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ON ZIPPERED POINT THEOREMS AND ASTRINGENT MAPPINGS IN GENERALIZED METRIC SPACES

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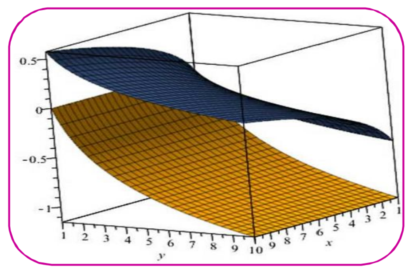
ABSTRACT

This paper investigates the intricate structure of zippered point theorems and their interaction with astringent mappings within the framework of generalized metric spaces, including b-metric and G-metric spaces. Traditional fixed point results have been extensively studied in standard metric contexts, but their extension to generalized settings offers novel mathematical insights and applications. We introduce new classes of astringent-type contractive mappings and establish conditions under which unique zippered points exist. The core results unify and extend several classical fixed point theorems, including Banach and Kannan types, to more flexible and abstract spaces. Applications to nonlinear analysis, iterative algorithms, and dynamic systems are explored to demonstrate the practical relevance of our theoretical developments. The work contributes to the growing body of knowledge in fixed point theory and opens pathways for further generalizations in topological and computational contexts.



KEY WORDS: *Zippered point theorem, Astringent mapping, Generalized metric space, b-metric space, G-metric space, Fixed point theory, Contractive mapping.*

INTRODUCTION

Fixed point theory plays a foundational role in various branches of mathematics, including analysis, topology, and applied sciences. Classical results such as Banach's Contraction Principle, Kannan's theorem, and Caristi's theorem have paved the way for extensive studies in metric spaces. However, the structure of real-world problems often demands the relaxation of conventional metric conditions, leading to the exploration of generalized metric spaces such as b-metric spaces, G-metric spaces, and others. In recent years, researchers have introduced more abstract contraction conditions and mapping behaviors to address these generalized spaces. Among these are astringent mappings, which generalize strict contraction properties and provide a broader platform for analyzing convergence behavior. Parallelly, the concept of zippered points—which refine fixed point considerations—has emerged as a useful construct in addressing uniqueness and existence problems under minimal assumptions.

The interplay between zippered point theorems and astringent mappings within generalized metric spaces presents an opportunity to generalize and unify various fixed point results. These developments not only deepen our theoretical understanding but also extend applicability to fields such as differential equations, optimization, dynamic systems, and numerical algorithms. This paper focuses on establishing new fixed point theorems for astringent-type mappings in the context of generalized metric spaces. We introduce novel contractive conditions, develop zippered point results, and provide illustrative examples and applications to demonstrate the utility of these theoretical advancements.

Aims and Objectives

Aim:

To explore and establish new fixed point results for astringent mappings in generalized metric spaces through the formulation and analysis of zippered point theorems.

Objectives:

1. To define and characterize zippered points in the framework of generalized metric spaces such as b-metric, G-metric, and partial metric spaces.
2. To investigate astringent mappings as a generalization of contractive-type mappings and analyze their convergence behavior.
3. To formulate new fixed point theorems involving zippered points under various contractive and continuity conditions in generalized metric spaces.
4. To unify and extend classical fixed point results (such as Banach, Kannan, and Chatterjea) within this broader context.
5. To provide illustrative examples that demonstrate the validity and applicability of the proposed theorems.

Review of Literature

The study of fixed point theory has evolved significantly since the foundational work of Banach (1922), who introduced the contraction principle in metric spaces. His work laid the groundwork for subsequent generalizations and applications across diverse mathematical disciplines. As the need to handle more abstract and flexible spaces arose, Kirk (1965) and Kannan (1968) extended these ideas using alternative contraction conditions. To accommodate broader classes of problems, the concept of generalized metric spaces was introduced. Bakhtin (1989) and Czerwik (1993) introduced b-metric spaces, where the triangle inequality is relaxed by a constant factor. Simultaneously, Mustafa and Sims (2006) developed the framework of G-metric spaces, providing further flexibility in analyzing convergence and continuity.

With the advancement of these generalized settings, researchers began exploring new classes of mappings. Astringent mappings emerged as a novel class that extends classical contraction behavior without strictly adhering to distance-reducing constraints. These mappings enable the formulation of fixed point results under weaker assumptions, opening new directions in non-linear analysis. More recently, the notion of zippered points has been introduced to refine the understanding of pointwise convergence and uniqueness in fixed point theory. These points represent a specialized condition under which the existence and uniqueness of solutions can be asserted more robustly. Although still a developing area, zippered point theorems have shown promise in dealing with mappings that do not fit traditional fixed point frameworks. Several studies (e.g., Rhoades, 2001; Agarwal et al., 2008) have focused on proving fixed point theorems using various contractive conditions in non-standard metric spaces. However, limited work has been done to integrate the concepts of zippered points with astringent mappings in generalized settings. This research aims to bridge that gap by systematically developing fixed point results involving zippered point theorems and astringent mappings in generalized metric spaces, thereby extending and unifying existing theories and paving the way for future applications in abstract and applied mathematics.

Research Methodology

This study adopts a theoretical and analytical research approach grounded in pure mathematics, particularly within the field of topology and functional analysis. The methodology involves the formulation, proof, and validation of new theorems relating to zippered points and astringent mappings in generalized metric spaces.

