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A MATHEMATICAL INVESTIGATION ON THE INFLUENCE OF THE POSITION AND METHOD OF PLUCKING A GUITAR STRING FOR THE DISTRIBUTION OF ENERGY BETWEEN FUNDAMENTAL TONE AND OVERTONES

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Abstract

This investigation aims to develop a mathematical model for prescribing the behavior of an idealized vibrating string. By mathematical model we can calculate theoretical amplitudes for the different harmonics, based on the extent and position of the pluck. In this paperwe are presenting an investigation on the influence of the position and method of plucking a guitar string for the distribution of energy between fundamental tone and overtones. It was done by recording sound samples from an acoustic guitar, and numerically analyzing the data using computer software. Conclusively, it is stated that there were more overtones produced when the string was plucked close to the edge of the guitar. Since the paper was a physics investigation, it examined the matter experimentally, presenting little to no mathematical explanations for the results.

Keywords: *acoustic guitar, Harmonics, amplitudes, Displacement functions, Fourier series*

1. INTRODUCTION

Recent years have seen incredible advances in physical model-based synthesis. In these tries, learning of the material science and acoustics of the instruments is a hypothetical beginning stage for the displaying. Certain disentanglements can make the models computationally proficient and they can then be actualized to keep running continuously on a PC. Since actualized physical models are gotten from the material science of the instruments, they bring about the amalgamation of especially practical instrumental sounds. In any case, if the physical model running on a PC is proposed to be played, then research must be stretched out to the entertainer's activity to see how to connect with the PC model. For the specific instance of the guitar, productive string amalgamation calculations exist and are constantly being enhanced (D.A Jaffe 1983) (M. Karjalainen 1993) (Smith 1993) (V. Valimaki 1996) (M.Karjalainen 1998). For the investigation partner, examination has been embraced trying to comprehend the connections between timbre subtleties and model (T. Tolonen 1997) , physical, expressive (C.Erkut 2000) (G.Cuzzucoli 1999) and psycho acoustical (N.Orio 1999) parameters. Among the parameters that can be removed, the culling point position on the string has a noteworthy impact on the timbre subtlety (O. S. Caroline Traube 2000). The left hand fingering is urgent as well. Truth be told, there are distinctive approaches to finger harmonies or play songs. A specific fingering will be picked on the grounds that it is ideal, effective and simple to hold, or in light of the fact that it sounds in a specific and fancied way. A few tones on a guitar can be played with up to five distinct mixes of string/fret. In this way, if a recording is the main data accessible, the fingering that was utilized by a specific entertainer is not generally evident or obvious. Among the instrumental motion parameters that add to the timbre of a guitar sound (P. D. Caroline Traube 2003), the area of the culling point along the string has a noteworthy impact. Culling a string near the scaffold creates a tone that is milder in volume, brighter and more keen. The sound is wealthier in high recurrence segments. This happens when playing the guitar sulponticello (P. D. Caroline Traube 2003). The other amazing is playing sultasto (P. D. Caroline Traube 2003), close or over the fingerboard, closer to the midpoint of the string. All things considered, the tone is louder, mellower, less rich in high recurrence segments. The nonpartisan position of the right hand is simply behind the sound opening. In light of the position of the right hand fingers, the low strings are typically culled further far from the scaffold than the higher ones (O. S. Caroline Traube 2000).

1.1Motivation

One of the primary inspirations for this paper was to attempt and clarify the disclosures, utilizing a numerical methodology. The exploration center is to attempt to infer a numerical model recommending the conduct of a vibrating string. Some streamlining suppositions should be connected to keep the scientific model reasonable. Toward the end of the exposition a few elucidations of the model are given.

2. Related work

In (O. S. Caroline Traube 2000) creator proposed a recurrence space strategy for evaluating the culling point on a guitar string from an acoustically recorded sign. It additionally incorporates a unique system for identifying the fingering point, in view of the culling point data. In (P. D. Caroline Traube 2003) creator proposed on the extraction of the excitation point area on a guitar string by an iterative estimation of the basic parameters of the ghostly envelope. This paper creator proposed a general technique to assess the culling point area, working into two stages: beginning from a measure identified with the autocorrelation of the sign as a first rough guess, a weighted slightest square estimation is utilized to refine a FIR brush channel deferral worth to better fit the deliberate ghostly envelope. In (C.Erkut 2000) creator proposed the adjustment of model parameters is a critical sub issue for reasonable model-based sound union. This paper portrays the modification of the adjustment procedure of an established guitar show, and extends the parameter extraction methodology to catch data about execution attributes, for example, the damping administrations, rehashed culls, vibrato qualities, distinctive fearlessness styles and element varieties.

3. Mathematical Model

The general arrangement of the one dimensional wave mathematical statement, with both ends fixed (Linhartn.d.), to be:

$$y(x,t) = \sum_i X_i(x)T_i(t) = \sum_{k=1}^{\infty} \sin\left(\frac{k\pi}{L}x\right)\left(\alpha_k \cos\left(\frac{ck\pi}{L}t\right) + \beta_k \sin\left(\frac{ck\pi}{L}t\right)\right) \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Where
$c=\sqrt{T/\mu}$
The natural frequencies are $\frac{ck\pi}{L}$

3.1 Imposing initial conditions

When plucking the string, it is removed by a distance *h* at position *d* from its equilibrium state. The shape of the string the moment it is plucked defines a function f(x).

