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MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEM IN FUZZY ENVIRONMENT: A COMPARATIVE ANALYSIS OF SOLUTION METHODS

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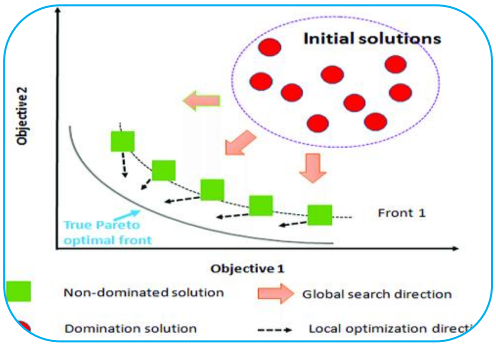
ABSTRACT

This paper aims to explore various solution methods for multi-objective linear programming (MOLP) problems within fuzzy environment. Fuzzy environment introduce uncertainty and imprecision, which are often represented by fuzzy sets and fuzzy numbers. Here, in this paper we are taking triangular fuzzy numbers into consideration and will compare different approaches of solving MOLP problems, including Zimmermann's Approach, Weighted Sum Approach, Fuzzy Goal Programming Approach and fuzzy decisive set approach.



KEYWORDS: *Multi-objective optimization, aspiration levels, expected value of fuzzy numbers, fuzzy goal programming.*

1. INTRODUCTION

Multi-objective programming problems (MOPPs) involve the simultaneous optimization of multiple conflicting objectives, reflecting the complexity and diversity of real-world decision-making scenarios. Traditional multi-objective programming seeks to find a set of solutions that best balances these competing goals. However, in many practical situations, the data and parameters involved are not precise but rather uncertain or vague. This is where fuzzy environments come into play. Fuzzy environments are characterized by the use of fuzzy set theory to model uncertainty and imprecision. Unlike traditional crisp sets where an element either belongs to a set or does not, fuzzy sets allow for partial membership, which is quantified by a membership function. This capability to handle partial belongingness, makes fuzzy set theory particularly suitable for modeling real-world problems where data is ambiguous, incomplete, or inherently imprecise.

In the context of multi-objective programming, fuzzy environments allow for the representation of fuzzy objectives, fuzzy constraints, and fuzzy coefficients. This means that instead of having fixed target values or constraints, these elements can be described by fuzzy sets, providing a more flexible and realistic framework for decision-making. The integration of fuzzy set theory into multi-objective programming has led to the development of various methods designed to solve these complex problems. Techniques such as fuzzy goal programming, fuzzy weighted sum methods, and fuzzy set decisive approach have been developed to find optimal or compromise solutions in fuzzy environments.

The study of multi-objective programming problems in fuzzy environments is crucial for fields such as engineering, economics, environmental management, and logistics, where decisions must often be made under uncertainty. By leveraging fuzzy set theory, decision-makers can better capture the challenges of real-world problems, leading to solutions that are not only mathematically sound but also practically viable.

This paper aims to explore the various methods and approaches developed to solve multi-objective programming problems in fuzzy environments, providing a comprehensive overview in this regard. The paper will also highlight the advantages and limitations of these methods, by giving a numerical illustration.

2. LITERATURE REVIEW

Multi-objective programming is a significant area in decision making under uncertainty, particularly when dealing with fuzzy environments where imprecise information and ambiguity prevails. The idea of fuzzy set theory was first proposed by Zadeh [19] which proves to be very efficient for dealing vagueness of data. Later, Bellman and Zadeh [1] together introduced decision-making in fuzzy environment. The integration of fuzzy set theory with multi-objective programming was introduced by Zimmermann [21] in 1978, who developed fuzzy decision-making frameworks to handle vague and imprecise information. Since then researchers have developed various methodologies to address MOLP problems in fuzzy environment.

