



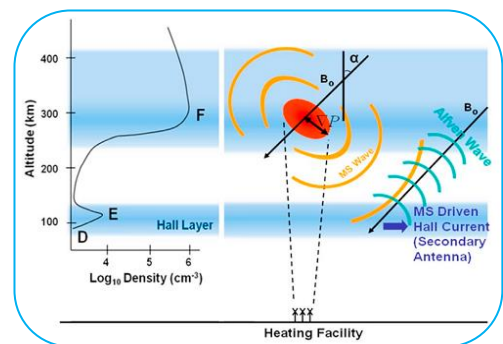
ULF WAVE AND THEIR GENERATION MECHANISM

M. A. Khan

Associate Professor, Dept. of Physics, G. F. College, Shahjahanpur.

ABSTRACT

Ultra low frequency (ULF) waves incident on the Earth are produced by processes in the magnetosphere and solar wind. These processes produce a wide variety of ULF hydromagnetic wave types that are classified on the ground as either Pi or Pc pulsations (irregular or continuous). Waves of different frequencies and polarizations originate in different regions of the magnetosphere. The location of the projections of these regions onto the Earth depends on the solar wind dynamic pressure and magnetic field. The study of ULF waves is a very active field of space research and much has yet to be learned about the processes that generate these waves.



KEYWORDS: cavity modes, file line resonances, MHD, magnetic storm, magnetosphere, pulsations.

INTRODUCTION

Ultra low frequency (ULF) waves incident in the Earth's environment are produced by processes in the magnetosphere and solar wind. These processes produce a wide variety of ULF hydromagnetic waves. Different frequencies of waves and polarizations originate in different regions of the magnetosphere. Ultra low frequency waves (magnetic pulsations) are caused by hydromagnetic waves that may be generated as a result of different types of plasma instabilities in the magnetosphere or on its boundary in a very complicated manner. In this chapter, the generation of hydromagnetic waves, their sources within and external to the magnetosphere and their propagation and modification within the magnetosphere and ionosphere are briefly discussed. A very good summary of these topics, with references to the most important publications dealing with ULF waves, has elegantly been reported by McPherron (2005) [1], and also presented in the books "Introduction to Space Physics", edited by Kivelson, M.G. and Russell, C.T. (1995) [2] and "Geomagnetic Micropulsations" by Jacobs (1970) [3]. The information given in this chapter is mainly cited from these publications and references contained therein.

Generation of ultra Low Frequency Waves

The study of ULF waves is a very active field of space research and much has yet to be learned about the processes that generate these waves. ULF waves are created by a variety of processes in magnetized plasma. Plasma is highly ionized gases threaded by a magnetic field. The processes that modify the equilibrium of the plasma and the magnetic field can serve as an energy source for waves. In 1942, H. Alfvén showed how plasma in a steady magnetic field in which waves of low enough frequency could propagate in fluids of high electrical conductivity can be produced. The direct confirmation of the

existence of the waves was difficult to obtain, as they decay rapidly in most experiments. The situation, however, is very different in problems of cosmic physics because of the enormous dimensions involved and one can experience these waves, as the decay rate can be small at these large spatial scales. Alfvén showed that at sufficiently low collision frequency, the charged particles simply gyrate around the magnetic field and travel along it. Any force that moves the charged particles also moves the magnetic field and vice versa. In this situation the field and plasma are “frozen together”. At low frequency electric field $\vec{E}$ and plasma flow velocity $\vec{V}$ are related by

$$\vec{E} = -\vec{V} \times \vec{B}$$

where $\vec{B}$ is magnetic field flux density.

So with the low frequency approximations, Faraday's law takes the form

$$\vec{\nabla} \times (\vec{V} \times \vec{B}) = \frac{\partial \vec{B}}{\partial t}$$

And it can be used to show any two particles initially connected by a field line which move with velocity $\vec{V}$ (perpendicular to $\vec{B}$) remain connected by the field line.