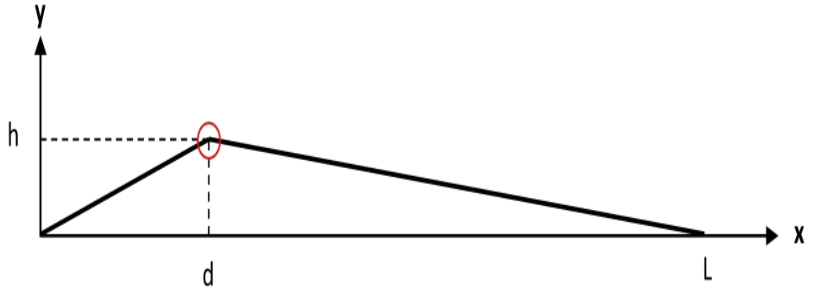


Figure: Initial conditions of the plucked string.

Since we are assuming that the string is motionless when released (no speed), the initial conditions are:

$$y(x,0) = f(x) \qquad \text{for all } 0 < x < L \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\frac{\partial}{\partial t} y(x,0) = 0 \qquad \text{for all } 0 < x < L \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Using (1) gives:

$$f(x) = y(x,0) = \sum_{k=1}^{\infty} \sin\left(\frac{k\pi}{L}x\right)\left(\alpha_k \cos\left(\frac{ck\pi}{L} \times 0\right) + \beta_k \sin\left(\frac{ck\pi}{L} \times 0\right)\right)$$

$$= \sum_{k=1}^{\infty} \alpha_k \sin\left(\frac{k\pi}{L}x\right) \qquad \ldots\ldots\ldots (4)$$

And

$$\frac{\partial}{\partial t} y(x,0) = \frac{\partial}{\partial t} \sum_{k=1} \sin\left(\frac{k\pi}{L}x\right)\left(\alpha_k \cos\left(\frac{ck\pi}{L}t\right) + \beta_k \sin\left(\frac{ck\pi}{L}t\right)\right)$$

$$= \sum_{k=1}^{\infty} \sin\left(\frac{k\pi}{L}x\right)\left(\alpha_k \frac{\partial}{\partial t} \cos\left(\frac{ck\pi}{L}t\right) + \beta_k \frac{\partial}{\partial t} \sin\left(\frac{ck\pi}{L}t\right)\right)$$

Using the chain rule for derivatives:

$$\frac{\partial}{\partial t}\cos\left(\frac{ck\pi}{L}t\right) = -\left(\frac{ck\pi}{L}\right)\sin\left(\frac{ck\pi}{L}t\right)$$

$$\frac{\partial}{\partial t}\sin\left(\frac{ck\pi}{L}t\right) = \left(\frac{ck\pi}{L}\right)\cos\left(\frac{ck\pi}{L}t\right)$$

At t=0:

$$\frac{\partial}{\partial t}\cos\left(\frac{ck\pi}{L}t\right)\bigg|_{t=0} = -\left(\frac{ck\pi}{L}\right)\sin\left(\frac{ck\pi}{L}\times 0\right) = 0$$

$$\frac{\partial}{\partial t}\sin\left(\frac{ck\pi}{L}t\right)\bigg|_{t=0} = \left(\frac{ck\pi}{L}\right)\cos\left(\frac{ck\pi}{L}\times 0\right) = \left(\frac{ck\pi}{L}\right)$$

Giving:

From (3):

$$\frac{\partial}{\partial t}y(x,0) = 0$$

$$\sum_{k=1}^{\infty}\beta_k\frac{ck\pi}{L}\sin\left(\frac{k\pi}{L}x\right) = 0$$

$$\frac{\partial}{\partial t}y(x,0) = \sum_{k=1}^{\infty}\sin\left(\frac{k\pi}{L}x\right)\left(\alpha_k\times 0 + \beta_k\frac{ck\pi}{L}1\right) = \sum_{k=1}^{\infty}\beta_k\frac{ck\pi}{L}\sin\left(\frac{k\pi}{L}x\right)$$

Implying $\beta_{k=0}$ for all *k*. This allows us to simplify (1) to:

$$y(x,t) = \sum_{k=1}^{\infty}\alpha_k\sin\left(\frac{k\pi}{L}x\right)\cos\left(\frac{ck\pi}{L}t\right) \qquad \ldots\ldots\ldots\ldots.. (5)$$

3.2 Fourier series

Any smooth function f(x) has a unique representation

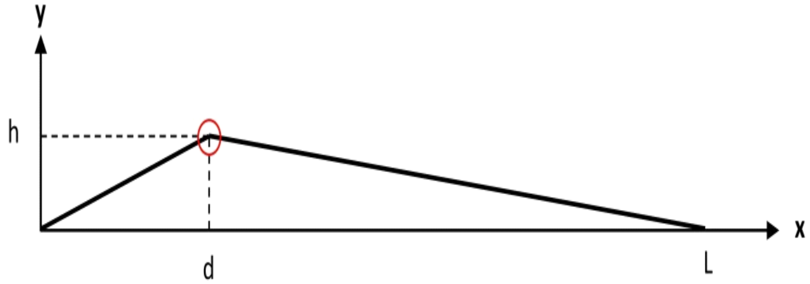
$$f(x) = \sum_{k=1}^{\infty}A_k\sin\left(\frac{k\pi}{L}x\right)$$

Where the coefficients are computed by

$$A_k = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{k\pi x}{L}\right)dx \qquad \ldots\ldots\ldots\ldots\ldots.. (6)$$

3 **Fourier series**

To find the coefficients α_k in (5) Fourier series will be used.
As noted earlier, the shape of the string at the moment it is plucked can be defined by a function f(x).



$$f(x) = \begin{cases} \frac{hx}{d}, & 0 \le x \le d \\ \frac{h(L-x)}{L-d}, & d < x \le L \end{cases}$$

Equation (4) is exactly in the form of a Fourier series. The coefficients are computed by (6), computing the integral from 0 to d and from d to L separately. Solving the integral is done by using the method integral by parts.

