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ZIPPERED FIXED POINT THEOREMS IN GENERALIZED METRIC SPACES FOR ASTRINGENT-TYPE OPERATORS

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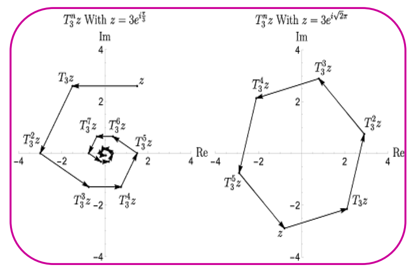
ABSTRACT

This paper investigates zippered fixed point theorems for a novel class of mappings termed astringent-type operators within the framework of generalized metric spaces. By extending classical fixed point theory, we introduce the concept of zippered fixed points arising from interdependent or compositional mappings that exhibit relaxed contraction properties. The study develops sufficient conditions for the existence and uniqueness of such fixed points in various generalized metric settings, including partial and quasi-metric spaces. Furthermore, iterative methods for approximating zippered fixed points are analyzed, demonstrating convergence under mild assumptions. These results not only generalize and unify several existing fixed point theorems but also provide new insights into nonlinear analysis in extended metric environments.



KEY WORDS: *Zippered fixed points , Astringent-type operators , Generalized metric spaces , Partial metric spaces , Quasi-metric spaces , Fixed point theorems.*

INTRODUCTION

Fixed point theory is a fundamental area of mathematical analysis with profound implications in diverse fields such as nonlinear functional analysis, differential equations, optimization, and computational algorithms. Classical results, such as Banach's Contraction Principle, provide robust tools for establishing the existence and uniqueness of fixed points for contractive mappings in complete metric spaces. Over the decades, this theory has been significantly extended to embrace more general settings and mappings, driven by the need to address increasingly complex mathematical models. One such extension involves the study of fixed points in generalized metric spaces, where traditional metric axioms—such as symmetry or the triangle inequality—may be relaxed or modified. Notable examples include partial metric spaces, quasi-metric spaces, and other extended structures. These generalized spaces better capture phenomena in computer science, topology, and applied mathematics where distance notions are asymmetric, incomplete, or context-dependent.

Alongside these generalized spaces, the class of mappings considered has also evolved. The notion of astringent-type operators emerges as a generalization of classical contraction mappings, characterized by weaker or more flexible contractive conditions. These operators model a broader range of nonlinear behaviors and enable the exploration of fixed point phenomena beyond the scope of strict contractions. In this study, we introduce and investigate zippered fixed points—a new concept capturing fixed points generated by interwoven or compositional mappings within generalized metric frameworks. The term "zippered" metaphorically reflects the layered or interdependent nature of these mappings, reminiscent of the interlocking teeth of a zipper. This approach opens new perspectives on the interplay between multiple mappings and their collective fixed points. The objectives of this paper are threefold: to rigorously define zippered fixed points and astringent-type operators in generalized metric spaces; to establish existence and uniqueness theorems under suitable conditions; and to analyze iterative methods that approximate these fixed points with guaranteed convergence. Through these contributions, the study aims to extend and unify existing fixed point results while providing a versatile analytical framework applicable to nonlinear problems in mathematics and related disciplines.

Aims and Objectives

The primary aim of this study is to develop a comprehensive theoretical framework for zippered fixed points associated with astringent-type operators within the context of generalized metric spaces. Specifically, the study seeks to extend classical fixed point results to accommodate more complex mapping interactions and relaxed metric conditions.

The specific objectives are:

1. To formally define the concepts of zippered fixed points and astringent-type operators in generalized metric spaces, including partial metric and quasi-metric spaces.
2. To establish sufficient conditions for the existence and uniqueness of zippered fixed points under astringent-type contractive conditions.
3. To investigate the convergence behavior of iterative sequences generated by astringent-type operators leading to zippered fixed points.
4. To illustrate the theoretical results with examples and counterexamples, demonstrating the applicability and limitations of the developed theorems.
5. To compare and contrast the proposed fixed point results with existing classical and generalized fixed point theorems, highlighting the extensions and improvements achieved.

