REVIEW OF RESEARCH
IMPACT FACTOR : 5.7631(UIF) UGC APPROVED JOURNAL NO. 48514 ISSN: 2249-894X

VOLUME - 8 | ISSUE - 1 | OCTOBER - 2018

THE ARL AND SDRL PERFORMANCE OF DISTRIBUTION-FREE CONTROL CHART FOR MONITORING PROCESS LOCATION

D. M. Zombade
Department of Statistics , Walchand College of Arts and Science, Solapur (MS), India.

ABSTRACT

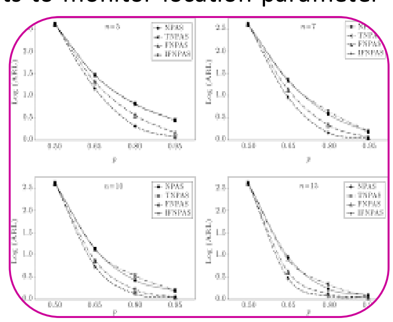
In this paper, the performance of Distribution-free control chart based on run statistic and sign statistic is compared with traditional $\overline{X}$ *chart in order to monitor the location. The proposed chart is distribution-free so that its application does not require the assumption of parametric model. The performance of the proposed chart is evaluated by simulating data from normal and non-normal distributions and compared with the traditional* $\overline{X}$ *chart. The ARL and SDRL performance indicating that proposed distribution-free charts are strong alternative for traditional* $\overline{X}$ *chart in terms of detecting shifts in process location under normal and non-normal process distributions.*

KEYWORDS: *ARL, SDRL, Sign test, runs test.*

1. INTRODUCTION

Most of the control charts are based on the assumption that the underlying distribution of the process is normal. In reality this assumption may not hold in all the situations. Therefore, it is necessary to suggest nonparametric control charts for monitoring processes which do not depend on the assumption of normality. Shewhart-type control charts for process location are widely used for purposes of determining whether a process is in control, for bringing an out-of-control process into control, and for monitoring a process to make sure that it stays in control. Control charts are used to monitor processes that manufacture products with a single quality characteristic of interest

The location parameter often monitored in distribution-free or nonparametric control charts. The problem of monitoring the location of a process is important in many applications. The location parameter could be the mean or the median or some percentiles of the distribution. In literature, several distribution-free or nonparametric control charts are proposed for monitoring location parameter of a univariate process. Amin et al. (1995) developed Shewhart and CUSUM control charts based on sign test statistic. Chakraborti et al. (2001) presented an extension overview of the literature on univariate nonparametric control charts. Many authors have developed nonparametric control charts to monitor location parameter of the process. Bakir (2004) developed a nonparametric Shewhart control chart for monitoring process center based on the signed-ranks of grouped observations. Chakraborti and Van de Wiel (2008) developed the control chart based on Mann-Whitney statistic for detecting location shifts. Das (2009) presented a comparison study of three non-parametric control charts to detect shift in location parameters. Khilare and Shirke (2010) developed a nonparametric synthetic control chart based on sign statistic to monitor shifts in process location. Yang and Cheng (2011) have proposed a nonparametric CUSUM chart to monitor the possible



small shifts in the process mean. Liu et al. (2015) proposed a sequential rank based nonparametric CUSUM control chart for detecting arbitrary magnitude of shifts in the location parameter. Zombade and Ghute (2018) presented Shewhart-type control charts for process location.

The purpose of this paper is to develop a distribution-free or nonparametric control chart for monitoring the location of a symmetric process. If underlying process distribution is non-normal, then we consider nonparametric control chart based on appropriate nonparametric test. Many nonparametric tests like sign, signed-rank has been proposed in the literature. In this paper, we use the run test statistic as a charting statistic for detecting location shifts of a symmetric process. The proposed distribution-free or nonparametric chart for monitoring the process location is based on runs computed within samples.