Integration by parts (Wikipedia)(encyclopedia n.d.):

$$\int_a^b u\,dv = [uv]_a^b - \int_a^b v\,du$$

Coefficients in (4):

$$A_k = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{k\pi x}{L}\right)dx = \frac{2}{L}\left[\underbrace{\int_0^d \frac{hx}{d}\sin\left(\frac{k\pi x}{L}\right)dx}_{\text{Part 1}} + \underbrace{\int_d^L \frac{h(L-x)}{L-d}\sin\left(\frac{k\pi x}{L}\right)dx}_{\text{Part 2}}\right]$$

Part: 1

Setting $u = \frac{hx}{d}$ and $dv = \sin\left(\frac{k\pi x}{L}\right) dx$

$$\int_0^d \frac{hx}{d} \sin\left(\frac{k\pi x}{L}\right) dx = \left[\frac{hx}{d}\left(\frac{-L}{k\pi}\right) cos\left(\frac{k\pi x}{L}\right)\right]_0^d - \int_0^d \left(\frac{-L}{k\pi}\right)\cos\left(\frac{k\pi x}{L}\right)\frac{h}{d}\, dx$$

$$= \left[\frac{-hLx}{k\pi d} cos\left(\frac{k\pi x}{L}\right)\right]_0^d - \int_0^d \frac{-hL}{k\pi d}\cos\left(\frac{k\pi x}{L}\right) dx$$

$$= \left[\frac{-hLx}{k\pi d} cos\left(\frac{k\pi x}{L}\right)\right]_0^d - \left(\frac{-hL}{k\pi d}\right)\left[\frac{L}{k\pi}\sin\left(\frac{k\pi x}{L}\right)\right]_0^d$$

$$= \frac{-hLd}{k\pi d} cos\left(\frac{k\pi d}{L}\right) - \left(\frac{-hL \times 0}{k\pi d}\right) cos\left(\frac{k\pi \times 0}{L}\right)$$

$$+ \frac{hL}{k\pi d}\frac{L}{k\pi}\left(\sin\left(\frac{k\pi d}{L}\right) - \sin\left(\frac{k\pi \times 0}{L}\right)\right)$$

$$= \frac{-hL}{k\pi} cos\left(\frac{k\pi d}{L}\right) + \frac{hL^2}{k^2\pi^2 d}\sin\left(\frac{k\pi d}{L}\right)$$

Part: 2

Setting $u = \frac{h(L-x)}{L-d}$ and $dv = \sin\left(\frac{k\pi x}{L}\right) dx$

$$\int_d^L \frac{h(L-x)}{L-d} \sin\left(\frac{k\pi x}{L}\right) dx = \left[\frac{h(L-x)}{L-d}\left(\frac{-L}{k\pi}\right) cos\left(\frac{k\pi x}{L}\right)\right]_d^L - \int_d^L \left(\frac{-L}{k\pi}\right)\cos\left(\frac{k\pi x}{L}\right)\frac{-h}{L-d}\, dx$$

$$= \left[\frac{-h(L-x)L}{(L-d)k\pi} cos\left(\frac{k\pi x}{L}\right)\right]_d^L - \int_d^L \frac{hL}{(L-d)k\pi}\cos\left(\frac{k\pi x}{L}\right) dx$$

$$= \left[\frac{-h(L-x)L}{(L-d)k\pi} cos\left(\frac{k\pi x}{L}\right)\right]_d^L - \frac{hL}{(L-d)k\pi}\left[\frac{L}{k\pi}\sin\left(\frac{k\pi x}{L}\right)\right]_d^L$$

$$= \frac{-h(L-L)L}{(L-d)k\pi} cos\left(\frac{k\pi L}{L}\right) - \frac{h(L-d)L}{(L-d)k\pi} cos\left(\frac{k\pi d}{L}\right)$$

$$- \frac{hL}{(L-d)k\pi}\frac{L}{k\pi}\left(\sin\left(\frac{k\pi L}{L}\right) - \sin\left(\frac{k\pi d}{L}\right)\right)$$

$$= \frac{hL}{k\pi} cos\left(\frac{k\pi d}{L}\right) + \frac{hL^2}{(L-d)k^2\pi^2}\sin\left(\frac{k\pi d}{L}\right)$$

Thus

$$A_k = \frac{2}{L}\left[\frac{-hL}{k\pi} cos\left(\frac{k\pi d}{L}\right) + \frac{hL^2}{k^2\pi^2 d}\sin\left(\frac{k\pi d}{L}\right) + \frac{hL}{k\pi} cos\left(\frac{k\pi d}{L}\right) + \frac{hL^2}{(L-d)k^2\pi^2}\sin\left(\frac{k\pi d}{L}\right)\right]$$

$$= \frac{2}{L}\left[\frac{hL^2}{k^2\pi^2 d}\sin\left(\frac{k\pi d}{L}\right) + \frac{hL^2}{(L-d)k^2\pi^2}\sin\left(\frac{k\pi d}{L}\right)\right]$$

$$= \frac{2}{L}\left(\frac{hL^2 \quad \times (L-d)}{k^2\pi^2 d \quad \times (L-d)} + \frac{hL^2 \quad \times d}{(L-d)k^2\pi^2 \quad \times d}\right)\sin\left(\frac{k\pi d}{L}\right)$$

$$= \frac{2}{L}\left(\frac{hL^3 - hL^2 d + hL^2 d}{(L-d)k^2\pi^2 d}\right)\sin\left(\frac{k\pi d}{L}\right)$$

$$= \frac{2hL^2}{(L-d)k^2\pi^2 d}\sin\left(\frac{k\pi d}{L}\right)$$

The solution:

$$A_k = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{k\pi x}{L}\right)dx = \frac{2hL^2}{\pi^2 k^2 d(L-d)}\sin\left(\frac{k\pi d}{L}\right) \qquad \text{………………} (7)$$