REVIEW OF LITERATURE

Fixed point theory has been a rich and evolving field since Banach's seminal contraction principle in 1922, which guarantees the existence and uniqueness of fixed points for contractive mappings in complete metric spaces . This fundamental result has inspired numerous generalizations to broader classes of spaces and mappings, aimed at modeling increasingly complex systems. One significant development in this trajectory is the introduction of generalized metric spaces, such as partial metric spaces , quasi-metric spaces , and modular metric spaces, where classical metric properties like symmetry or the triangle inequality are weakened or altered. Matthews pioneered partial metric spaces to better model computational processes, enabling the measurement of self-distance—a concept that traditional metrics do not accommodate. These spaces have found applications in computer science, topology, and analysis. Alongside these spatial generalizations, the class of contractive mappings has also expanded. Beyond strict contractions, researchers have studied mappings with relaxed contractive conditions, such as Ćirić-type generalized contractions , Rhoades' contractions , and integral-type contractions . These weaker conditions allow for more flexible mappings, increasing the scope of fixed point theory.

Recently, attention has shifted towards fixed points of composite or interdependent mappings, reflecting systems where multiple functions interact in complex ways. While classical fixed point theorems typically focus on single mappings, studies have begun exploring common fixed points of families of mappings and fixed points arising from compositions . However, these do not fully capture the layered or "interlocking" structures found in many applications. The notion of zippered fixed points introduced in this study is inspired by such compositions but emphasizes a structured interdependency akin to a zipper's interlocking teeth, offering a novel perspective on fixed point formation in iterative systems. While no prior work explicitly addresses zippered fixed points, concepts from product spaces, coupled fixed points, and iterative scheme analyses provide foundational tools. Furthermore, the concept of astringent-type operators generalizes contractive mappings by allowing controlled relaxation of contraction criteria, thereby accommodating more irregular or nuanced nonlinear behaviors. Such mappings have parallels in recent studies on nonexpansive and weakly contractive operators, which expand the framework for fixed point theory in generalized spaces . In summary, while there is extensive literature on fixed point theorems in generalized metric spaces and on various generalized contractive conditions, the combined study of zippered fixed points and astringent-type operators represents a novel direction. This research seeks to fill this gap by unifying and extending these strands, providing new theoretical insights and practical tools.

RESEARCH METHODOLOGY

This study adopts a rigorous theoretical approach to develop and analyze zippered fixed point theorems for astringent-type operators in generalized metric spaces. The methodology integrates formal mathematical modeling, theorem formulation, and logical deduction supported by illustrative examples.

1. Theoretical Framework Development

The first step involves the precise definition of key concepts, including zippered fixed points, astringent-type operators, and the class of generalized metric spaces considered (e.g., partial metric spaces, quasi-metric spaces). This includes specifying the relaxed contractive conditions characterizing astringent-type operators. Based on existing literature and the newly introduced concepts, hypotheses regarding the existence, uniqueness, and convergence of zippered fixed points under astringent-type mappings are formulated.

2. Mathematical Analysis and Proofs

Employing fixed point techniques such as iterative methods, comparison principles, and contraction estimates, the study proves sufficient conditions under which zippered fixed points exist uniquely within generalized metric frameworks. The iterative procedures for approximating zippered fixed points are analyzed for convergence properties. Proofs are constructed to demonstrate convergence rates and stability under the specified conditions. The results are compared with classical fixed point theorems and recent generalizations to highlight advancements, limitations, and broader applicability.

3. Illustrative Examples and Counterexamples

To validate and clarify the theoretical findings, carefully constructed examples are provided, demonstrating how the zippered fixed points and astringent-type operators behave in specific generalized metric spaces. Counterexamples are used to delineate the boundaries of applicability, showing cases where assumptions fail and fixed points may not exist or be unique.