2. CONTROL CHART BASED ON SIGN TEST STATISTIC [NP-S]

Let $X_i = (X_{i1}, X_{i2}, \ldots, X_{in})$ be a sample taken at i[th] time point. The distribution of observations will assume to be continuous with location θ. These observations represent measurements of process characteristic. Let θ_0 and σ_0 be the desired process center and the process standard deviation respectively. Without loss of generality, we assume that $\theta_0 = 0$ and $\sigma_0 = 1.$ we are interested in detecting shifts in θ. We are interested in detecting shifts in θ. Now to assess whether process is in-control with respect to θ is equivalent to testing the null hypothesis,

$$H_0 : \theta = 0 \text{ against } H_1 : \theta \neq 0. \tag{2.1}$$

When the distribution of observations is normal, Shewhart $\overline{X}$ chart is most widely used to detect shifts in process location. The Shewhart $\overline{X}$ chart is based on sample means $\overline{X}_1, \overline{X}_2, \ldots$, where $\overline{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{ij}$ (Montgomery 2005). This chart signals that θ has shifted from θ_0 at first i for which $\overline{X}_i$ falls outside $\theta_0 \pm \frac{k\sigma}{\sqrt{n}}$, where k is a constant chosen to achieve a specified in-control ARL. When an upper control limit only is used, the ARL of the $\overline{X}$ chart in positive direction is

$$ARL(\theta) = \frac{1}{P\left(\overline{X} \geq \theta_0 + \frac{k\sigma}{\sqrt{n}} \mid \theta\right)} \tag{2.2}$$

Procedure based on sign test statistic require that, at any time t, each observation from the sample be compared with target value θ_0 and number of observations above and below θ_0 be recorded for each sample.

Define,

$$\text{Sign}(X_{ij}) = \begin{cases} 1, & \text{if } X_{ij} > \theta_0 \\ 0, & \text{if } X_{ij} = \theta_0 \\ -1, & \text{if } X_{ij} < \theta_0 \end{cases} \tag{2.3}$$

where X_{ij} is the j[th] observation in the i[th] sample.

Let $S_i = \sum_{j=1}^{n} \text{Sign}(X_{ij} - \theta_0), i = 1, 2, 3, \ldots$ where S_i is the just difference between the number of observations above θ_0 and number of observations below θ_0 in the i[th] sample. A random variable $T_i = \frac{S_i + n}{2}$ gives the number of positive signs in the sample of size n and has binomial distribution with parameters *n* and $p = P[X_{ij} > \theta_0]$, when $p = 0.5$ in the in-control case and θ_0 is in-control location of process.

For sign control chart, $\text{ARL}(\theta)$ for two-sided control chart is

$$\text{ARL}(\theta) = \frac{1}{P}$$

where P is the detecting power of sign chart,

$$P = P\,[|S_i| > \text{UCL} \mid \theta]$$

The ARL for one-sided control limit in the positive direction is,

$$\text{ARL}(\theta) = \frac{1}{P(S_i \geq \text{UCL} \mid \theta)} \quad (2.4)$$

We consider S as the control statistic for the nonparametric control chart for monitoring process location and the chart is referred as nonparametric S (denoted NP-S) chart. The chart operates by plotting S values on the chart with *UCL*. When the plotted point lies above the *UCL,* it indicates that there has been a change in location θ, and the process is considered to be out-of-control.

3. CONTROL CHART BASED ON RUN TEST STATISTIC [NP-R]

In this Section, distribution-free or nonparametric control chart based on run statistic is proposed for monitoring the location of a continuous symmetric process distribution. The proposed chart is distribution-free or nonparametric so that its application does not require the assumption of parametric model (such as normality).

A run is defined as a succession of two or more identical symbols which are followed and preceded by different symbols or no symbol at all. At each inspection point, a nonparametric run statistic is computed using *n* observations $X_1, X_2, \ldots, X_n$.

Define $$\eta_j = S(X_{Dj}) = \begin{cases} 1, & \text{if } X_{Dj} > 0, \; j = 1, 2, 3, \ldots, n \\ 0, & \text{otherwise} \end{cases}, \quad (3.1)$$

where D_j is the antirank of $|X|_{(j)}$. Hence D_j labels the X which corresponds to the j^{th} order absolute value.