Alfvén's model for wave generation is summarized in Fig. 1.1 [4]. Consider an infinite volume of fully ionized hydrogen. As shown in Fig. 1.1(a), imagine that a force is applied to the plasma perpendicular to $\vec{B}$ displacing a rectangular section of plasma with velocity $\vec{V}$. As the charges begin to move they experience a Lorentz force $\vec{F} = q(\vec{V} \times \vec{B})$, where q is the charge in Coulombs. The electrons move to the left side of the moving slab, and the protons to the right side as shown in Fig. 1.1(b). This polarization of the charges creates an electric field $\vec{E}$ orthogonal to both $\vec{V}$ and $\vec{B}$. If the slab were in a vacuum, the charges would be trapped at the edges of the column and the force of the electric field would eventually stop any further transfer of charge. At this point the electric force would be equal and opposite to the Lorentz force so that,

$$q\vec{E} = -q(\vec{V} \times \vec{B})$$

$$\text{or} \quad \vec{E} = (\vec{V} \times \vec{B})$$

However, in plasma the charges can flow through the surrounding fluid in an attempt to neutralize the polarization. This charge motion creates an electrical current density $\vec{J}$ as shown in the Fig. 1.1(c). As these current flows across the magnetic field above and below the moving slab, it exerts a force on the gas, $\vec{F} = (\vec{J} \times \vec{B})$. The direction of the force is the same as the initial motion of the slab. As a result, the plasma above and below begins to move.

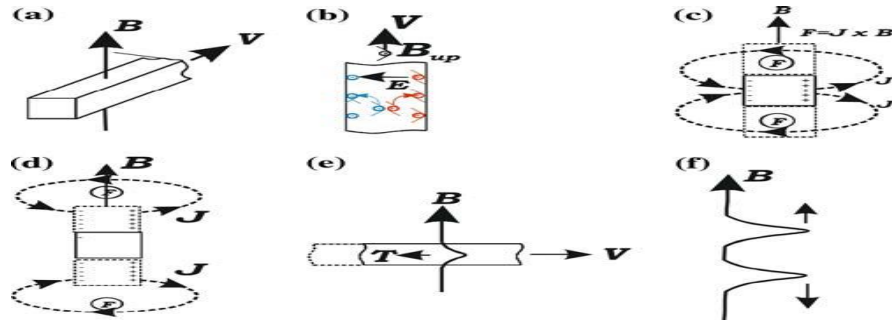


Fig.1. The generation of an Alfvén wave by displacing a slab of plasma threaded by a magnetic field [4] [1].

Similarly these two moving slabs up and down the initial slab become polarized, driving currents that cause slabs further up and down the initial slab to start moving as seen in the Fig. 1.1(d). Clearly, the initial disturbance is propagating in both directions along with the magnetic field away from the initial disturbance. The displacement of the slab distorts the magnetic field that is frozen into the plasma, which is clear from the Fig. 1.1(e). A tension develops due to bending of field, which causes a restoring force that brings the slab to a stop and then returns it towards its initial location. As this happens the moving slabs up and down the initial slab distort the field line in the same way so that two pulses appear to propagate away from the origin as illustrated in Fig. 1.1(f). These pulses are called Alfvén waves.

MHD waves are found as solutions to the basic fluid equations, Maxwell's equations and Ohm's law. In a magnetized plasma the two electromagnetic waves are joined together with sound wave by the "frozen in" magnetic field, and there are three solutions to the basic equations. A low frequency approximation to the basic equations is called magnetohydrodynamics (MHD). The wave solutions to these equations are called MHD waves. The fluids equations are

$$\frac{\partial}{\partial t} \rho + \vec{\nabla} \cdot (\rho \vec{V}) = 0 \quad \text{(Equation of continuity)} \quad (i)$$

$$\frac{\partial}{\partial t} \vec{V} = -\vec{\nabla} p + \vec{J} \times \vec{B} \quad \text{(Equation of motion)} \quad (ii)$$

$$\frac{p}{\rho^\gamma} = \text{constant} \quad \text{(Equation of state)} \quad (iii)$$

Here, ρ is the plasma density, $\vec{V}$ is the velocity, p is the pressure, $\vec{J}$ is current density, $\vec{B}$ is the magnetic flux density and γ is the ratio of specific heats. The Maxwell's equations are:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \text{(Gauss law in electrostatics)} \quad (iv)$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \text{(Gauss law in magnetostatics)} \quad (v)$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \vec{B} \quad \text{(Faraday's law)} \quad (vi)$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \vec{E} \quad (\text{Ampere's law}) \quad (\text{vii})$$

In the problems of cosmic physics with low frequency approximations, displacement currents are negligible in comparison to conduction current. So the second term proportional to the electric field can be dropped. Thus we get,

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \quad (\text{Ampere's law in MHD limits}) \quad (\text{viii})$$

Here, ϵ_0 and μ_0 are the permittivity and permeability of free space, respectively.

Also, Ohm's law in the frame in which the plasma velocity is measured can be written as

$$\vec{J} = \sigma (\vec{E} + \vec{V} \times \vec{B}) \quad (\text{ix})$$

where σ is the electrical conductivity.