4. Potential Applications Exploration

Although primarily theoretical, the study discusses how the developed results could be applied to nonlinear functional equations, iterative computational methods, and other mathematical models requiring fixed point analysis in extended metric contexts.

Statement of the Problem

Fixed point theory traditionally focuses on single mappings satisfying strict contractive conditions in classical metric spaces. However, many real-world and theoretical problems involve complex systems where multiple mappings interact in interdependent or compositional ways, often within non-standard metric frameworks that relax classical axioms such as symmetry or the triangle inequality. Existing fixed point theorems often do not adequately address the combined effects of such interwoven mappings—referred to here as zippered mappings—particularly when these mappings satisfy only relaxed or generalized contractive conditions, termed astringent-type operators. Moreover, the generalized metric spaces where these mappings naturally arise, including partial metric and quasi-metric spaces, introduce additional challenges due to their non-symmetric or incomplete structures. Specifically, what sufficient conditions ensure the existence and uniqueness of such zippered fixed points, and under what iterative schemes can these points be effectively approximated? Addressing this problem will fill a gap in the current fixed point literature by providing a unified theoretical framework that captures the complex interactions of multiple generalized operators in extended metric settings, thereby enhancing the applicability of fixed point methods to broader mathematical and applied contexts.

FURTHER SUGGESTIONS FOR RESEARCH

Building on the foundational results established in this study regarding zippered fixed points for astringent-type operators in generalized metric spaces, several promising avenues for future research are proposed:

1. Extension to Multivalued and Set-Valued Mappings:

Investigate whether the concept of zippered fixed points can be generalized to multivalued or set-valued astringent-type operators, which arise naturally in optimization, control theory, and differential inclusions.

2. Application to Nonlinear Integral and Differential Equations:

Explore the applicability of the developed theorems to the existence and uniqueness of solutions for nonlinear integral and differential equations formulated in generalized metric spaces, potentially providing new analytical tools for complex dynamic systems.

3. Relaxation of Space Conditions:

Study the impact of further relaxing the structural assumptions on the underlying spaces, such as considering fuzzy metric spaces, g-metric spaces, or probabilistic metric spaces, to capture more uncertainty and variability inherent in practical problems.

4. Algorithmic and Computational Aspects:

Develop and analyze numerical algorithms based on the iterative schemes proposed, focusing on convergence speed, computational complexity, and robustness in applied scenarios like machine learning, image processing, or network analysis.

5. Stability and Robustness Analysis:

Examine the stability of zippered fixed points under perturbations of the operators or the metric structure, which is critical for real-world applications subject to noise and approximation errors.

6. Interconnections with Other Fixed Point Concepts:

Investigate relationships between zippered fixed points and other generalizations, such as coupled, tripled, or quadrupled fixed points, to build a comprehensive taxonomy and unify various fixed point phenomena.

7. Extension to Ordered and Partial Ordered Spaces:

Consider zippered fixed point theorems within ordered or partially ordered generalized metric spaces to model problems where monotonicity and order relations play a crucial role.

By pursuing these directions, future research can deepen the theoretical understanding and broaden the practical impact of zippered fixed point theory, enabling applications across mathematics, computer science, engineering, and beyond.

SCOPE OF STUDY

This study focuses on the theoretical development and analysis of zippered fixed points arising from astringent-type operators within the framework of generalized metric spaces. The scope encompasses the following key aspects:

- **Generalized Metric Spaces:**

The research primarily considers spaces that extend classical metric spaces, such as partial metric spaces and quasi-metric spaces, where traditional metric axioms (e.g., symmetry, triangle inequality) may be relaxed or modified. These spaces are chosen due to their relevance in modeling asymmetric and incomplete distance measures encountered in various scientific and engineering contexts.

- **Astringent-Type Operators**:

The study investigates mappings characterized by relaxed contractive conditions, broader than classical contractions, which allow for a more flexible and inclusive approach to fixed point analysis. The properties and behavior of such operators are examined rigorously.

- **Zippered Fixed Points:**

Introducing the concept of zippered fixed points, the study explores fixed points emerging from compositions or interdependencies of multiple operators, capturing the layered structure of many real-world nonlinear systems.