The displacement function then becomes:

$$y(x,t) = \sum_{k=1}^{\infty}\frac{2hL^2}{\pi^2 k^2 d(L-d)}\sin\left(\frac{k\pi d}{L}\right)\sin\left(\frac{k\pi}{L}x\right)\cos\left(\frac{ck\pi}{L}t\right) \qquad \text{………………} (8)$$

3.4 Harmonics

Equation (8) states that the displacement function is a sum of terms called modes or harmonics. Each mode represents a harmonic motion with different wavelength. For a fixed timet_1:

$$y(x,t_1) = \sum_{k=1}^{\infty}\frac{2hL^2}{\pi^2 k^2 d(L-d)}\sin\left(\frac{k\pi d}{L}\right)\cos\left(\frac{ck\pi}{L}t_1\right) \times \sin\left(\frac{k\pi}{L}x\right)$$

Implying that the each mode is a constant timessin$(k\pi x/L)$. As x runs from 0 to L, the argument ofsin$(k\pi x/L)$runs from 0 to kπ , which is$k$ half- periods of sin.

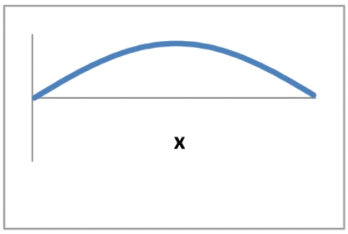


Fig: Amplitude for k=1

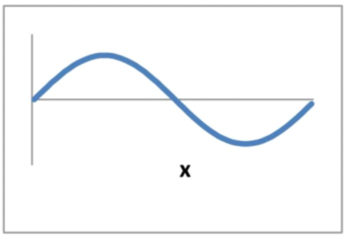


fig: Amplitude for k=2

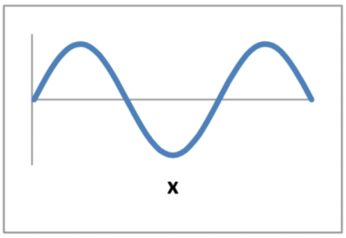


fig: Amplitude for k=3

Similarly, for any fixed position x1:

$$y(x_1,t) = \sum_{k=1}^{\infty} \frac{2hL^2}{\pi^2 k^2 d(L-d)} \sin\left(\frac{k\pi d}{L}\right) \sin\left(\frac{k\pi}{L}x_1\right) \times \cos\left(\frac{ck\pi}{L}t\right)$$

Implying that each mode is a constant times$\cos(ck\pi t/L)$. As t increases from 0 to 1 s, the argument of $\cos(ck\pi t/L)$increases by$\frac{ck\pi}{L}$, which is$\frac{ck}{2L}$ cycles. For mode k=1 (the fundamental tone), the frequency is$\frac{c}{2L}$ cycles per second. For mode k = 2 (the second harmonic), the frequency is $2\frac{c}{2L}$ cycles per second. For mode k = 3 (the third harmonic), the frequency is $3\frac{c}{2L}$ cycles per second etc.

Since c=$\sqrt{T/\mu}$ the frequency of oscillation of a string decrease with the density and increase with the tension or by shortening the string.

3.5 Coefficients

The coefficients in (7) found by Fourier transformation gives the amplitude of each harmonic, i.e. computed by inserting k=1, k=2, k=3, etc. By superposition lemma the total displacement is formed by the sum of the harmonics.

To illustrate the principle, we consider the following example: L = 0.64, h = 0.005 and d = 0.16, giving the following amplitudes according to (7):

k	A_k
1	3.821E-03
2	1.351E-03
3	4.246E-04
4	4.138E-20
5	-1.528E-04

We compute the graphs using Excel for t = 0 as an example. The first harmonic:

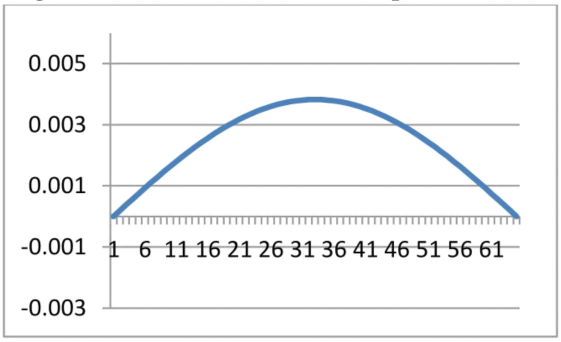
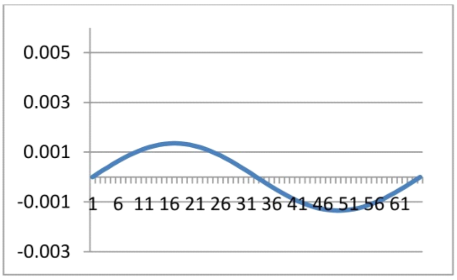


Figure: First harmonic at t=0.

The second harmonic and the sum of the first two harmonics:



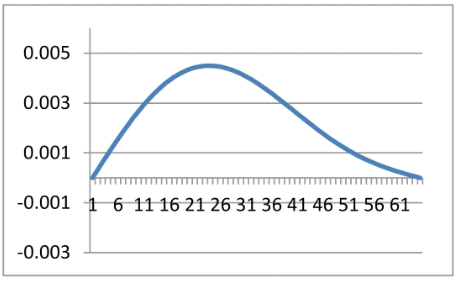
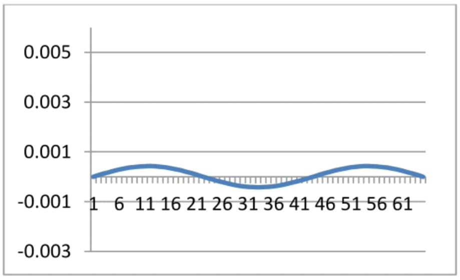


Fig: Second harmonic at t=0. Fig:Sum of the two first harmonics at t=0.