Let us assume that the plasma is initially at rest, which means that there are neither flows nor electric fields, and also assume that no currents are flowing. The wave perturbations introduce finite but small $\vec{E}$, $\vec{u}$ and $\vec{J}$. The magnetic field, mass density and pressure also changes. So the velocity of plasma and magnetic field can be written in terms of initial (background) values and small perturbations as

$$\vec{V} = 0 + \vec{u}, \quad \vec{B} = \vec{B}_0 + \vec{b},$$

Similarly the density and pressure can be written as

$$\rho = \rho_0 + \delta\rho, \quad \text{and} \quad p = p_0 + \delta p$$

All of the perturbed quantities, $\vec{b}$, $\delta\rho$, δp , $\vec{u}$, $\vec{E} = -\vec{u} \times \vec{B}$ and $\vec{J} = \vec{\nabla} \times \vec{b} / \mu_0$ are assumed to be small enough that only terms linear in any of them are retained and squares or high powers and cross products will be dropped. Putting these expressions into the preceding equations and after several substitutions and algebraic manipulation, we get a set of four equations.

$$\rho_0 \frac{\partial \vec{u}}{\partial t} = -\vec{\nabla} p + \frac{1}{\mu_0} [(\vec{\nabla} \times \vec{b}) \times \vec{B}_0] \quad (\text{x})$$

$$\vec{\nabla} p = \vec{\nabla} \rho = \gamma \frac{p_0}{\rho_0} \vec{\nabla} \delta\rho \quad (\text{xi})$$

$$\frac{\partial}{\partial t} \delta\rho + \rho_0 (\vec{\nabla} \cdot \vec{u}) = 0 \quad (\text{xii})$$

$$\frac{\partial}{\partial t} \vec{b} = (\vec{B}_0 \cdot \vec{\nabla}) \vec{u} = \vec{B}_0 (\vec{\nabla} \cdot \vec{u}) \quad (\text{xiii})$$

For a plane wave propagating with wavelength λ and frequency f , all oscillating quantities can be taken as proportional to

$$e^{i\vec{k}\cdot\vec{r}} e^{-i\omega x} = e^{i\vec{k}\cdot\vec{r}-i\omega x}$$

where $k = 2\pi/\lambda$, is the wave number and $\omega = 2\pi f$, is the angular frequency of the wave and $\vec{r}$ is the position vector. Again by substituting and simplifying, we get a set of four algebraic equations. These four equations can be solved by obtaining a single vector equation for the velocity perturbation of the plasma. This equation is

$$\omega^2 \vec{u} - \gamma \frac{p_0}{\rho_0} \vec{k}(\vec{k} \cdot \vec{u}) - \frac{1}{\mu_0 \rho_0} \left\{ B_0^2 \vec{k}(\vec{k} \cdot \vec{u}) - \vec{B}_0(\vec{k} \cdot \vec{u})(\vec{k} \cdot \vec{B}_0) - \vec{k}(\vec{k} \cdot \vec{B}_0)(\vec{B}_0 \cdot \vec{u}) \right\} + (\vec{k} \cdot \vec{B}_0)^2 \vec{u} \Bigg\} = 0 \quad (\text{xiv})$$

For simplifying this equation we can choose a special coordinate system in which the magnetic field $\vec{B}_0$ lies along with the z-axis, and the wave number $\vec{k} = k\vec{n}$ with $\vec{n}$ the direction of propagation, lies in the y-z plane along the unit vector $\vec{n}$ at an angle θ to the z-axis and can also choose two characteristic velocities. First $s^2 = \gamma(p_0/\rho_0)$ is the square of the sound speed and the second is the square of the Alfvén velocity given as $V_A^2 = B_0^2/\mu_0 \rho$. By taking these choices, the equation (14) gives a vector equation corresponding to the three elements of the plasma velocity $\vec{u}$ and by expressing it in matrix form; the equation appears much simpler as

$$\begin{pmatrix} \left(\frac{\omega^2}{k^2} - V_A^2 \cos^2 \theta \right) & 0 & 0 \\ 0 & \left(\frac{\omega^2}{k^2} - s^2 \sin^2 \theta - V_A^2 \right) & -s^2 \sin \theta \cos \theta \\ 0 & -s^2 \sin \theta \cos \theta & \left(\frac{\omega^2}{k^2} - s^2 \cos^2 \theta \right) \end{pmatrix} \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = 0$$