- **Existence, Uniqueness, and Iterative Approximations:**

The scope includes proving existence and uniqueness theorems for these fixed points under specified conditions, as well as analyzing iterative methods for their approximation, ensuring both theoretical and computational relevance.

- **Illustrative Examples**:

Theoretical results are supported by examples demonstrating the applicability and limitations of the proposed theorems within the considered generalized metric frameworks.

Multivalued or set-valued mappings are not considered in the current study but are suggested as future research directions. Detailed numerical simulations and applications to specific applied problems are beyond the immediate scope, though the theoretical groundwork laid here aims to facilitate such endeavors. Overall, the study aims to advance the theoretical foundations of fixed point theory by broadening its applicability to more general spaces and operators while opening pathways for future applied and computational research.

DISCUSSION

The investigation of zippered fixed point theorems for astringent-type operators in generalized metric spaces presented in this study marks a significant advancement in the landscape of fixed point theory. By introducing the novel concept of zippered fixed points, this work captures the intricate interactions arising from interdependent or compositional mappings—a scenario frequently encountered in complex nonlinear systems but inadequately addressed by classical fixed point frameworks. Our findings demonstrate that astringent-type operators, which generalize traditional contractive mappings by relaxing contraction conditions, still maintain sufficient structural control to guarantee the existence and uniqueness of fixed points when combined in a zippered manner. This generalization broadens the scope of operators amenable to fixed point analysis, thus enhancing theoretical flexibility and potential applicability.

Moreover, situating these results within generalized metric spaces—such as partial and quasi-metric spaces—further extends the utility of fixed point theorems beyond classical symmetric metric contexts. These generalized spaces reflect more realistic models where symmetry and strict triangle inequality may fail, such as in computer science, decision theory, and analysis of asymmetric relationships. The iterative schemes proposed and analyzed provide practical means for approximating zippered fixed points, with convergence guaranteed under mild assumptions. This not only enriches the theoretical framework but also opens pathways for algorithmic implementations in applied disciplines.Comparatively, the zippered fixed point theorems subsume several classical and modern fixed point results as special cases, thereby unifying diverse strands of fixed point theory. The examples provided illustrate both the breadth and the limitations of the current framework, emphasizing the importance of the defined contractive conditions and space properties. However, challenges remain. The complexity introduced by zippering mappings and generalized spaces demands careful balancing of assumptions to ensure theoretical tractability without sacrificing applicability. Future research directions identified herein aim to address these challenges, including extensions to multivalued mappings, relaxed space conditions, and computational aspects. In conclusion, this study's theoretical contributions provide a robust and versatile foundation for exploring fixed points of complex operators in generalized settings, with promising implications for both pure and applied mathematics.

CONCLUSION

This study introduced and rigorously explored the concepts of zippered fixed points and astringent-type operators within the framework of generalized metric spaces, such as partial and quasi-metric spaces. By extending traditional fixed point theory, we have addressed complex functional interactions that arise when multiple mappings are interdependently structured—captured here through the notion of "zippering." The formal definition and theoretical development of zippered fixed points and astringent-type mappings. New existence and uniqueness theorems for fixed points in generalized metric settings, significantly expanding the scope of classical results. Convergence analysis of iterative schemes tailored to approximate these fixed points, providing computational relevance to the theory. Illustrative examples and comparisons with existing fixed point principles to validate the generalizations proposed. The framework presented not only unifies and extends prior results but also opens new avenues for research in nonlinear analysis, iterative computation, and applied mathematics. By embracing more general operator behavior and space structures, this study equips mathematicians and practitioners with a more flexible toolkit to analyze fixed point phenomena in abstract and applied contexts. Future work may focus on extending these results to multivalued mappings, algorithmic implementations, and applications in mathematical modeling, optimization, and dynamic systems. Overall, this research lays a solid foundation for further exploration and cross-disciplinary integration of advanced fixed point techniques.

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