The third harmonic and the sum of the first three harmonics:



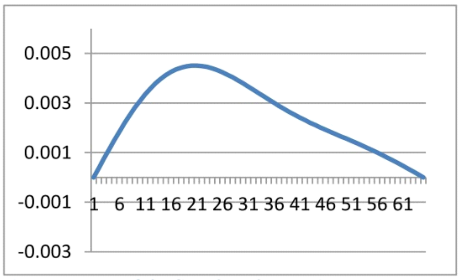


Figure: Third harmonic at t=0.Figure: Sum of the first three harmonics at t=0

As more harmonics are added, the total displacement function as the accumulated sum of harmonics becomes more and more similar to the initial displacement function *f(x)*

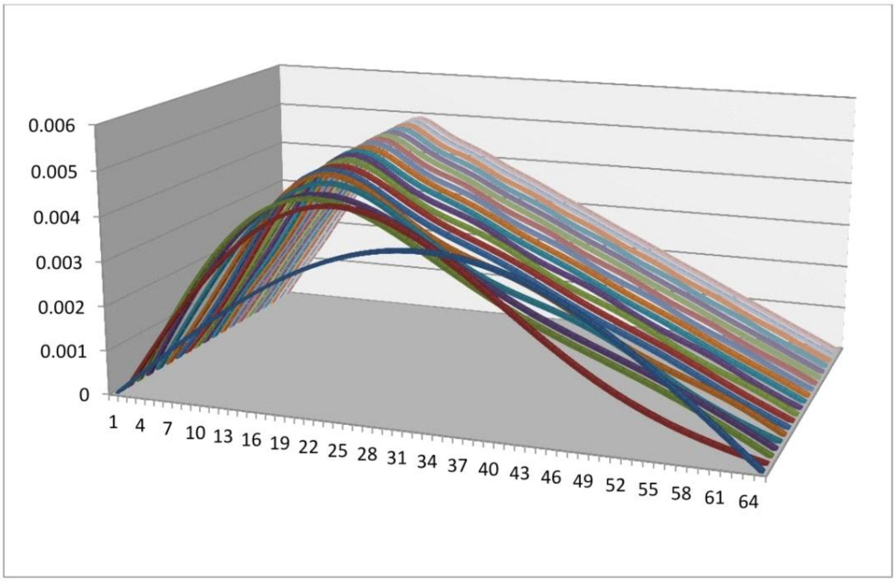


Figure: Accumulated sum of the first 20 harmonics for d = 0.16.

3.6 Amplitude distribution depending on position of plucking

To find the influence of the distribution of amplitudes for the harmonics based on the position of plucking, we compute amplitudes according to (7) for some example values of d. L and h are fixed (L = 0.64, h = 0.005).

***d* = 0.32**

Plucking at the middle of the string (d = 0.32) should produce the following amplitude distribution:

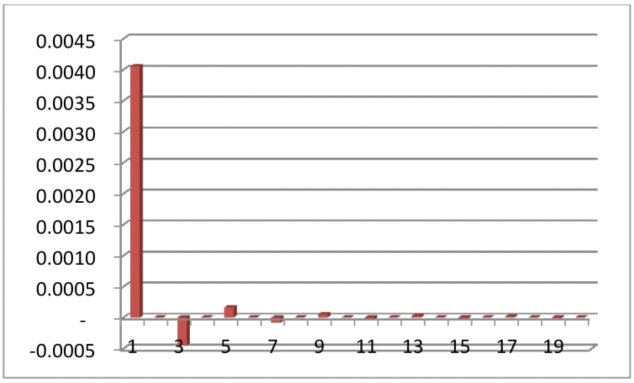


Figure: Amplitudes for d = 0.32.

Here the fundamental harmonic is very dominant, producing a "pure" sound with low amplitudes on overtones. Even-numbered harmonics (k = 2, k = 4, etc.) are completely missing.

***d* = 0.16**

Plucking at the regular playing area of the guitar (ca$d$ = 0.16) produces a slightly different amplitude distribution.

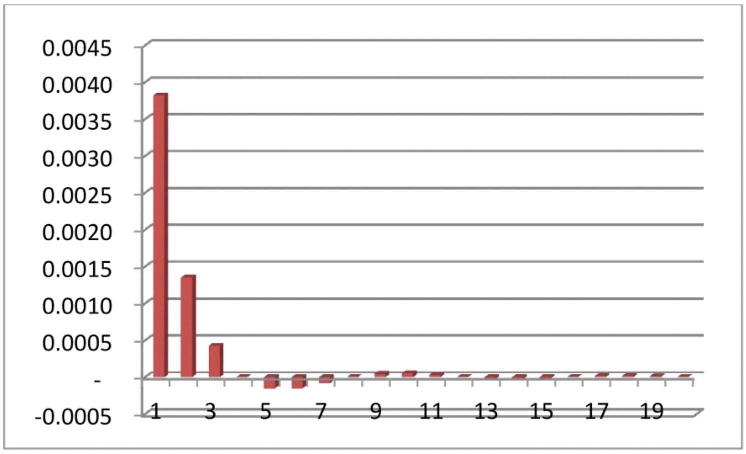


Figure: Amplitudes for d = 0.16.

The fundamental tone is still dominant, but the five first overtones would influence the tone quality.

d = 0.05

Plucking close to the nut or the bridge of the guitar should produce the following amplitude distribution.

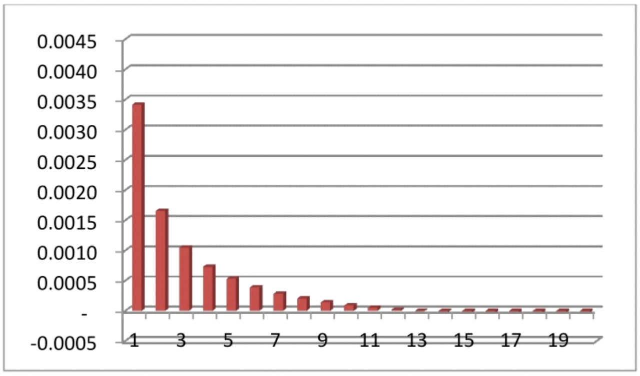


Figure: Amplitudes for d = 0.05.