Define $I_1 = 1$

$$I_j = \begin{cases} 1, & \text{if } \eta_{j-1} \neq \eta_j, \ j = 2,3,....,n. \\ 0, & \text{if } \eta_{j-1} \neq \eta_j \end{cases} \tag{3.2}$$

Define the partial sums,

$$r_i = \sum_{j=1}^{i} I_j \ , i = 1,2,....,n. \tag{3.3}$$

Naturally $r_i \leq r_j$ for $i < j$ and r_n is the total number of runs in the sequence. Test statistic based on runs (Corzo (1989)) is given as

$$R = \frac{1}{r_n} \sum_{j=1}^{n} \delta_j r_j \tag{3.4}$$

where

$$\delta_j = \begin{cases} 1, & \text{if } \eta_j = 1, \\ & \qquad j = 1, 2, 3, ..., n. \\ -1, & \text{if } \eta_j = 0 \end{cases} \tag{3.5}$$

Note that R includes the number of runs until every element of the dichotomized succession, increasing their value when $\eta_j = 1 \ (\delta_j = 1, \text{runs of ones})$ and decreasing when $\eta_j = 0 \ (\delta_j = -1, \text{runs of zeros})$ the large value of R indicate greater number of runs of ones and it is an indication that $\theta > 0$. Additionally the inverse of total number of runs $\frac{1}{r_n}$ is used as a factor of standardization.

We consider R as the control statistic for the proposed distribution-free or nonparametric control chart for monitoring process location and the chart is referred as nonparametric R (denoted NP-R) chart. The chart operates by plotting R values on the chart with *UCL*. When the plotted point lies above the *UCL,* it indicates that there has been a change in location θ, and the process is considered to be out-of-control.

4. PERFORMANCE COMPARISONS

The performance of a control chart is measured in terms of the run length distribution. As the run length distribution is skewed to right, the various summary measures such as mean, standard deviation and the quartiles are considered to characterize the distribution. In this Section, performance of the proposed NP-R chart is compared with NP-S chart and the traditional $\overline{X}$ chart. We determined ARL values by simulation when process was operating under normal, double exponential and uniform with mean zero and variance one. The uniform distribution is considered as process distribution to see the effect of a light tailed distribution and double exponential distribution is considered to see the effect of heavy tailed distribution on the performance of the NP-R chart. Equation (3.6), (3.7) and (3.8) respectively gives probability density functions of normal distribution with location θ and scale σ, uniform distribution with location θ and scale λ and double exponential distribution with location θ and scale λ.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(\frac{x-\theta}{\sigma}\right)^2, \ -\infty < x < \infty \text{ and } \sigma > 0 \tag{3.6}$$

$$f(x) = \frac{1}{2\lambda}, \ \theta - \lambda < x < \theta + \lambda \text{ and } \lambda > 0 \tag{3.7}$$

$$f(x) = \frac{1}{2\lambda} \exp\left(\frac{|x-\theta|}{\lambda}\right), \ -\infty < x < \infty \text{ and } \lambda > 0 \tag{3.8}$$

To achieve standard deviation of 1, we choose $\sigma = 1$ for normal distribution, $\lambda = \sqrt{3}$ for uniform distribution and $\lambda = \frac{1}{\sqrt{2}}$ for double exponential distribution.

Consider a process where quality characteristic of interest X is distributed with location θ and standard deviation σ. Let θ_0 and σ_0 be the in-control values of θ and σ respectively. When a shift in process location occurs, we have change from the in-control value θ_0 to the out-of-control value $\theta_1 = \theta_0 + \delta, (\delta > 0)$. Therefore, when control chart for location is employed, the process shifts are measured through $\delta = \frac{|\theta_1 - \theta_0|}{\sigma_0}$, where θ_1 is location and θ_0 is in-control location. When $\delta = 0$, the process is in-control. Computer programs written in C language are used to study the performance of the charts under study. The in-control and out-of-control ARL and SDRL values of the $\overline{X}$, NP-S and NP-R charts are computed using 10000 simulations for sample size of *n* = 5 and 10.

Table 1 to Table 3 provide the ARL and SDRL values of the $\overline{X}$ chart, NP-S chart and proposed NP-R chart with sample sizes *n* =5 and 10 when the underlying process data actually follows normal, double exponential and uniform distributions respectively. It is obvious that both ARL (SDRL) decrease as δ increase. This indicates that larger shifts can be detected quicker and will result in smaller spread in the run length distribution.