In 1984, Tanaka and Asai [15] proposed their work on fuzzy linear programming problem with fuzzy numbers. They have treated all the parameters defining the problem as triangular fuzzy numbers. Slowinski [14] and M. Sakawa [13] proposed an interactive decision making approach for solving a MOLP with fuzzy parameters and its application in the field of water supply planning management. Sakawa applied this approach of interactive decision-making to solve multi-objective linear fractional programming problem within fuzzy environment. Interactive decision making approach involves decision maker in the optimization process by interactively adjusting membership functions and parameters based on their preferences and judgements. A new approach for solving MOLP problems were given by Mohanty and Vijayaraghavan [12], where they solved the problem by transforming it into its equivalent goal programming problem with appropriate priorities and aspiration levels. H. Kuwano [7] also worked in this direction and solved fuzzy MOLP problem through goal programming approach. While dealing with fuzzy parameters, most often the calculations become tedious and lenthy, so to overcome this challenge, Heilpern [5] proposed the concept of expected value of fuzzy numbers. Through this concept the fuzzy numbers are transformed into its equivalent crisp form which are easy to handle and claims better solutions.

The research progresses and many researchers came up with different approaches for solving MOLP problems or linear programming problems such as by defining linear membership function [3], [17], and generalized fuzzy goal optimization [8] etc. Cohon [2] published a book on multi-objective programming where he discusses about the various aspects of MOLPPs and its solution methods. Jimenaz [6] and Wu et.al [18] both proposed the pareto optimal solution for MOLP problem but Wu made it with fuzzy goals. Pareto optimal solution or efficient solution is the compromise solution for the MOLP problem, since the achievement of optimal solution in such cases are not possible. Different authors have suggested different concepts to solve MOLP problem and discusses its application in various fields like agriculture, supply chain etc. [4], [16].

Recently, Mishra and Singh [10], [11] proposed a new method of solving fully fuzzy MOLP problem using its equivalent goal programming problem and also gave its application in choosing optimal land allocation in the field of agricultural production. They have also proposed a linear fractional programming procedure for MOLP problem in agriculture sector where the objectives are in ratio form, is scalarized and converted to MOLP using a novel approach [9].

3. PRELIMINARIES

3.1 Definition: Fuzzy set

Let X be a universal set, then $\tilde{A}$ is called fuzzy set if it is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associate each element x in X to a real number from [0 1].
This function is known as the membership function.

3.2 Definition: Fuzzy Number

A fuzzy set $\tilde{A}$ in R is said to be a fuzzy number, if it satisfies the following conditions

- $\tilde{A}$ is normal.
- $\tilde{A}_\alpha$ is a closed interval for every $\alpha \in [0\ 1]$.
- The support of $\tilde{A}$ is bounded.

3.3 Definition: Triangular Fuzzy Number

A fuzzy number $\tilde{A}$ is called triangular fuzzy number (TFN), if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < b, x > c \\ \dfrac{x-b}{a-b} & b \le x \le c \\ \dfrac{c-x}{c-a} & a < x \le c \end{cases}$$

Where the TFN, $\tilde{A}$ is denoted by the triplet (b, a, c).

4. Fuzzy Multi-objective Linear Programming Problem

Fuzzy multi-objective linear programming (FMOLP) is an extension of multi-objective linear programming that incorporates uncertainty and imprecision in the objectives and constraints. In a typical multi-objective programming problem, we aim to optimize multiple, often conflicting, objectives simultaneously. The fuzzy multi-objective programming problem can be formulated as follows

Let $x = (x_1, x_2, \dots, x_n)$ be the vector of decision variables. Consider m fuzzy objectives represented by fuzzy sets. The mathematical formulation of an FMOLP is as follows

$$Maximize\ \tilde{F}(x) = (\tilde{f_1}(x), \tilde{f_2}(x), \dots, \tilde{f_m}(x))$$

Subject to $\quad g_i(x)\ \{\le, \ge, \approx\}\ \tilde{b_i} \quad$ for $i = 1,2, \dots, p$

To solve an FMOLP, techniques such as fuzzy goal programming, fuzzy weighted sum approach and Zimmermann's approach may be employed. These methods help in determining solutions that are acceptable under the fuzzy constraints and provide a balance among the conflicting fuzzy objectives. Fuzzy set decisive approach is considered by Veeramani [17] which is an iterative method. Here we considered the numerical example taken by him and try to solve it through the above stated methods. We claim that the solution obtained through the above stated methodologies is better in comparison with Veeramani approach.