In this case the overtones will be clearly more visible, even up to the 10^{th} overtone. This would create a 'bright' tone quality.

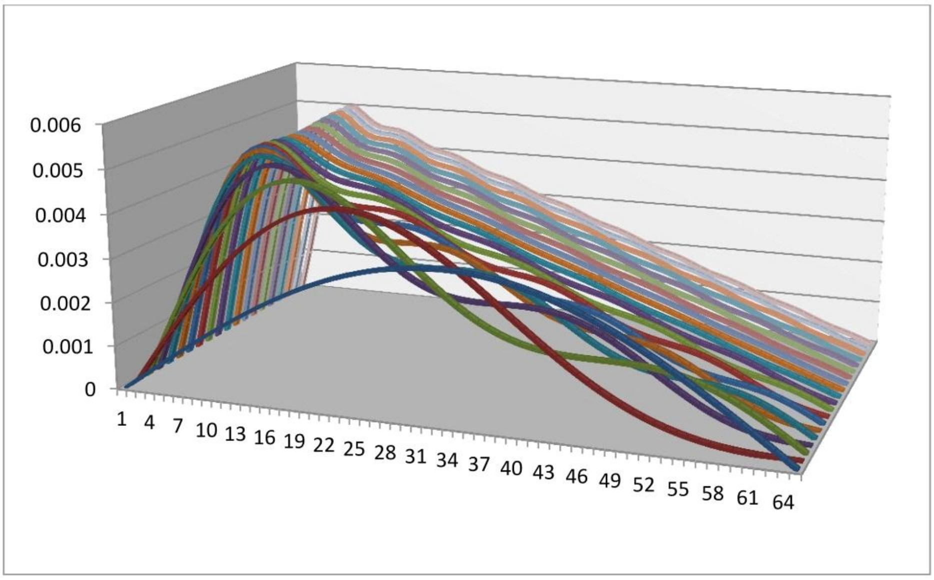


Figure: Accumulated sum of the first 20 harmonics for d = 0.05.

4 Experiment setup

A microphone was connected to a PC running the following software tools:

a. Audacity for recording from the microphone

b. Spectra Scope for analyzing the sound files and producing frequency amplitudes

c. MS Excel for examining and analyzing the output frequency tables

Using Audacity and recoding to a file enabled full control over the start and end times of the recording, avoiding transients at the beginning and distortion due to fading. Samples of 1.04 seconds were used.

Spectra Scope uses an implementation of a Fast Fourier Transform (FFT) to produce a frequency plot of the amplitudes given in dB. This allowed us to derive the relative energies in the fundamental tone and overtones. Spectra Scope supported export of frequency tables.
Excel was used to import and then analyze the frequency tables output form Spectra Scope. To reduce sources of error, dBv values from ca 10 Hz below to ca 10 Hz above the actual frequency were summarized. Each group always contained eight values.

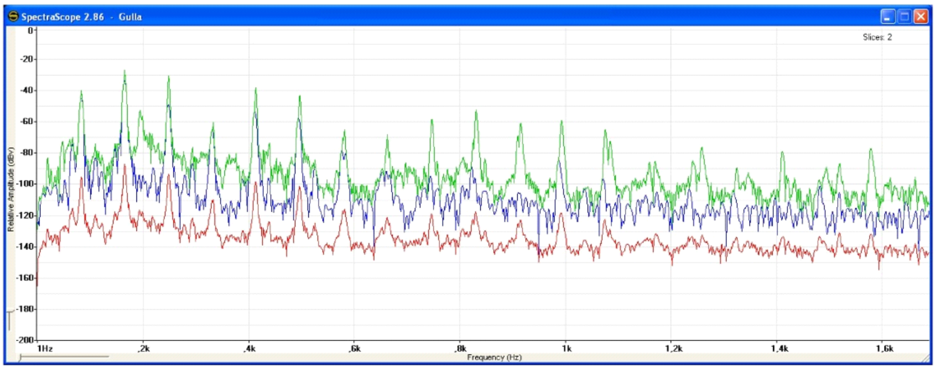
In order to get reproducible and consistent results, the position of plucking was accurately measured and tagged on the guitar. The sideways offset of 0.5 cm was also indicated.

Figure: Plucking the guitar string.

4.1 Measurements

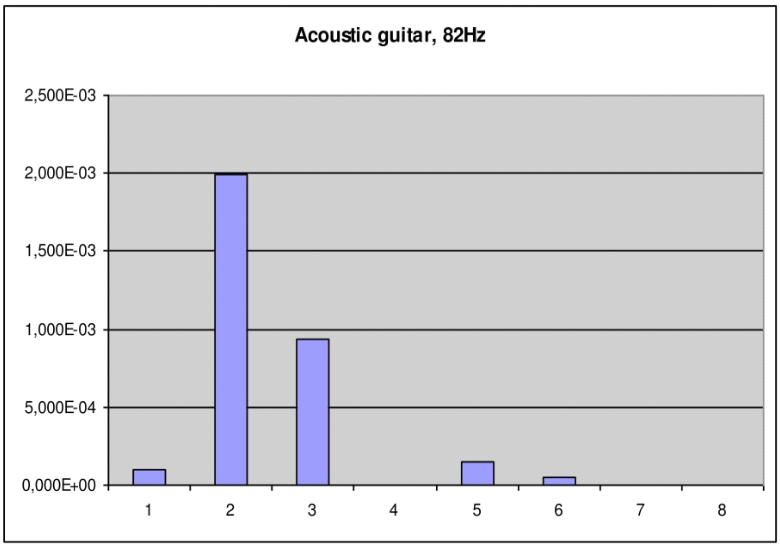
4.1.1 Acoustic guitar, 82Hz, d=0.16, plucking with plastic plectrum

Using a high-quality microphone connected to a laptop, a recording of 1.04 second was done. Below the amplitude-to-frequency diagram of SpectraScope is shown.