This set of equations has three solutions corresponding to three different modes of wave propagation. The first wave is the Alfvén wave, also called shear Alfvén wave, transverse or guided mode. It has only an x-component of the perturbation velocity (i.e. $u_x \neq 0$, & $u_y = u_z = 0$) orthogonal to the plane containing the ambient field and the direction of propagation. In this condition the first equation requires that

$$\frac{\omega^2}{k^2} - V_A^2 \cos^2 \theta = 0$$

$$\text{or} \quad V_{ph}^2 = \frac{\omega^2}{k^2} = V_A^2 \cos^2 \theta \quad (\text{xvi})$$

where $V_{ph} = V_A \cos \theta$ is phase velocity of shear Alfvén wave. The wave energy propagates along the direction of the Poynting flux vector:

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{b},$$

This direction will be along the ambient field ($\pm \vec{B}_0$) in the shear Alfvén wave. Thus the shear Alfvén wave propagates with the phase velocity $V_A \cos \theta$ and sets the fluid into motion in the direction perpendicular to the plane containing the propagation vector $\vec{k}$ and the background field $\vec{B}_0$. As the wave magnetic perturbation is transverse to the ambient field so the magnetic pressure does not change in the wave and also the wave Poynting flux is always along $\vec{B}_0$ and thus the ambient magnetic field strictly guides the energy flow and information contents.

The other two wave modes are blended together. One has a speed that is fast compared to the shear Alfvén wave, and the other has a speed that is slow. The fast wave or compressional wave is a combination of pressure and magnetic field fluctuations. When the fast wave propagates perpendicular to the background field it is seen as alternating compressions and rarefactions of both the field and plasma density. The Poynting flux vector is in the direction of propagation (at an arbitrary angle relative to $\vec{B}_0$). Thus

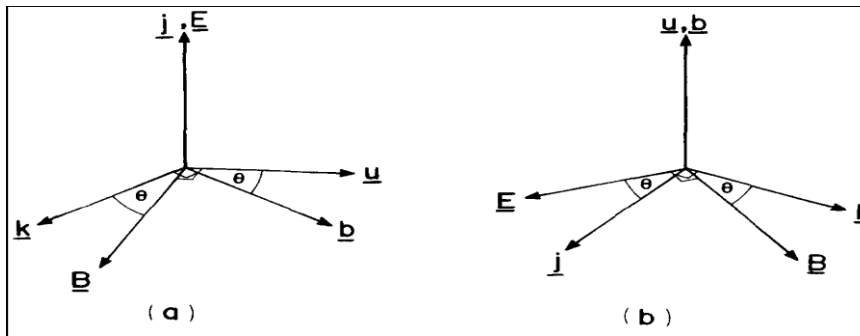


Fig. 1.2 Relationship between the directions of the ambient magnetic field, B_0 , and the hydromagnetic wave parameters for (a) a fast mode and (b) a shear Alfvén mode in a uniform, cold plasma [5].

the fast (compressional) mode-wave can carry energy flow and information contents in any direction. The dispersion relation of fast wave is

$$V_{ph}^2 = \frac{\omega^2}{k^2} = V_A^2 \quad (\text{xvii})$$

The slow wave is closest to being a pure sound wave. Each of these waves has unique polarization properties with the electric, magnetic, and plasma velocity fluctuations being oriented in different directions relative to the direction of wave propagation and background field. Fig. 1.2 illustrates the differences in polarization of the fast and shear Alfvén modes in a uniform, cold plasma [5]-[6].

In general, the ULF waves seen at the ground originate in space as either fast mode, or shear Alfvén mode, or a combination of the two. They propagate along or across the magnetic field until they reach the ionosphere. At the ionosphere they drive electrical currents that radiate pure electromagnetic waves into the neutral atmosphere. Thus magnetic pulsations measured on the ground are MHD waves converted to purely electromagnetic waves in the ionosphere[7].

FINDINGS

The logical investigation of micropulsations, magnetic pulsations, and ULF waves has a long history. Recent instruments and satellite examination have uncovered an exceptionally perplexing scientific categorization of waves that arrive at the earth's surface. These waves are created by measures as distant as the Sun and as close as the ionosphere. There are many sources of waves both external and internal to the magnetosphere [8]. The actual magnetosphere is a full cavity and waveguide for waves that spread through the system. A large number of the waves presently have acknowledged clarifications as far as actual cycles in the solar wind or magnetosphere. Information on the wave qualities and the present status of space climate can give one a smart thought of what waves are probably going to happen at specific latitude and nearby time. The waves are demonstrative of the conditions in space. Estimations on the ground can be utilized to distantly detect the conditions in the magnetosphere and solar wins. They can likewise be utilized to test the subsurface conductivity construction of the Earth as those at this gathering are well aware processes that produce these waves.

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