5. Techniques for solving the FMOLP

METHOD-1: Zimmermann's Approach

Zimmermann's approach to solving Fuzzy Multi-Objective Linear Programming (FMOLP) problems is a prominent method that addresses the challenge of optimizing multiple fuzzy objectives under fuzzy constraints. This approach is particularly useful in situations where the objectives and constraints are not precisely known but are instead expressed using fuzzy sets. Zimmermann introduces the concept of membership functions to quantify the degree of satisfaction of the fuzzy objectives and constraints. Let $\tilde{F}_k^{max}$ and $\tilde{F}_k^{min}$ be the maximum possible value and minimum possible value respectively of the kth objective function then, the linear membership function is formulated as below

$$\mu_k(\tilde{F}_k) = \begin{cases} 1 \ \tilde{F}_k \geq \tilde{F}_k^{min} \\ (\tilde{F}_k - \tilde{F}_k^{min})/(\tilde{F}_k^{max} - \tilde{F}_k^{min}) \ for \ \tilde{F}_k^{min} \leq \tilde{F}_k \leq \tilde{F}_k^{max} \\ 0 \ for \ \tilde{F}_k \leq \tilde{F}_k^{min} \end{cases}$$

$$\text{for } k = 1,2,\dots,m$$

Then the Zimmermann's max-min approach states that the fuzzy multi-objective linear programming problem may transformed into its equivalent linear programming problem as

$$\max \lambda$$

$$\text{Such that } \lambda \leq \mu_k(\tilde{F}_k) \text{ for } k = 1,2,\dots,m$$

$$\text{And} \qquad g_i(x)\ \{\leq, \geq, \approx\}\ \tilde{b}_i \qquad \text{for } i = 1,2,\dots,p$$

$$x \geq 0$$

Zimmermann's approach provides a systematic way to handle fuzzy multi-objective linear programming problems by transforming fuzzy sets into quantifiable membership functions and aggregating these into a composite objective function. This method allows for an effective optimization process by balancing multiple fuzzy objectives while ensuring that fuzzy constraints are satisfied to a desired degree.

Method-2: Weighted Sum Approach

The Weighted Sum Method is a popular approach for solving Fuzzy Multi-Objective Linear Programming (FMOLP) problems. This method simplifies the problem of optimizing multiple fuzzy objectives by aggregating them into a single objective function through a weighted sum. It is particularly useful when dealing with problems where the objectives are of different priorities and constraints are expressed in fuzzy terms.

To combine multiple fuzzy objectives into a single linear objective, the weighted sum method assigns a weight w_k to each objective $\tilde{F}_k$. The composite objective function is given as

$$Max\ Z = \sum_{k=1}^{m} w_k\, \tilde{F}_k$$

Subject to $\quad g_i(x)\ \{\leq, \geq, \approx\}\ \tilde{b}_i \qquad$ for $i = 1,2,\dots,p$

The Weighted Sum Method is a straightforward and effective technique for solving FMOLP problems. It simplifies the problem by aggregating multiple fuzzy objectives into a single objective function using weighted sums, allowing the decision maker to have different priority objectives.

Method-3: Fuzzy Goal Programming Approach

The Fuzzy Goal Programming (FGP) approach is a method for solving Fuzzy Multi-Objective Linear Programming Problems (FMOLPPs) that extends traditional goal programming to handle fuzzy goals and constraints. It aims to find a solution that satisfies the fuzzy goals as closely as possible while meeting fuzzy constraints.

For solving FMOLP problems, it is first change to goal programming by solving objective separately, considering each as single LPP. By doing this we get an aspiration level for every goals. Then by using achievement function, the fuzzy goal programming approach can be mathematically formulated as follows

$$Min \sum_{k=1}^{m} (d_k^- + d_k^+)$$

$$\text{Subject to constraints} \quad \tilde{F}_k + d_k^- - d_k^+ = \theta_k \qquad \text{for } k = 1,2,\dots,m$$

$$\text{And} \quad g_i(x)\ \{\le,\ge,\approx\}\ \tilde{b_i} \ \text{ for } i = 1,2,\dots,p$$

Where d_k^+ and d_k^- is the positive and negative deviations from the kth goal. For a fuzzy goal, $\tilde{F}_k$, d_k^+ represents how much the goal is exceeded and d_k^- represents how much the goal is underachieved. θ_k is the acceptable level of satisfaction for each goal.

