -1$. For the satisfaction of the current value of DE, it should be fine-tuned [9-11].

In recent years many dark energy models have been studied including quintessence scalar field models [12, 13], tachyon field [14, 15], K-essence [16-18], phantomfield [19-21], Chaplygin gas [22, 23], Quinton[24] andso on [25-27]. Various DE cosmologies (isotropic) having early deceleration and delayed acceleration was previously reviewed by Bamba et al. [28]. The increasing expansion with the phantom /quintessence characteristic in detail together with cosmography test has been represented by cosmological model like Holographic dark energy, coupled dark energy, scalar field theory, $f(T)$ gravity, $f(R,T)$ gravity, $f(R)$ gravity and ΛCDM cosmological model. These models have been studied by Bamba et al. [28]. The scalar tensor theory of gravitation consisting of five dimensional DE was thoroughly investigated by Reddy et al. [29].



Toinvestigate the problem of dark energy,holographic dark energy model provides a more simple and reasonable frame. The principle known as the holographic principle [25,27, 30-31] emerged as a new paradigm in quantum gravity in relation to black hole physics, it was initially put forward by Hooft [32] and latter extended by Fischer and Susskind [33] to string theory. According to this principle, the entropy of system scale on its surface area and not with its volume. Thus if we take whole universe in consideration the energy created by vacuum when related to holographic principle can be used as DEwhich is known as holographic dark energy. These new studies of string theory and black hole can provide a new solution of DE problem.Though, it is the most radical modification and proposed a relation $\rho_\Lambda = H^2$, which is the relation between ρ_Λ (holographic dark energy density) as well as H (Hubbleparameter). It can't support toplay a part of present accelerated expansion of the universe [34,35].A holographic density of the form $\rho_\Lambda \approx \alpha H^2 + \beta H$, where α and β are constant and H is Hubble parameter has been put forward by Granda and Oliverson [36] but it should satisfy the restication imposed by the current observational data. Granda and Oliverson's [37] new model of dark energy not only represent the accelerated expansion of the universe but also it is consistent with current observational data. A scalar phantom-nonphantom transition model and generalised holographic dark energy have been discussed by Nojiri and Odintsov [38].

An important role in the early stage of the universe was played by anisotropy thereby making it necessary to study the homogeneous as well as anisotropic cosmological models. So the models based on Bianchi type universe are considered to be spatially homogeneous model of cosmology that is in general anisotropic. The holographic dark energy is permitted to interact with dark matter are the models which are studied under special class[39-50].In literature, many authors have studied holographic dark energy model in Bianchi type's space-time in GR and alternative theories.Recently Sarkar [51-53] has been studied Bianchi type-I and V universe with the help of non-interacting holographic DE using linearly varying deceleration parameter. Adhav et. al. [54, 55] have investigated holographic dark energy model in Bianchi type-I and V space-time. The holographic dark energy model in Bianchi type-VI0with the help of scalar tensor theory has been investigated by Reddy et. al.[56]. Mete et.al. [57] explored LRS Bianchi type-I universe filled with interacting cold dark matter and holographic dark energy. Interacting dark fluids in Bianchi type-I universe with variable deceleration parameter has been constructed by Mete et.al.[58].

This motivates us to investigate interactingholographic dark energy (DE)and(CDM) in LRS Bianchi type-V universe.

2. METRIC AND FIELD EQUATIONS

The LRS Bianchi type-V line element can be written as

$$ds^2 = dt^2 - A^2dx^2 - e^{-2x}B^2(dy^2 + dz^2\ ), \tag{1}$$

where A and B are the cosmic scale factors and functions of the cosmic time t only .
The Einstein's field equations in GR are ($8\pi\, G = 1$ and $c = 1$)

$$R_{ij} - \frac{1}{2}g_{ij}R = -({}^mT_{ij} + {}^\Lambda T_{ij})\,. \tag{2}$$

The matter tensor for holographic dark energy and cold dark matter
(pressureless i.e. $w_m = 0$) are

$${}^\Lambda T_{ij} = (\rho_\Lambda + p_\Lambda)u_iu_j + g_{ij}p_\Lambda\,, \tag{3}$$

$${}^mT_{ij} = \rho_m\ u_i\ u_j\,. \tag{4}$$

Here ρ_Λ and p_Λ denote the energy density and pressure of holographic dark energy and ρ_m is the energy density of dark matter.

