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THE LAPLACE-BELTRAMI PROBLEM IN COUPLED DOMAINS

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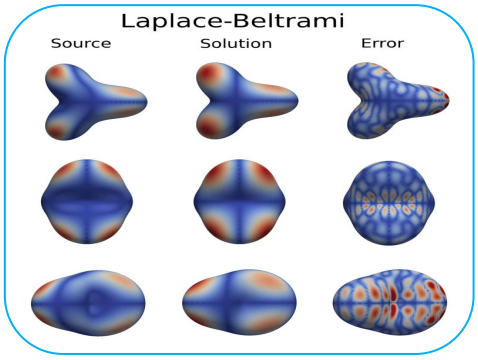
ABSTRACT :

The laplace –Beltrami problem in two coupled domains using the finite element method.the key idwa is quite similar to the laplace – beltrami problem on a single spherical surface, Which is explained in the previous but now we have two domains with the continuity of pressure and the total flux on the boundary of internal sub – domains.the coupled- sphere problem ensure that the numerical method behaves with optimal efficiency in this case with modified boundary conditions, we verify the numerical results on a model problen with a known analytical solution by checking the convergence of the numerical method.



KEY WORDS: *COUPLED DOMAINS, LAPLACE –BELTRAMI ,NUMERICAL EXAMLES.*

INTRODUCTION :

The laplace-Beltrami Dirichlet boundary –value problem to obtain surface permeability of porous matrix elements, which is one the main ingredients of the super-fast diffusion model in porous media. We have thoroughly validated and demonstrated the use of the methodology by means of numerical simulations of the laplace – Beltrami problem with the help of a specifically adopted surface finite element technique using simplified examples. We have shown,that surface permeability of a sinhle element or a small cluster of the elements of a porous matrix can be accurately calculated by the developed method.

THE PROBLEM IN TWO COUPLED DOMAINS.

We consider the problem of two particles coupled through the common boundaries we are solving the following laplace-Beltrami problem.with dirichlet boundary conditions and two continuity conditions due to the conservation of mass. Next, the weak formulation of the mathematical model will be derived for the numerical anlysis.

THE LAPLACE –BELTRAMI COUPLED BOUNDARY VALUE PROBLEM .

In this section we are going to describe the weak Formulation of the coupled laplace Beltrami boudary value problem set on the truncated spheres which will be later solved numerically by the surface finite element method using the galerkin formulation.

NUMERICAL EXAMLES:

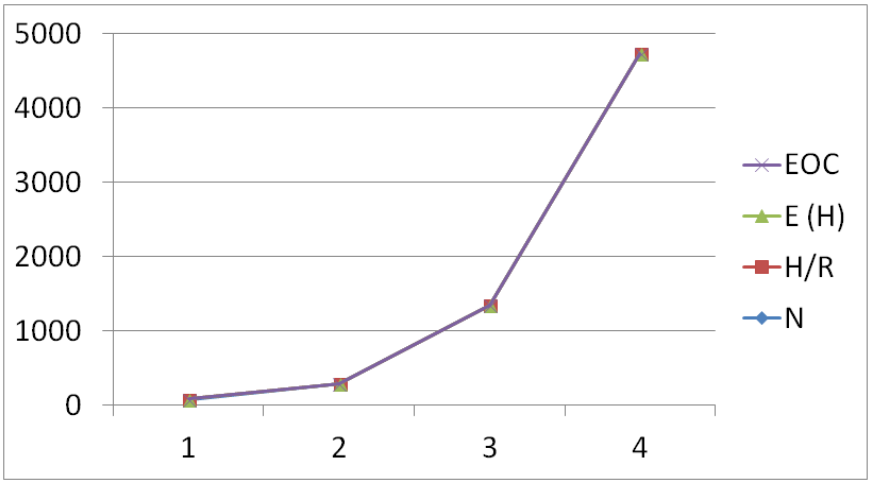
Here the surface finite element approximation to the laplace beltrami problem in two coupled domains correspponding to the equation will be presented in all examples the lenear element discretisatin has been used the experimental order of convergence is calculated.the presentation of the simulation will be split between two case one with the azimuthal symmetrical case one with the case of arbitrary oriented boundaries. The numerical examples are illustrated next topic.

THE AZIMUTHAL SYMMETRICAL CASE IN COUPLED DOMAINS:

We start our tests with a relatively low resolution of just 80 node approxima tion of gradually increasing the number of nodes to a maximum value 296992 to observe numerical convergence of the procedure. For each resolution we define the largest value of the mesh size h/r the results of non dimensional pressure are presented in order to show the convergence of the finite element approximation we also show the error measured in the H^1 semi norm formula.