The Fuzzy Goal Programming approach provides a structured way to handle FMOLPPs by translating fuzzy goals into quantifiable deviations and minimizing these deviations while satisfying fuzzy constraints. This method allows for a flexible and realistic optimization process, reflecting the inherent uncertainties and imprecision in real-world decision-making problems.

6. $\beta - feasibility$ of constraint set

In FMOLPPs, β-feasibility is a concept used to handle the feasibility of constraints when dealing with fuzzy constraints. β-feasibility provides a way to measure and ensure that a solution meets the constraints to an acceptable degree, given that these constraints are expressed in fuzzy terms. β-feasibility measures the degree to which a solution satisfies the fuzzy constraints, given a certain threshold $\beta \in [0\ 1]$.

The decision vector $x = (x_1, x_2, \dots, x_n)$ is said to be β –feasible for the given problem, if x verifies constraints at least in a degree β. The mathematical expression is given as

$$g_i(x) \le_\beta \tilde{b_i} \qquad\qquad \text{for } i = 1,2,\dots,p$$

Where $\le_\beta$ is defined as

$$\Omega_\beta = \begin{cases} g_i^R - \beta(g_i^R - g_i^L)x \le b_i^L + \beta(b_i^R - b_i^L) \\ x \ge 0 \end{cases} \qquad \text{for } i = 1,2,\dots,p$$

Where Ω_β is the set of all β- feasible decision vectors.

This method provides a way to ensure that solutions adhere to acceptable levels of satisfaction for all fuzzy constraints. This approach allows for practical handling of fuzzy constraints by translating them into measurable and manageable criteria.

7. Numerical Illustration

For numerical illustration, we have taken the problem from the paper proposed by Veeramani [17], where they have applied the set decisive approach to solve the below FMOLP problem.

$$\text{Max } Z_1(x) = 10x_1 + 11x_2 + 15x_3 \qquad\qquad \text{P(1)}$$

$$\text{Max } Z_2(x) = 4x_1 + 5x_2 + 9x_3$$

s.t.

$$\tilde{1}x_1 + \tilde{1}x_2 + \tilde{1}x_3 \leq \widetilde{15}$$

$$\tilde{7}x_1 + \tilde{5}x_2 + \tilde{3}x_3 \leq \widetilde{80}$$

$$\tilde{3}x_1 + \widetilde{4.4}x_2 + \widetilde{10}x_3 \leq \widetilde{100}$$

The fuzzy parameters are defined as

$\tilde{1} = (0,1,2)$ $\widetilde{15} = (10, 15, 20)$ $\tilde{7} = (3, 7, 11)$ $\tilde{5} = (2, 5, 8)$ $\tilde{3} = (2, 3, 4)$

$\widetilde{80} = (40, 80, 120)$ $\widetilde{4.4} = (2.4\ 4.4\ 6.4)$ $\widetilde{10} = (6, 10, 14)$

$\widetilde{100} = (70, 100, 130)$

Now, for constraint set we are taking $\beta -$ feasibility, and for numerical illustration here we are setting $\beta = 0.5$ we get,

$$\text{Max } Z_1(x) = 10x_1 + 11x_2 + 15x_3 \qquad \text{P(2)}$$

$$\text{Max } Z_2(x) = 4x_1 + 5x_2 + 9x_3$$

s.t.

$$(0 + \beta)x_1 + (0 + \beta)x_2 + (0 + \beta)x_3 \leq 20 - 5\beta$$

$$(3 + 4\beta)x_1 + (2 + 3\beta)x_2 + (2 + \beta)x_3 \leq 120 - 40\beta$$

$$(2 + \beta)x_1 + (2.4 + 2\beta)x_2 + (6 + 4\beta)x_3 \leq 130 - 30\beta$$

Now, we have solved the above MOPP by taking one objective at a time

For $\beta = 0.5$

We have,

$$\text{Max } Z_1(x) = 10x_1 + 11x_2 + 15x_3 \qquad \text{P(3)}$$

$$\text{Max } Z_2(x) = 4x_1 + 5x_2 + 9x_3$$

s.t.