The dBv values for different frequencies were imported into Excel for further processing.

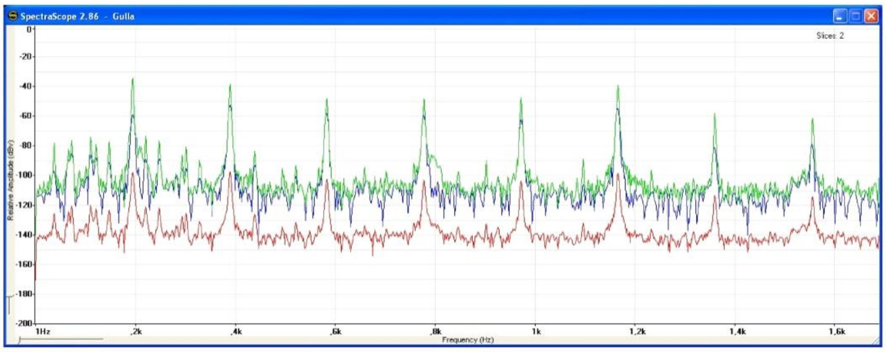
Tone	Ideal frequency (Hz)	Actual frequency (Hz) for max amplitude	Amplitude (dBv) from SpectraScope	Amplitude
Fundamental	82	82	-39.9	1.023E-04
Overtone 1	164	164	-27.0	1.995E-03
Overtone 2	246	247	-30.3	9.333E-04
Overtone 3	328	332	-60.3	9.333E-07
Overtone 4	410	413	-38.3	1.479E-04
Overtone 5	492	496	-43.2	4.786E-05
Overtone 6	574	581	-64.7	3.388E-07
Overtone 7	656	662	-68.2	1.514E-07



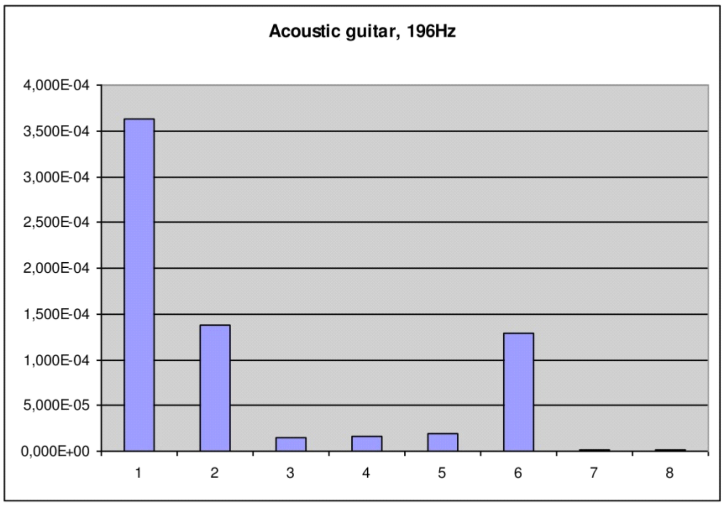
As observed from the diagram above, the amplitude for the fundamental tone is quite low for 82Hz. In an acoustic guitar the sound is mainly produced by the guitar body and not the string itself(J.Pelc n.d.). A hypothesis could be that the body was not able to reproduce such low tones.

4.1.2 Acoustic guitar, 196 Hz, d=0.16, plucking with plastic plectrum

Instead of plucking the low E string, the experiment was redone with the G string at approximately 196 Hz.



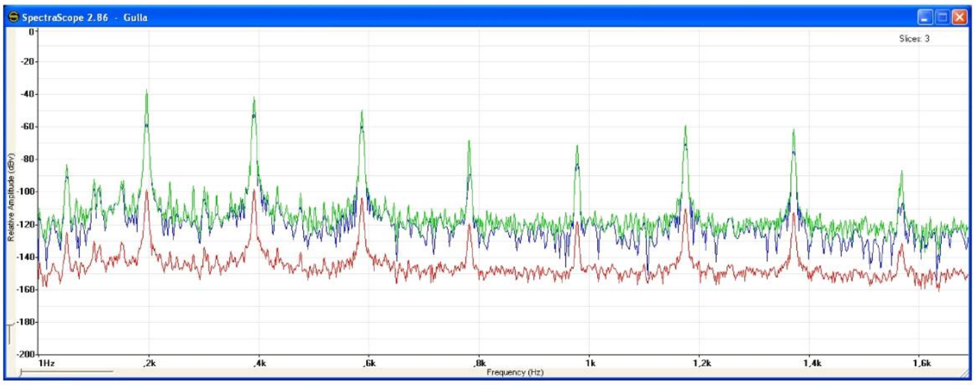
Tone	Ideal frequency (Hz)	Actual frequency (Hz) for max amplitude	Amplitude (dBv) from SpectraScope	Amplitude
Fundamental	196	193	-34.4	3.631E-04
Overtone 1	392	388	-38.6	1.380E-04
Overtone 2	588	582	-48.2	1.514E-05
Overtone 3	784	776	-48.0	1.585E-05
Overtone 4	980	971	-47.1	1.950E-05
Overtone 5	1176	1165	-38.9	1.288E-04
Overtone 6	1372	1359	-58.0	1.585E-06
Overtone 7	1568	1555	-61.1	7.762E-07