MESH SIZE

I	N	H/R	E (H)	EOC
1	80	0.9078	0.0415	**-**
2	288	0.4815	0.0221	**0.8048**
3	1344	0.1996	0.0109	**0.8048**
4	4736	0.1226	0.0058	**1.2812**
5	18304	0.0611	0.0030	**0.9694**
6	74504	0.0307	0.0015	**1.0076**
7	296992	0.0154	0.0007	**1.0176**

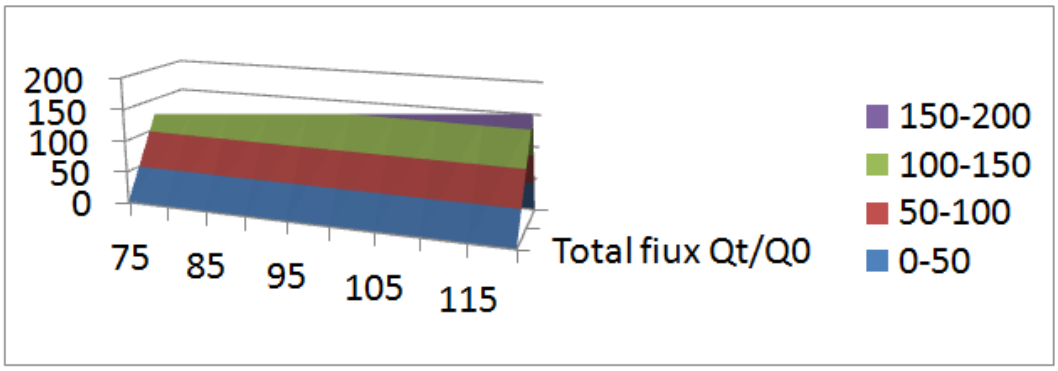


THE CASE OF ARBITRARY ORIENTED BOUNDARIES IN COUPLED DOMAINS:

One way to understsnd the coefficient of permeability in case of the two connected domain is to observe the total flux as function of the angle so the next topic will focus on the value of total flux with respect to different positions of the boundary . in this work,the boundary.are fixed while the boundary rotated around X-axis, anticlockwise for a which using the surfase finite element method to solve the system of the laplace beltrami equations on truncated spheres we are able to find out the total flux and hence the permeability of the particles.

Two couples table

Angle (alpa) deg	Total fiux Q_t/Q_0	Angle (alpa) deg	Total fiux Q_t/Q_0
75	1.5619	125	1.0881
80	1.4657	130	1.0711
85	1.3896	135	1.0561
90	1.3280	140	1.0454
95	1.2772	145	1.0343
100	1.2346	150	1.0255
105	1.1940	155	1.0167
110	1.1680	160	1.0072
115	1.1311	165	1.0024
120	1.1080	170	1.0022



Tabilated values of the non dimensional liquid total flux Qt/Q0 in case of arbitrary oriented boundaries for two coupled domains with different tilt angle alpa the value of the non dimensional capillary pressure on the boundaries are fixed at alpa 1/U0=0.8 and alpa 2/u0 =0.2 on $DR^1_{1,2.}$ The

Data are from triangulation level 7 (the maximum mesh size h/r =0.018) here q0 is the normalized total flux in the azimuthally symmetric case u0=2y/R cos 0c is the cotact angle is the surface tension.from 135 degree to 75 degree at 170 degree interval.

If we look at the data on the table we can see the value non dimensional liquid total flux Qt / Q0 Drops that result in the increasing the tilt angle alpa and the boundary contours at the connected domains move away from each other. In order to understand the overall trend in data , we now show the dependence of the total flux and hence the permeability.as a function of the tyilt angle alpa in case of two couple domains the graph is already illustrated.

CONCLUSION:

We have demonstrated how the coefficeint of permeability of a few coupled element of the porous matrix can be evaluated on the basis of the solution of the laplace beltrami equation in the subsequent coupled domains the idea is similar with the one used to obtain permeability of a single element but with an addition of two coupling boundary conditions. Namely the continuity of pressure and the total flux on the boundary conditions.namely the continuity of pressure and the total fiux on the boundary of the internal sub domains in azimuthally symmetric case we were able to estimate the value of the errors.we have shown that the error converges with diminishing of the triangulation size the optimal convergence rate observed implies that our method is efficient for solving the leplace beltami rquations with the new set of the boundary conditions.

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