$$0.5x_1 + 0.5x_2 + 0.5x_3 \leq 17.5$$

$$5x_1 + 3.5x_2 + 2.5x_3 \leq 100$$

$$2.5x_1 + 3.4x_2 + 8x_3 \leq 115$$

$$x_1, x_2, x_3 \geq 0$$

By Using MATLAB (online free version) and we get solution for first objective as,

$$x_1 = 0, x_2 = 26.2821, \; x_3 = 3.2051, \; \text{Max } Z_1(x) = 337.1795.$$

Similarly, solve for second objective and we get,

$$x_1 = 0, x_2 = 26.2821, \; x_3 = 3.2051, \text{Max } Z_2(x) = 160.2564.$$

Method 1: Zimmermann's Approach

Now, we have considered the obtained maximum values of the objective considering only one objective at a time as the parameter with some flexibility i.e. Max Z_1 as fuzzy parameter, So, define the membership function of objective Z_1 by setting lower limit to 300,
we get,

$$\mu_{z_1(x)} = \begin{cases} 0 & ; z_1 \leq 300 \\ \frac{z_1 - 300}{37.18} & ; 300 \leq z_1 \leq 337.18 \\ 1 & ; z_1 \geq 337.18 \end{cases}$$

Similarly for objective Z_1 by setting lower limit to 150, we can construct the membership function as

$$\mu_{z_2(x)} = \begin{cases} 0 & ; z_2 \leq 150 \\ \frac{z_2 - 150}{10.26} & ; 150 \leq z_2 \leq 160.26 \\ 1 & ; z_2 \geq 160.26 \end{cases}$$

Then by using the Zimmermann's approach for compromise solution, we have

$$\text{Max } \lambda \qquad \text{P(4)}$$

s.t.

$$\lambda \leq \frac{Z_1 - 300}{37.18}$$

$$\lambda \leq \frac{Z_2 - 150}{10.26}$$

$$0.5x_1 + 0.5x_2 + 0.5x_3 \leq 17.5$$

$$5x_1 + 3.5x_2 + 2.5x_3 \leq 100$$

$$2.5x_1 + 3.4x_2 + 8x_3 \leq 115$$

$$x_1, x_2, x_3 \geq 0$$

We get the equivalent LPP of MOPP as,

$$\text{Max } \lambda \qquad \text{P(5)}$$

s.t.

$$-0.27x_1 - 0.3x_2 - 0.4x_3 + \lambda \leq -8.07$$

$$-0.4x_1 - 0.5x_2 - 0.88x_3 + \lambda \leq -14.62$$

$$0.5x_1 + 0.5x_2 + 0.5x_3 \leq 17.5$$

$$5x_1 + 3.5x_2 + 2.5x_3 \leq 100$$

$$2.5x_1 + 3.4x_2 + 8x_3 \leq 115$$

$$x_1 , x_2, x_3, \lambda \geq 0$$

Solving the above we obtained the solution as,

$$x_1 = 0, x_2 = 26.2821, \ x_3 = 3.2051 , \ \lambda = 1$$

For this solution,

$$\text{Max } Z_1(x) = 337.18, \ \text{Max } Z_2(x) = 160.26$$

Method 2: Weighted Sum Method

Here also we have taken $\beta -$ feasibility of the constrained set, and for the numerical solution we set $\beta = 0.5$.

Let Z_1 and Z_2 be two objectives then by using weighted sum method the objective of the MOPP becomes

$$Z(x) = \sum_{i=1}^{2} w_i Z_i = w_1 Z_1 + w_2 Z_2$$

As the two objectives are of same nature, we can take $w_1 = w_2 = 0.5$

So, the MOP problem reduces to,

$$\text{Max } 7x_1 + 8x_2 + 12x_3 \qquad \text{P(6)}$$

s.t.

$$0.5x_1 + 0.5x_2 + 0.5x_3 \leq 17.5$$

$$5x_1 + 3.5x_2 + 2.5x_3 \leq 100$$

$$2.5x_1 + 3.4x_2 + 8x_3 \leq 115$$

$$x_1 , x_2, x_3 \geq 0$$

By solving the above LPP, we get solution as,

$$x_1 = 0, x_2 = 26.2851, \ x_3 = 3.2051$$

$$\text{Max } Z_1(x) = 337.18, \ \text{Max } Z_2(x) = 160.26$$

Method 3: Goal Programming method

Here also we have taken $\beta -$ feasibility of the constrained set, and for the numerical solution we set $\beta = 0.5$.

