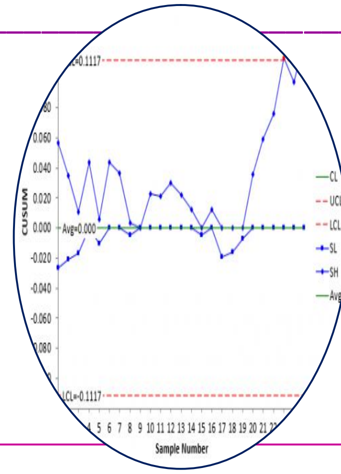
REVIEW OF RESEARCH

ISSN: 2249-894X
IMPACT FACTOR : 3.8014 (UIF)
VOLUME - 6 | ISSUE - 10 | JULY - 2017



A NONPARAMETRIC CUSUM-TYPE CONTROL CHART FOR PROCESS LOCATION

D. M. Zombade
Department of Statistics ,
Walchand College of Arts and Science, Solapur (MS), India.

ABSTRACT

In this paper, nonparametric CUSUM-type control chart for monitor the process location of a continuous process distributions. The proposed nonparametric CUSUM-type control chart for process location based on run test statistic. The proposed nonparametric CUSUM-type control chart is investigated using a simulation study and is compared under normal, uniform and Laplace process distributions. The proposed nonparametric CUSUM-type control chart is more efficient for detecting small shifts in process location under normal than uniform and Laplace distributions.

KEY WORDS: ARL, Run test, Control chart, Process location, CUSUM chart.

INTRODUCTION:

Shewhart-type control charts for location are widely used for purposes of determining whether a process is in control, for bringing an out-of-control process into control, and for monitoring a process to make sure that it stays in control. Univariate control charts are used to monitor processes that manufacture products with a single quality characteristic of interest. Most of the control charts are based on the assumption that the underlying distribution of the process is normal. In reality, this assumption may not hold in all the situations.

Therefore, it is necessary to suggest nonparametric control charts for monitoring processes which do not depend on the assumption of normality. The location and scale of a process are two main parameters often monitored in nonparametric control charts. The problem of monitoring the location of a process is important in many applications. The location parameter could be the mean or the median of the distribution. In literature, some nonparametric CUSUM-type control charts are proposed for monitoring location parameter of a univariate process distribution.

LITERATURE

In literature, several nonparametric control charts are proposed for monitoring location parameter of a univariate process. Amin et al. (1995) developed Shewhart and CUSUM control charts based on sign test statistic. Chakraborti et al. (2001) presented an extension overview of the literature on univariate nonparametric control charts. Chakraborti and Van de Wiel (2008) developed the control chart based on Mann-Whitney statistic for detecting location shifts. Das (2009) presented a comparison study of three nonparametric control charts to detect shift in location parameters. Khilare and Shirke (2010) developed a nonparametric synthetic control chart based on sign statistic to monitor shifts in process location. Pawar and Shirke (2010) developed a nonparametric Shewhart-type synthetic control chart based on signed-rank statistic to monitor shifts in the known in control process location. For monitoring the location of a univariate continuous process, some nonparametric CUSUM charts have been developed. Bakir and Reynolds (1979) developed a nonparametric CUSUM chart to monitor process center based on within group signed ranks. McDonald (1990) proposed a CUSUM procedure based on sequential ranks. Amin et al. (1995) developed nonparametric CUSUM control chart for grouped data based on sign test statistic. Li et al. (2010) proposed a nonparametric CUSUM control chart based on well known Mann-Whitney test statistic for monitoring the unknown location of a process. Yang and Cheng (2011) have proposed a nonparametric CUSUM chart to monitor the possible small shifts in the process mean. Liu et al. (2015) proposed a sequential rank based nonparametric CUSUM control chart for detecting arbitrary magnitude of shifts in the location parameter.