For the metric (1), the Einstein's field equations (2)using equations (3) and (4)convert to following set of equations

$$\frac{2\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}-\frac{1}{A^2}=-p_\Lambda\ , \tag{5}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}-\frac{1}{A^2}=-p_\Lambda\ , \tag{6}$$

$$\frac{\dot{B}^2}{B^2}+\frac{2\dot{A}\dot{B}}{AB}-\frac{3}{A^2}=\rho_\Lambda+\rho_m\ , \tag{7}$$

$$\frac{-2\dot{B}}{B}+\frac{2\dot{A}}{A}=0\ , \tag{8}$$

where an overheaddot (˙)indicates derivative with respect to time *t*.

We assumed that both components i.e. holographic dark energy and dark matter interact with each other and do not conserve separately in such a manner that balance equationsin form as;

$$\dot{\rho}_m+\left(\frac{\dot{V}}{V}\right)\rho_m=Q\ , \tag{9}$$

$$\dot{\rho}_\Lambda+\left(\frac{\dot{V}}{V}\right)(1+w_\Lambda)\rho_\Lambda=-Q\ . \tag{10}$$

here w_Λ denotes equation of state parameter (EoS) for holographic dark energy is given by $w_\Lambda=\frac{p_\Lambda}{\rho_\Lambda}$ .The quantity $Q>0$ denotes the interaction between the dark energy components. Wetterich [12] introduced models featuring an interacting matter-dark energy and first used by Horvat[60] alongside the holographic dark energy. Although this expression for interacting term may looks purely phenomenological. Continuity equations (9) and (10) suggest that $Q\propto {}^1\!/_t$ i.e. the interacting term (Q) proportional to a quantity with unit of inverse time. This occurred due to choosing H and Q term and is motivated purely by mathematical simplicity. To Form energy density any combination of dark matter and dark energy can be expressed phenomenological in form such as [59, 61]

$$Q=3b^2H\rho_m=b^2\frac{\dot{V}}{V}\rho_m\ , \tag{11}$$

where b^2 is known as coupling constant.The relation considered by Cai and Wang [62] for interacting phantom DE and DM is same as above relation usedfor neglecting the coincidence problem.
The energy density of dark matter is given by equations (9) and (11) as

$$\rho_m = \rho_0 V^{(b^2-1)},(12) \qquad (2.14)$$

where $\rho_0 > 0$ is an integrating constant.
With the help of equations (11) and (12), the interacting term given as

$$Q = 3\,\rho_0 b^2 H V^{(b^2-1)}. \qquad (13)$$

3. Cosmological Solutions

Integrating equation (8), we obtain

$$A = nB\ , \qquad (14)$$

where $n > 1$ is constant of integration.
Subtracting equation (5) from equation(6) we obtain,

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\ddot{A}}{A} = 0. \qquad (15)$$

Using equation (14) in the equation (15) and then on integrating, we obtain the value of scale factors as

$$A = n(c_1 t + c_2)^{\frac{1}{2}}\ , \qquad (16)$$

$$B = (c_1 t + c_2)^{\frac{1}{2}}\ , \qquad (17)$$

where $c_1 > 0$ and c_2 are real constants of integration.
The volume scale factor *V* is defined and obtained as,

$$V = AB^2 = n(c_1 t + c_2)^{\frac{3}{2}}. \qquad (18)$$

Mean hubble parameter defined and obtained as,

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{c_1}{2(c_1 t + c_2)}. \qquad (19)$$

Using equation (18) in equations (12) and (13), we get energy density of dark matter and interacting term as,

$$\rho_m = \rho_0 [n]^{(b^2-1)} (c_1 t + c_2)^{\frac{3(b^2-1)}{2}}, \qquad (20)$$

$$Q = \frac{3}{2} b^2 c_1 \rho_0 [n]^{(b^2-1)} (c_1 t + c_2)^{\frac{3b^2-4}{2}}. \qquad (21)$$

From equation(7) using equations (16), (17) and (20), the holographic dark energy density is given as,

$$\rho_{\Lambda} = \frac{3c_1^2}{4(c_1t+c_2)^2} - \frac{3}{n^2(c_1t+c_2)} - \rho_0 n^{(b^2-1)}(c_1t+c_2)^{\frac{3}{2}(b^2-1)} \tag{22}$$