Table 1. ARL (SDRL) comparison of various charts under normal distribution.

Shift δ	n = 5			n =10		
	$\overline{X}$ UCL=0.83	NP-S UCL=5.0	NP-R UCL=5.0	$\overline{X}$ UCL=0.9794	NP-S UCL=10.0	NP-R UCL=10.0
0.0	31.46 (30.13)	32.04 (31.74)	31.57 (30.33)	1039.81 (1047.22)	1020.40 (1042.49)	1022.65 (1023.51)
0.2	12.40 (10.71)	15.46 (14.90)	15.47 (13.79)	144.77 (145.70)	235.68 (234.16)	235.35 (236.41)
0.4	5.98 (5.76)	8.25 (7.84)	8.27 (7.75)	30.13 (29.20)	67.41 (65.60)	67.15 (66.50)
0.6	3.31 (2.77)	5.02 (4.52)	4.98 (4.45)	8.89 (6.09)	24.81 (24.18)	24.55 ((23.09)
0.8	2.13 (1.55)	3.29 (2.73)	3.27 (2.72)	3.52 (2.88)	10.72 (10.19)	10.78 (10.27)
1.0	1.55 (0.92)	2.38 (1.80)	2.37 (1.80)	1.91 (1.36)	5.55 (5.13)	5.64 (5.12)
1.5	1.07 (0.27)	1.41 (0.76)	1.42 (0.77)	1.05 (0.32)	2.01 (1.41)	2.01 (1.42)
2.0	1.03 (0.18)	1.12 (0.37)	1.00 (0.00)	1.00 (0.00)	1.26 (0.57)	1.00 (0.00)

Table 2. ARL (SDRL) comparison of various charts under double exponential distribution.

Shift δ	n = 5			n = 10		
	$\overline{X}$ UCL=0.85	NP-S UCL=5.0	NP-R UCL=5.0	$\overline{X}$ UCL=1.055	NP-S UCL=10.0	NP-R UCL=10.0
0.0	32.72 (31.74)	31.50 (31.00)	31.86 (31.52)	1025.45 (1013.39)	1023.32 (1041.46)	1025.28 (1037.79)
0.2	21.27 (19.63)	18.63 (17.83)	16.82 (14.84)	206.29 (204.21)	111.69 (111.52)	113.84 (113.68)
0.4	14.51 (12.58)	11.34 (10.65)	10.61 (10.10)	47.31 (46.50)	27.76 (28.18)	28.10 (26.73)
0.6	9.79 (7.31)	7.28 (6.84)	7.27 (4.32)	13.68 (11.93)	11.01 (10.52)	10.99 (8.75)
0.8	6.81 (3.56)	4.77 (4.25)	5.37 (4.84)	4.94 (4.41)	5.85 (5.36)	5.75 (5.23)
1.0	4.87 (4.34)	3.27 (2.67)	4.17 (3.64)	2.31 (1.74)	3.62 (3.11)	3.64 (3.10)
1.5	2.49 (1.93)	1.43 (0.78)	2.57 (2.01)	1.09 (0.31)	1.86 (1.25)	1.88 (1.29)
2.0	1.56 (0.93)	1.00 (0.00)	1.90 (1.31)	1.00 (0.00)	1.34 (0.68)	1.35 (0.69)

Table 3. ARL (SDRL) comparison of various charts under uniform distribution.

Shift δ	*n* = 5			*n* = 10		
	$\overline{X}$ UCL=1.43	NP-S UCL=5.0	NP-R UCL=5.0	$\overline{X}$ UCL=0.315	NP-S UCL=10.0	NP-R UCL=9.0
0.0	31.28 (29.75)	31.54 (31.14)	32.11 (30.81)	1026.35 (1047.41)	1021.83 (1023.64)	1022.78 (1027.32)
0.2	3.22 (2.67)	18.59 (18.19)	18.72 (17.27)	7.07 (6.55)	346.23 (348.29)	345.76 (346.56)
0.4	1.27 (0.59)	11.31 (10.80)	11.11 (10.60)	1.27 (0.58)	127.63 (127.44)	127.49 (126.72)
0.6	1.01 (0.10)	7.27 (6.77)	7.17 (6.65)	1.00 (0.00)	52.85 (52.36)	51.85 (51.28)
0.8	1.00 (0.00)	4.85 (4.32)	4.79 (4.26)	1.00 (0.00)	23.29 (23.20)	23.07 (21.73)
1.0	1.00 (0.00)	3.29 (2.78)	3.30 (2.75)	1.00 (0.00)	10.72 (10.21)	10.61 (10.10)
1.5	1.00 (0.00)	1.41 (0.76)	1.43 (0.78)	1.00 (0.00)	1.98 (1.39)	2.00 (1.41)
2.0	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)	1.00 (0.00)