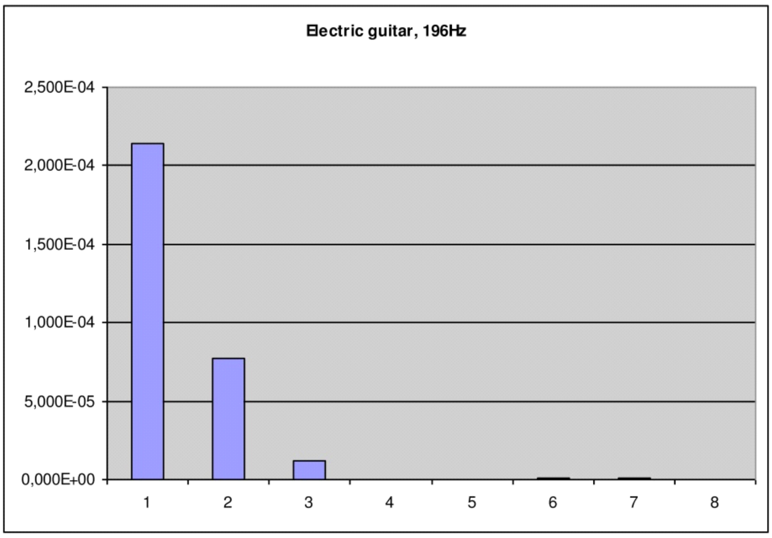
This graph matches the predicted distribution by the mathematical model much closer.

4.1.3 Electric guitar, 196Hz, d=0.16, plucking with plastic plectrum

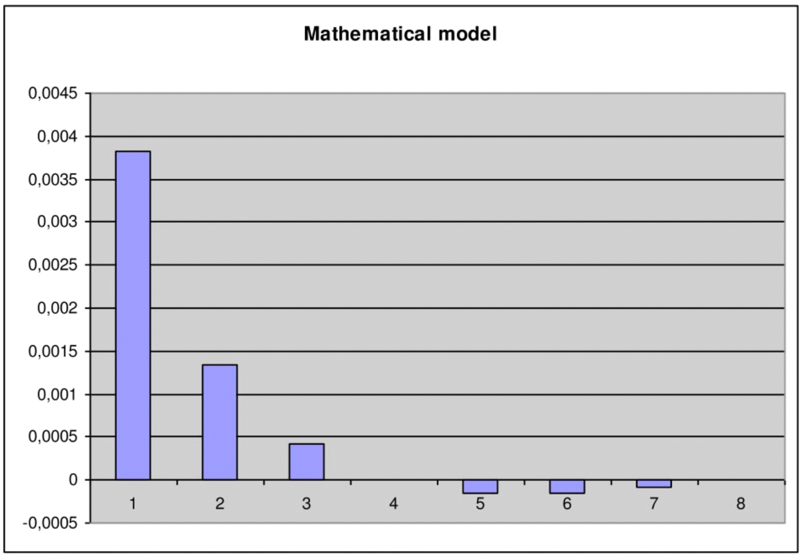
To eliminate the resonance effect of the acoustic guitar body completely, an electric guitar was used. A digital external sound was used to amplify the signal produced by the electric guitar.



Tone	Ideal frequency (Hz)	Actual frequency (Hz) for max amplitude	Amplitude (dBv) from SpectraScope	Amplitude
Fundamental	196	195	-36.7	2.138E-04
Overtone 1	392	390	-41.1	7.762E-05
Overtone 2	588	586	-49.3	1.175E-05
Overtone 3	784	781	-68.2	1.514E-07
Overtone 4	980	978	-71.0	7.943E-08
Overtone 5	1176	1174	-59.1	1.230E-06
Overtone 6	1372	1372	-61.2	7.586E-07
Overtone 7	1568	1567	-86.8	2.089E-09



For comparison the corresponding amplitudes computed by formula (7) are shown below.



As can be observed the relative distribution of amplitudes for the different harmonics for an actual electrical guitar quite closely resemble the distribution predicted by the mathematical formula.

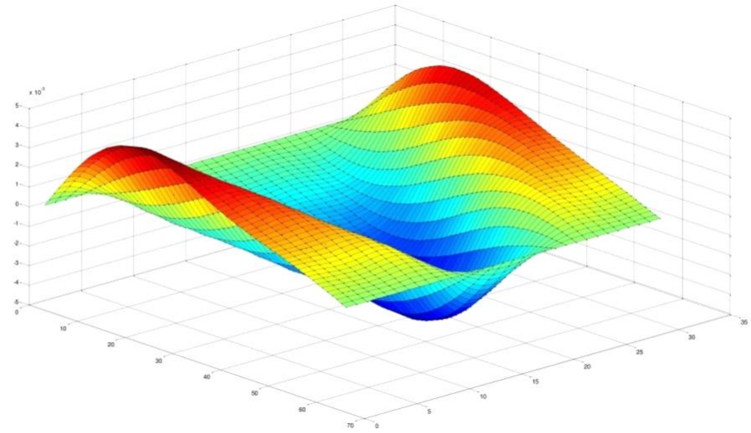


Figure: Displacement functions of time and position for the three first harmonics.

5. CONCLUSION

Equation (8) defines the displacement from equilibrium for any segment of the string (denoted by position x) at any point of time (denoted by t). Due to the structure of the formula, the complex function can more easily be understood as a superposition of modes or harmonics. Each harmonic defines a tone corresponding to the possible standing waves given by the string length, with frequency depending on string tension and density. The amplitudes of each harmonic (7) correspond to the coefficients derived using Fourier series. Based on this mathematical model we are able to compute theoretical amplitudes for the fundamental tone and the overtones dependent on the extent (h) and position (d) of the plucking of the string. When plucking close to the middle of the string, the amplitude of the fundamental tone is dominant, producing a clean, sine-like tone. When plucking closer to the edge of the string, more of the potential energy released goes into higher-pitch harmonic tones, producing a sharper sound. This also is also consistent with observations using a real guitar.

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