From equations (5)-(6), the pressure of holographic dark energy is given by using equations (16) and (17) as,

$$p_{\Lambda} = \frac{c_1^2}{4(c_1t+c_2)^2} + \frac{1}{n^2(c_1t+c_2)}. \tag{23}$$

With the help of equations (23) and (24), the EoS parameter of holographic dark energy is defined and obtained as,

$$w_{\Lambda} = \frac{p_{\Lambda}}{\rho_{\Lambda}}$$

$$w_{\Lambda} = \frac{\dfrac{c_1^2}{4(c_1t+c_2)^2} + \dfrac{1}{n^2(c_1t+c_2)}}{\dfrac{3c_1^2}{4(c_1t+c_2)^2} - \dfrac{3}{n^2(c_1t+c_2)} - \rho_0 n^{(b^2-1)}(c_1t+c_2)^{\frac{3}{2}(b^2-1)}}. \tag{24}$$

The coincidence parameter $\bar{r} = \rho_m / \rho_{\Lambda}$, which is the ratio of two energies density i.e. the ratio of dark matter energy density to the dark energy density is given by,

$$\bar{r} = \frac{\rho_0 [n]^{(b^2-1)}(c_1t+c_2)^{\frac{3(b^2-1)}{2}}}{\dfrac{3c_1^2}{4(c_1t+c_2)^2} - \dfrac{3}{n^2(c_1t+c_2)} - \rho_0 n^{(b^2-1)}(c_1t+c_2)^{\frac{3}{2}(b^2-1)}}. \tag{25}$$

5. STATEFINDER DIAGNOSTIC

Though, there have been several dark energy models constructed to describe and interpret the cosmic acceleration, to differentiate between those competing dark energy models is required. For this purpose, the parameter pair {*r*, *s*}known as 'Statefinder diagnostic' was proposed by Sahni et al. [63], gives the universe's expansion dynamics through a (expansion factor) which is considered to be natural companion of 'q'(deceleration parameter).

The Statefinder parameter {*r*, *s*} is as follows,

$r = \dfrac{\dddot{a}}{aH^3}$ and $s = \dfrac{r-1}{3(q-1/2)}$.

The geometrical diagnostic pair known as 'Statefinder' depends upon metric describing space time and expansion factor. The $s-r$ plane containing trajectories which corresponds to other cosmological models show qualitatively different behaviors. A fixed point $\{s,r\}\big|_{\Lambda CDM} = \{0,1\}$ shown in a Fig.2 corresponds to flat ΛC CDM. A good way which is provided for demonstration of 'distance' of this model from ΛCDM, depart a given dark energy model from the fixed point [64]. For differentiation amongst a large variety of DE models including quintessence, cosmological constant, the Chaplygin gas, interacting dark energy modelsand braneworld models, the statefinder may prove to be successful tool [59, 65,66].

$r = 3$ and $s = \frac{2(r-1)}{r}$.

6. DISCUSSION & CONCLUSION:

Here we discussed interacting holographic DE and DM in LRS Bianchi type-V space-time. Equation of state of DE $w_\Lambda < 0$ is go along with the decay of DE component into pressureless DM in the holographic DE model by engaging the apparent horizon as the IR cutoff [67].In this model as $t \to 0$, $v \to$ constant and $t \to \infty$, $v \to \infty$ that there is no Big-Ban type of initial singularity and universe expand as time increases.

For Fig.1 As $t \to 0$, $\omega_\Lambda \to \infty$, As time increases ω_Λ starts from phantom region and for some finite time, it reaches to $\omega_\Lambda = -1$ (cosmological constant), which indicate the model reduces to ΛCDM. After some finite timethen it goes in to quintessence region $(-1 < \omega_\Lambda < -1/3)$.The available observation in cosmology, especiallythe three year WMAP data [4], the SNeIa data [68,69] and the SDSS data [70] pointed out that the $\wedge$CDM model or the model reducible to $\wedge$CDM are present as a standard model in cosmology which is similar to the present universe.The Fig. 2 shows the *s-r* plane having evolving trajectory for the corresponding model is different from other dark energy models. We aspire that the future high accurate observations will be able for determination of these statefinder parameters as well as explore the DE's nature. Also for any choice of constants, we cannot avoid the coincident problem in our model.