The rest of the chapter is organized as follows. A short introduction of CUSUM chart for monitoring process location in Section 2 and a brief introduction nonparametric Run Test for Location presented in section 3.In section 4 The proposed nonparametric CUSUM-type control chart for monitoring process location based on run test statistic is presented. The performance of proposed nonparametric CUSUM-type chart is evaluated and compared under the normal, uniform and Laplace distribution in Section 5.Some conclusions are given in Section 6.

2. CUSUM chart for process Location

Let *X* denote the process variable being measured and suppose that *X* has a normal distribution with mean μ and standard deviation σ. Let μ_0 be in-control value for μ and σ_0 be in-control value for σ. Usually, two symmetric CUSUM charts are used to detect two-sided mean shifts. For detecting positive shifts in μ, a one-sided (upper) CUSUM consists in computing recursively the sequence $C_i^+,\ i \ge 1$, where

$$C_0^+ = 0\ ;\ C_i^+ = \max[\,0,\ C_{i-1}^+ + (\overline{X}_i - \mu_0) - k\,], i \ge 1 \qquad (1)$$

Where *i* is subgroup number; $\overline{X}_i$ is the mean of study variable *X*, μ_0 is the target mean of the study variable *X* and *k* a positive constant is the reference value of the CUSUM scheme. The CUSUM signals a change in process mean as soon as C_i^+ exceeds a control limit $h > 0$, interpreting that process mean has shifted upward.

To control downward shifts in the process mean μ, a one-sided (lower) CUSUM consists in computing recursively the sequence $C_i^-,\ i \ge 1$, where

$$C_0^- = 0\ ;\ C_i^- = \max[\,0,\ C_{i-1}^- - (\overline{X}_i - \mu_0) - k\,],\ i \ge 1 \qquad (2)$$

The CUSUM chart using this statistic would signal whenever signals a change in process mean as soon as C_i^- exceeds $C_i^- < -h$, where $h > 0$, interpreting that process mean has shifted downward. For a two-sided CUSUM chart, the two charting statistics C_i^+ and C_i^- are plotted against a single control limit *h*. The starting value for both plotting statistics is usually taken as $C_0^+ = C_0^- = 0.$ A signal is given if $C^+ > h$ or $C^- < -h$.

3. NONPARAMETRIC RUN TEST FOR LOCATION

In this Section, we briefly review the run test described by Varon (2010). Let $X_1, X_2, ..., X_n$ be a subgroup sample of size *n* > 1 from a distribution with location θ and standard deviation σ. It is assumed that these observations are independent and have a continuous distribution symmetric about location (median or median) θ. Let θ_0 denote the target known value of the process location. Without loss of generality, we assume that $\theta_0 = 0$ and $\sigma_0 = 1$. A test for the hypothesis $\mathrm{H}_0 : \theta = 0$ versus $\mathrm{H}_1 : \theta > 0$, based on runs has been discussed by Varon (2010). A run is defined as a succession of two or more identical symbols which are followed and preceded by different symbols or no symbol at all. At each inspection point, a nonparametric run statistic R is computed using a subgroup sample $X_1, X_2, ..., X_n$. For the construction of runs, the variable η_j is defined as

$$\eta_j = S(X_{Dj}) = \begin{cases} 1 , & \text{if } X_{Dj} > 0 , \; j = 1,2,3,....,n \\ 0 , & \text{otherwise} \end{cases} , \qquad (3)$$

where D_j is the antirank of $|X|_{(j)}$ such that $|D_j| = |X|_{(j)}$. Hence D_j labels the X which corresponds to the j^{th} order absolute value. Then the sequence $\eta_1, \eta_2, ..., \eta_n$ is a dichotomized sequence. The changes in the dichotomized succession are identified with the following indicators:

Define $I_1 = 1$

$$I_j = \begin{cases} 1 , & \text{if } \eta_{j-1} \neq \eta_j , \; j = 2,3,....,n. \\ 0 , & \text{if } \eta_{j-1} = \eta_j \end{cases} \qquad (4)$$