As both the objectives are of maximization type, we can set the aspiration level for the objectives by considering one objective at a time and solving the MOPP as LPP, we get
two LPP's

$$\text{Max } Z_1(x) = 10x_1 + 11x_2 + 15x_3 \qquad P(7)$$

$$\text{s.t.} \quad 0.5x_1 + 0.5x_2 + 0.5x_3 \le 17.5$$

$$5x_1 + 3.5x_2 + 2.5x_3 \le 100$$

$$2.5x_1 + 3.4x_2 + 8x_3 \le 115$$

$$x_1, x_2, x_3 \ge 0$$

$$\text{Max } Z_2(x) = 4x_1 + 5x_2 + 9x_3 \qquad P(8)$$

$$\text{s.t.} \quad 0.5x_1 + 0.5x_2 + 0.5x_3 \le 17.5$$

$$5x_1 + 3.5x_2 + 2.5x_3 \le 100$$

$$2.5x_1 + 3.4x_2 + 8x_3 \le 115$$

$$x_1, x_2, x_3 \ge 0$$

We get the aspiration level of both the objectives as,

$$\text{Max } Z_1 = 337.1795, \ \text{Max } Z_2 = 160.2564$$

To find the compromise solution, the goal Programming technique has been used.
The achievement function is formulated as,

$$\text{Min } d_1^- - d_1^+ + d_2^- - d_2^+ \qquad P(9)$$

s.t.

$$10x_1 + 11x_2 + 15x_3 + d_1^- - d_1^+ = 337.1795$$

$$4x_1 + 5x_2 + 9x_3 + d_2^- - d_2^+ = 160.2564$$

$$0.5x_1 + 0.5x_2 + 0.5x_3 \le 17.5$$

$$5x_1 + 3.5x_2 + 2.5x_3 \leq 100$$

$$2.5x_1 + 3.4x_2 + 8x_3 \leq 115$$

$$x_1 , x_2, x_3 \geq 0$$

By solving above on MATLAB (free online version), we get

$$x_1 = 0, x_2 = 26.2821, \ \ x_3 = 3.2051,$$

$$d_1^- = d_1^+ = 101.488, d_2^- = d_2^+ = 105.76$$

For this solution

$$Z_1 = \ 337.1795 \ \text{ and } \ Z_2 = \ 160.2564$$

Table-1: Comparison of solution obtained by different approach

Objectives	Zimmermann's Approach	Weighted Sum Approach	Fuzzy Goal Programming Approach	Fuzzy Set Decisive Approach
Z_1	337.1795	337.1795	337.1795	139.1
Z_2	160.2564	160.2564	160.2564	80.12

8. CONCLUSION

In the comparative study of solving techniques for Fuzzy Multi-Objective Linear Programming Problems (FMOLPPs), several approaches stand out for their distinct methodologies and applications. Here, we summarize and compare four prominent techniques: Zimmermann's approach, the Weighted Sum Method, the Fuzzy Goal Programming (FGP) approach, and the Fuzzy Set Decisive Approach.

Zimmermann's approach simplifies the problem by aggregating multiple fuzzy objectives into a single function. It is particularly effective when a clear aggregation of objectives is possible and when the degree of satisfaction for each objective can be precisely quantified. However, it might be less effective in highly complex or non-linear fuzzy environments.

The Weighted Sum Method transforms the problem of optimizing multiple fuzzy objectives into a single-objective problem by using a weighted sum of membership functions. It excels in scenarios where objectives can be linearly aggregated and when relative importance of objectives is well understood.

This technique focuses on minimizing deviations from fuzzy goals, providing a flexible framework to handle multiple fuzzy objectives and constraints. FGP is particularly useful in cases where a balance between goal satisfaction and constraint compliance is crucial.

The Fuzzy Set Decisive Approach employs fuzzy set theory to evaluate the feasibility of solutions based on their membership values in relation to the fuzzy constraints. This method might prove to be lengthy and can takes number of iterations.

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