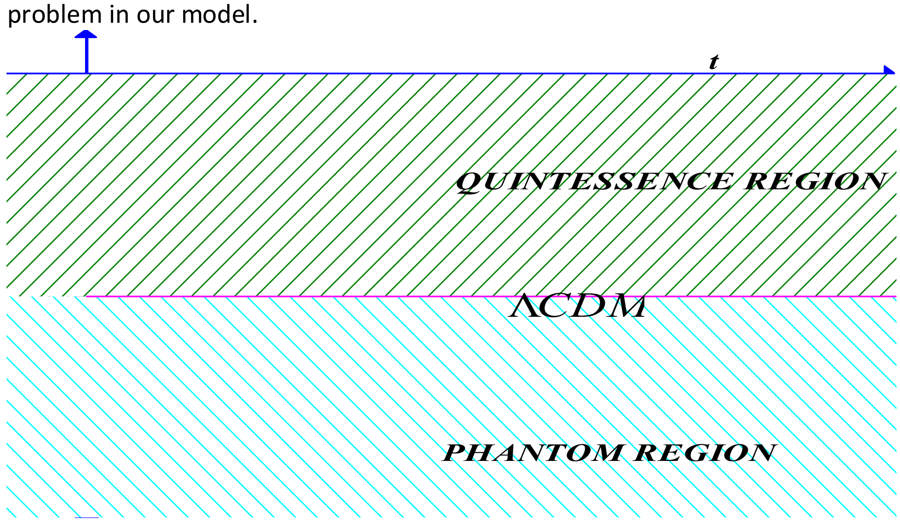


Fig.1:Evolution of EoS parameter (w_Λ).

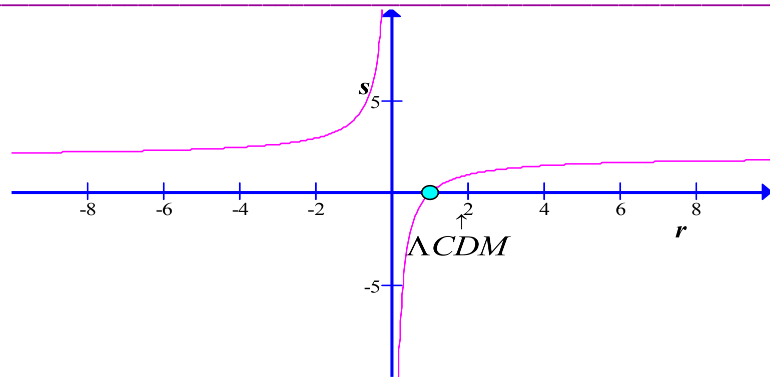


Fig.2- Statefinder parameters *s v*/s r.

Acknowledgement: One of the authors (V.M. Ingle) would like to thank the UGC, New Delhi, India for providing financial assistance under CSIR/JRF scheme where part of this work was done.

REFERENCES:

[1] Riess, A.G.,Filippenko, A.V.,et al.(1998), "Observational evidence from supernovae for an accelerating universe and a cosmological constant", The Astronomical Journal, vol. 116, no. 3, pp. 1009.
[2] Perlmutter, S.,(1999),"Measurements of Ω and Λ from 42 high-redshift supernovae", The Astrophysical Journal, vol.517, no. 2, pp. 565.
[3] Bennett,C. L., Halpern, M.,Hinshaw, G.,Jarosik, N.,Kogut, A.,et.al.,(2003) "First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Preliminary maps and basic results", The Astrophysical Journal, vol. 148, no.1, pp. 1-27.
[4] Spergel, D.N.,et. al.,(2003), "First-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Determination of cosmological parameter", Astrophysical Journal Supplement Series, vol. 148, no. 1, pp. 175-194.
[5] Tegmark,M.et. al.,(2004),"Cosmological parameters from SDSS and WMAP", Physical review D, vol. 69, no. 10, Article ID 103501, pp.28 .
[6] Tegmark, M.et. al.,(2004), "The three-dimensional power spectrum of galaxies from the Sloan Digital Sky Survey", The Astrophysical Journal, vol. 606, no. 2,pp. 702-740.
[7] Peebles,P. J. E. ,Ratra, B.,(2003),"The cosmological constant and dark energy", Reviews of Modern Physics, vol. 75, no. 2, pp. 559.
[8] Padmanabhan,T., (2003),"Cosmological constant-the weight of the vacuum", Physics Report, vol. 380, no. 5-6, pp. 235-320.
[9]Weinberg,S.,(1989), "The Cosmological constant problem", Reviews of Modern Physics, vol. 61, no. 1.
[10] Carroll,S. M., (2001), "The Cosmological constant", Living Reviews in Relativity, vol. 4, no.1.
[11] Copeland, E.J., Sami, M. ,Tsujikawa,S.,(2006), "Dynamics of dark energy", International Journal of Modern Physics D, vol. 15, no. 1, pp. 1753-1936 .
[12] Wetterich, C.,(1988), "Cosmology and the fate of dilatation symmetry", Nuclear Physics B, vol. 302, no. 4, pp. 668-696.
[13] Ratra, B., Peebles,P. J. E.,(1988), "Cosmological consequences of a rollinghomogeneous scalar field", Physical Review D, vol. 37, no. 12, p. 3406.
[14] A. Sen,A.,(2002), "Rolling Tachyon", JournalHigh Energy Physics 04, vol. 2002, pp.19.
[15] Padmanabhan, T. Chaudhury, T.R.,(2002),"Can the clustered dark matter and the smooth dark energy arise from same scalar field?", Physical Review D, vol .66, no. 8, Article ID 081301, pp.4 .
[16] Chiba, T., T. Okabe,T. and Yamaguchi,M.,(2000), "Kinetically driven quintessence", Physical Review D, vol. 62, no. 2, Article ID 023511, pp.18 .

[17] Armendariz-Picon, C.,Mukhanov, V.and Paul J. Steinhardt,(2000) "Dynamical Solution to the problem of a small cosmological constant and late time cosmic acceleration", Physics Review Letters, vol. 85, no. 21, p.4438.
[18]Armendariz-Picon, C., Mukhanov, V. and Paul J. Steinhardt,(2001) "Essential of K-essence", Physics Review D, vol. 63, no. 10, Article ID 103510, pp.14.
[19] Caldwell,R.R., (2002) "A phantom menace? Cosmological consequences of a dark energy component with super-negative equation of the state", Physics Letters B, vol. 545, no. 1-2, pp. 23.
[20] Nojiri, S.,Odinstov, S.D.,(2003) "Quantum Desitter cosmology and phantom matter", Physics Letter B, vol. 562, no. 3-4, pp.147.
[21] Nojiri, S., Odinstov, S.D.,(2003) "Desitterbrane universe induced by phantom and quantum effects ", Physics Letter B, vol. 565, no.1, pp. 1.
[22] Kamenshchik, K.,Moschella, U.,Pasquier,V.,(2001) "An alternative to quintessence", Physical Letter B, vol. 511, no. 2-4, pp. 265.
[23] Bento, M.C.,Bertolami, O.,Sen,A.A.,(2002) "Generalized chyplygin gas, accelerated expansion and dark energy-matter unification", Physical Review D, vol. 66, Article ID 043507, pp. 5.
[24] Elizalde, E.,Nojiri, S.,Odintsov, S.D.,(2004) "Late-time cosmology in a (phantom) scalar-tensor field theory: dark energy and cosmic speed up," Physical Review D, vol. 70, no. 4, Article ID 043539, pp. 48.
[25] Cohen, A.G., Kaplan, D.B., Nelson, A.E.,(1999) "Effective field theory, Black holes and the cosmological constant," Physical Review Letters, vol. 82, no. 25, p. 4971.
[26] Horava, P., Minic, D.,(2000) "Probable values of cosmological constant in a holographic theory", Physical Review Letters, vol. 85, no. 8, p. 1610.
[27] Li, M., (2004)"A model of holographic dark energy", Physics Letter B, vol. 603, no.1-2, pp. 1-5.
[28]Bamba,K., Capuzziello, S., Nojiri, S. ,Odinstov, S.D.,(2012) "Dark energy cosmology: the equivalent description via different theoretical models and cosmography tests", Astrophysics and space science, vol. 342, no. 1, pp. 155-228.
[29]Reddy, D.R.K.,Satyanarayana, B., R. L. Naidu,R.L.,(2012) " Five dimensional dark energy model in scalar-tensor theory of gravitation",Astrophysics and space science, vol. 339, no. 2, pp. 401-405.
[30] Bekenstein, J.D.,(1973) "Black hole and Entropy", Physical Review D, vol. 7, no. 8, p. 2333.
[31] Bousso, R.,(2002) "The holographic principle", Review of modern physics, vol. 47, no. 3, p. 825.
[32] Hooft, G.,(1993) "Dimensional Reduction in Quantum Gravity", General relativity and constant cosmology, 9310026.
[33]Fischler, W., Susskind, L.,(1998) "Holography and Cosmology", High energy physics-theory, 9806039.
[34] Susskind, L.,(1995) "The world as a hologram", Journal of Mathematical Physics, vol. 36, pp. 6377-6396.
[35]Witten,E.,(1998) "Anti desitter space and holography", Advance Theoretical Mathematics Physics, vol. 2,pp. 253-291.
[36] Granda, L. N., Oliveros,A.,(2008) "Infrared cut-off proposal for the holographic density", Physics Letters B, vol. 669, no. 5, pp. 275-277.
[37] Granda, L. N., Oliveros ,A.,(2009) "New infrared cut-off for the holographic scalar fields modelsof dark energy," Physics Letters B, vol. 671, no. 2, pp.199-202.
[38] Nojiri, S. ,Odinstov, S.D.,(2006) "Unifying phantom inflation with late-time acceleration: scalar phantom-non-phantom transition model and generalized holographic dark energy", General Reletivity and Gravitation, vol. 38, no.8, pp. 1285-1304.
[39] Wang, B.,Gong, Y.,Abdalla, E.,(2005)"Transition of the dark energy equation of the state in an interacting holographic dark energy model", Physics Letters B, vol. 624, no. 3-4, pp.141-146.
[40] Carvalho, F. C., Saa, A.,(2004) "Nonminimal coupling, exponential potential and $w<-1$ regime of dark energy", Physical Review D, vol. 70, no. 8, Article ID 087302, pp. 5.
[41] Gong, Y., (2004)"Extended holographic dark energy", Physical Review D, vol. 70, no. 6, Article ID 064029, pp. 3.