6. CONCLUSIONS

Examinations of Table 1 to Table 3, In-control ARL and SDRL values of the $\overline{X}$, NP-S and NP-R control charts are almost identical under normal, double exponential and uniform process distributions. Out-of-control ARL and SDRL values of proposed NP-R chart and NP-S chart are smaller than that of the $\overline{X}$ chart for double exponential process distribution. Therefore, NP-R chart and NP-S chart perform better than the $\overline{X}$ chart when underlying process distribution is heavy tailed. Out-of-control ARL and SDRL values of the $\overline{X}$ chart are smaller than that of the NP-R and NP-S charts when underlying process distributions are normal and uniform. Therefore, NP-R and NP-S charts are not efficient to detect shifts in process location as compared to the $\overline{X}$ chart when underlying process distributions are normal and light tailed. Out-of-control ARL and SDRL values of proposed NP-R chart and that of the NP-S chart are almost similar when underlying process distributions are normal, double exponential and uniform for all considered shifts in process location. Hence her we can conclude that proposed Distribution-free control charts namely NP-R chart and NP-S chart is good alternative to traditional $\overline{X}$ chart.

REFERENCES

1. R. W. Amin, M. R. Reynolds Jr., S. T. Bakir; Nonparametric quality control charts based on the sign statistic, Communications in Statistics-Theory and Methods, 24(6), 1597-1623 (1995).
2. S.T. Bakir, M. R. Reynolds Jr.; A nonparametric procedure for process control based on within group ranking, Technometrics, 21, 175-183 (1979).
3. S. Chakraborti, M. A. Graham; Nonparametric control charts, Encyclopedia of Statistics in Quality and Reliability, John Wiley, New York, 2007.
4. S. Chakraborti, P. Van der Lann, M. Van de Wiel; A class of distribution-free control charts, Applied Statistics, 53, 443-462 (2004).
5. S. Chakraborti, P. Van der Lann, S. T. Bakir; Nonparametric control charts: An overview and some results, Journal of Quality Technology, 33, 304-315 (2001).

6. N. Das; A multivariate nonparametric control chart based on sign test, Quality Technology and Quantitative Management 6(2), 155-169 (2009).
7. V. B. Ghute, D. T. Shirke; Bivariate nonparametric synthetic control chart based on sign test, Journal of Industrial and system Engineering 6(2), 108-121 (2012b).
8. L. Liu, J. Zhang, X. Zi; Dual nonparametric CUSUM control chart based on ranks, Communications in Statistics- Theory and Methods, 44, 756-772 (2015).
9. D. McDonald; CUSUM procedure based on sequential ranks, Naval Research Logistics, 37(5), 627-646 (1990).
10. C. A. Mc Gilchrist, K. D. Woodyer; A note on a distribution-free CUSUM technique, Technometrics, 17(3), 321-325 (1975).
11. E. S. Page; (1954), Continuous inspection schemes, Biometrika, 41, 100-115 (1954).
12. M. J. R. Varon; Ph. D. Dissertation on Nonparametric test based on runs for a single sample location problem (2010). RL: http://kops.ub.uni-konstanz.de/volltexte/2010/11634.
13. S. Yang, S. W. Cheng; A new nonparametric CUSUM mean chart, Quality and Reliability Engineering International, 27, 867-875 (2011).
14. D.M. Zombade and V. B. Ghute; Shewhart- type nonparametric control chart for process location, Communications in Statistics - Theory and Methods, (online print), 1-14(2018) .