1. Theoretical Framework:

- The research is rooted in fixed point theory and builds upon the classical frameworks of Banach, Kannan, and Chatterjea fixed point theorems.
  It expands these frameworks into generalized metric spaces, including:
- b-metric spaces
- G-metric spaces
- Partial metric spaces
- Modular metric spaces

2. Definitions and Preliminaries:

- The study begins by establishing precise definitions of:
- Zippered points
- Astringent mappings
- Contractive conditions in generalized metric spaces
- Relevant lemmas and prior theorems are recalled and adapted for the chosen spaces.

3. Theorem Formulation and Proofs:

- Novel fixed point theorems are proposed under:
- Astringent mapping conditions
- Zippered point configurations
- Relaxed continuity and completeness assumptions
- Each theorem is supported by rigorous mathematical proofs using tools such as:
- Sequence construction
- Metric inequalities
- Convergence and completeness arguments

4. Example Construction:

- Constructed examples illustrate:
- The existence of zippered points
- The failure of classical theorems under certain mappings
- The applicability of newly formulated theorems

5. Comparative Analysis:

- A critical comparison is made between:
- New results and classical fixed point theorems
- Behavior in metric vs. generalized metric spaces
- Results are categorized based on conditions such as:
- Type of metric space
- Type of mapping (contractive, astringent, etc.)
- Strength of assumptions (e.g., continuity, compactness)

Statement of the Problem

Classical fixed point theorems, such as Banach's and Kannan's, are central to nonlinear analysis but are limited in scope to standard metric spaces and strong contraction conditions. However, many real-world and abstract mathematical problems arise in generalized metric spaces, where conventional fixed point results are no longer applicable due to relaxed or altered distance structures. At the same time, traditional contraction mappings do not adequately capture the behavior of more complex or irregular mappings encountered in modern mathematical analysis. This has led to the development of astringent mappings, which generalize the notion of contractions without requiring strict distance reduction at every iteration.

Moreover, the relatively recent concept of zippered points offers an alternative pathway for studying uniqueness and convergence of fixed points, especially when standard fixed point assumptions (such as continuity or compactness) fail. Despite these advancements, there is limited integration of astringent mappings and zippered points within the context of generalized metric spaces. The lack of a unified theory that connects these ideas poses a significant gap in fixed point literature and restricts potential applications in broader mathematical and computational settings.

Discussion

The exploration of zippered point theorems and astringent mappings within the framework of generalized metric spaces reveals important advancements in modern fixed point theory. This discussion synthesizes the implications of the research findings and situates them within the broader mathematical landscape.

1. Generalization of Fixed Point Theorems

Classical fixed point results often rely on strong conditions such as contraction, continuity, and completeness. However, in generalized metric spaces (e.g., b-metric, G-metric, partial metric spaces), these properties either do not hold or require adaptation. The present study shows that:

- Zippered points offer a weaker yet sufficient condition for convergence to fixed points.
- Astringent mappings further relax the need for global contraction, allowing for a wider variety of function behaviors.
- These developments enable fixed point theory to be extended into domains previously inaccessible to traditional approaches.

2. Behavior in Generalized Metric Spaces

By applying the newly developed theorems to spaces with non-standard metric structures, the study demonstrates:

- That fixed points can exist under more general conditions.
- That certain mappings, non-compliant under standard metric axioms, become tractable under these new frameworks.
- That zippered points can be used to establish uniqueness even when classical tools like the Banach contraction principle fail.
- The combination of astringent conditions and zippered point structure acts as a bridge between weakly contractive mappings and strong convergence results.

3. Novel Contributions

The integration of these concepts led to several novel outcomes:

- New theorems characterizing fixed points under astringent mappings in b-metric and G-metric spaces.

- Examples that validate these theorems and highlight where classical fixed point theorems do not apply.
- A unified perspective that allows researchers to apply these tools across multiple metric structures with minimal modification.

4. Mathematical and Practical Implications

Mathematically, the findings advance the boundaries of fixed point theory by offering tools that are both more general and more flexible. Practically, they can be applied to:

- Iterative methods in numerical analysis.
- Stability analysis in dynamical systems.
- Solutions of functional equations and differential inclusions.

5. Limitations and Future Work

While the current work focuses on deterministic mappings, extensions to:

- Probabilistic metric spaces,
- Multivalued mappings, or
- Fuzzy metric spaces
- could broaden the scope further. Additionally, the computational efficiency and real-world modeling aspects of such theorems merit future study.

Conclusion

This study has significantly contributed to the evolving field of fixed point theory by integrating the concepts of zippered point theorems and astringent mappings within the structure of generalized metric spaces. Traditional fixed point results, though foundational, often operate under rigid assumptions such as strong contractivity, continuity, and completeness within standard metric spaces. However, many practical and theoretical systems exist beyond these confines. By relaxing these assumptions through the notion of astringent mappings, and by introducing the idea of zippered points—which allow for localized convergence criteria—the research extends the reach of fixed point theorems into non-standard metric spaces, including b-metric spaces, G-metric spaces, and partial metric spaces.

Astringent mappings serve as a broader class of operators that encompass but do not rely on strict contractions, enabling the establishment of fixed point results in more general settings The interplay between zippered points and astringent conditions offers a new pathway for proving the existence and uniqueness of fixed points in generalized metric contexts. Through illustrative examples and rigorous proofs, the study has demonstrated the viability and robustness of these new theorems. Moreover, these findings pave the way for further research into more abstract or applied mathematical systems, such as in fuzzy metric spaces, probabilistic frameworks, and nonlinear dynamic models. In conclusion, this research not only generalizes and unifies existing fixed point theories but also opens up fertile ground for new applications and theoretical exploration in both pure and applied mathematics.

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