[42] Gong, Y., Zhang,Y.Z., (2005) "Holography and holographic dark energy model", Classical and quantum gravity, vol. 22, no. 22, pp. 489.

[43] Guberina, B. ,Horvat, R. ,Nikolic, H., (2005)"Generalized holographic dark energy and IR cutoff problem", Physical Review D, vol. 72, no. 12, Article ID.125011, pp. 13.

[44] Guberina, B. ,Horvat, R. , Nikolic, H., (2006) "Dynamical dark energy with a constant vacuum energy density ,"Physical Letters B, vol. 636, no.80, pp. 80-85.

[45] Guo, Z.K.,Ohta, N. , Zhang, Y.Z., (2005) " Parameterization of quintessence and its potential", Physical Review D, vol. 72, no. 2, Article ID 023504, pp. 8.

[46]Sadjadi,H.M., (2007) "The particle versus the future event horizon in an interacting holographic dark energy model", Journal of cosmology and Astroparticle physics, vol. 2007, pp. 12.

[47]Banerjee, N.,Pavon, D. (2007) "Holographic dark energy in Brans-Dicke theory", Physical Letters B, vol. 647, no. 5-6, pp. 477-481.

[48] Kim, H. , Lee, H. W.,Myung, Y.S., (2006)"Equation of state for an interacting holographic dark energy model", Physics Letters B, vol. 632, no. 5-6, pp. 605-609.

[49] Zimdahl, W.,Pavon,D., (2003)"Scaling cosmology", General Relativity and Gravitation, vol. 35, no. 3, pp. 413-422.

[50] Zimdahl, W., (2005) "Interacting dark energy and cosmological equation of state", International Journal of Modern Physics, vol. 14, no. 12, pp. 2319-2325.

[51] Sarkar, S., (2004)"Holographic dark energy with linearly varying deceleration parameter and generalized chaplygin gas dark energy model in Bianchi type-I universe", Astrophysics and Space Science, vol. 349, no. 2, pp. 985-993.

[52] Sarkar,S.,(2014) "Holographic dark energy with linearly varying deceleration parameter and escaping big rip singularity of the bianchi type-V universe", Astrophysics and Space Science, vol. 352, no. 2, pp. 859-866.

[53] Sarkar,S., (2014) "Interacting holographic dark with variable deceleration parameter and tachyon scalar field dark energy model in LRS Bianchi type-II universe", Astrophysics and Space Science, vol. 350, no. 2, pp. 821-829.

[54]Adhav,K.S.,Tayde, G.B.,Bansode, A.S., (2014) "Interacting dark matter and holographic dark energy in an anisotropic universe", Astrophysics and space science, vol. 353, no.1, pp. 249-257.

[55] Adhav,K.S., Munde, S.L.,Tayade, G.B.,Bokey,V.D., (2015) "Interacting dark matter and holographic dark energy in Bianchi type-v universe",Astrophysics and space science, vol. 359, no.1, p.24.

[56] Reddy, D.R.K., Anita, S. ,Umadevi, S., (2016)"Holographic dark energy model in Bianchi type VI_0 universe in a scalar-tensor theory of gravitation with hybrid expansion law", Canadian Journal of Physics, vol. 94, no. 12, pp. 1338-1343.

[57] Mete,V.G., Murade, P.B., Bansod, A.S., (2017) "LRS Bianchi type-I universe filled with interacting cold dark matter and holographic dark energy" , African review of physics, vol.12, pp.125-129.

[58] Mete,V.G., Bokey, V.D., Bawane, V.S., (2018) " Interacting dark fluid in Bianchi type-I universe with variable deceleration parameter", Int. J. of current advanced research, vol. 7 , 6(H), pp.13609.

[59]Guo, Z.K. , Ohta, N. , Tsujikawa, S., (2004)"Probing the coupling between dark components of the universe", Physical Review D, vol. 76, no. 2, Article ID 023508, pp 9.

[60] Horvat, R., (2004) "Holography and a variable cosmological constant", Physical Review D, vol. 70, no. 8,Article ID 087301, pp.7.

[61]Amendola,L. , Campos, G.C., Rosenfeld, R., (2007) "Consequence of dark matter-dark energy interaction on cosmological parameters derived from type Ia supernova data", Physical Review D, vol. 75, no. 8, Article ID 083506, pp. 16.

[62]Cai, R.G., Wang, A., (2005) "Cosmology with interaction between phantom Dark energy and Dark matter and the coincidence problem", Journal of Cosmology and Astroparticle Physics, vol. 2005.

[63] Sahni, V.,Saini, T.,Starobinsky, D.,Alam, A.A., (2003) "Statefinder- A New geometrical diagnostic of dark energy ", Journal of experimental and Theoretical physics letters, vol. 77, no. 5, pp. 201-206.

[64]Alam, U.,Sahni, V.,Sahni, T. D.,Starobinsky, A.A., (2003) "Exploring the expanding universe and dark energy using the statefinder diagnostic", Monthly notices of the royal astronomical society, vol. 344, no. 4, pp. 1057-1074.
[65] Farrar, G.R., Peebles, P.J.E., (2004) "Interacting dark matter and dark energy",Astrophysical Journal, vol. 604, no.1, pp. 1-11.
[66] Amendola,L., (2003) "Acceleration at z>1?", Monthly notice of the royal astronomical society, vol.342, no.1, pp.221-226.
[67] Pavon, D.,Zimdahl,W., (2005) "Holographic dark energy and cosmic coincidence", Physics Letter B, vol. 628, no. 3-4, pp. 206-210.
[68] Riess, A. G.,Strolger, L.G., et al., (2004) "Type Ia Supernovae Discoveries at $z > 1$ from the Hubble Space Telescope: Evidence for Past Deceleration and Constraints on Dark Energy Evolution" Astrophysical Journal, vol. 607, no. 2, pp. 665-687.
[69] Astier, P., Guy, J.,Regnault, N. , Pain,R., Aubourg, E., et. al., (2006) "The Supernova Legacy Survey: measurement of Ω_M, Ω_Λ and w from first year data set", Astronomy and Astrophysics, vol.447, no. 1, pp. 31-48.
[70] D. J. Eisenstein, I. Zehavi, D. W. Hogg,D.W., et.al., (2005) "Detection of baryon acoustic peak in the large-scale correlation function of SDSS luminous red galaxies", The Astrophysical Journal, vol. 633, no. 2, pp. 560-574.

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