The number of runs until the j^{th} element of the dichotomized succession is obtained through the following partial sums:

$$r_i = \sum_{j=1}^{i} I_j \; , i = 1,2,....,n. \qquad (5)$$

Naturally $r_i \leq r_j$ for $i < j$ and r_n is the total number of runs in the sequence. Test statistic based on runs is given as

$$R = \frac{1}{r_n} \sum_{j=1}^{n} \delta_j r_j \qquad (6)$$

Where

$$\delta_j = \begin{cases} 1 , & \mathit{if} \; \eta_j = 1, \\ -1 , & \mathit{if} \; \eta_j = 0 \end{cases} \qquad j = 1, 2, 3, ..., n. \qquad (7)$$

Note that *R* includes the number of runs until every element of the dichotomized succession, increasing their value when $\eta_{\mathrm{j}} = 1\ (\delta_{\mathrm{j}} = 1, \text{runs of ones})$ and decreasing when $\eta_{\mathrm{j}} = 0\ (\delta_{\mathrm{j}} = -1, \text{runs of zeros})$ the large value of *R* indicate greater number of runs of ones and it is an indication that $\theta > 0$. Additionally the inverse of total number of runs $\frac{1}{r_n}$ is used as a factor of standardization. It should be noted that the statistic *R* takes values between –*n* and *n*. Large values of *R* indicate a positive shift where as small value indicate a negative shift. For $\theta > 0$, it is expected that *R* takes large positive values. Accordingly H_0 is rejected for large values of *R*.

4. NONPARAMETRIC CUSUM-TYPE CHART FOR LOCATION

In this Section, we develop a nonparametric CUSUM chart for monitoring location of a process. Let *X* denote the process variable being measured and suppose that *X* has a continuous symmetric distribution with location parameter θ. Here we have to monitor location parameter θ through control charting. Let θ_0 be in-control or target value of θ. The location parameter of a distribution under study is usually unknown in practice and need to be estimated from the analysis of the preliminary samples taken when the process is assumed to be in-control. The proposed nonparametric CUSUM charting technique for detecting a change in location θ from in-control value θ_0 to some out-of-control value θ_1 is based on first transforming the observed data into a nonparametric run statistic *R* and then applying the CUSUM chart on the transformed statistic (the chart is referred as CUSUM-type chart). The proposed CUSUM-type chart is constructed by accumulating the statistics $R_1, R_2, \dots$ sequentially from each sample subgroup $X_1, X_2, \dots, X_n$ of size *n*. When detection of shift in location θ (from its specified value θ_0) in only one direction (up or down) is of interest, a one-sided CUSUM chart is desirable. When the objective is to detect increase in θ an upper one-sided CUSUM uses the plotting statistic

$$S_i^+ = \max[0,\ S_{i-1}^+ + (R_i - \theta_0) - k] \tag{8}$$

where starting value of plotting statistic is $S_0^+ = 0$ and *k* is the reference value of the CUSUM scheme. The statistic S_i^+ is plotted on the chart along with control limit *h*. If any S_i^+ plots on or outside the control limit *h*, the process is declared out-of-control and search for assignable causes is started, otherwise, the process is considered in-control and control procedure continues. When the objective is to detect decrease in θ lower one-sided CUSUM uses the plotting statistic

$$S_i^- = \max[0,\ S_{i-1}^- - (R_i - \theta_0) - k] \tag{9}$$

where starting value of plotting statistic is $S_0^- = 0$ and *k* is the reference value of the CUSUM scheme. The statistic S_i^- is plotted on the chart along with control limit *h*. If any S_i^- plots on or outside the control limit -*h*, the process is declared out-of-control and search for assignable causes is started.

When the objective is to detect both increase and decrease in θ from θ_0, a two-sided *CSM-R* chart uses both of the statistics S_i^+ and S_i^- simultaneously. A signal is given if $S_i^+ > h$ or $S_i^- < -h$. We assume that the objective of monitoring the process is to detect any special cause that changes location θ from θ_0. Although detecting decrease in location θ may be of interest in some applications, here we focus on more important problem of detecting increase in the process location θ. The upper one-sided *CSM-R* chart can be constructed as follows:

Step 1. Collect a subgroup sample $X_i = (X_{i1}, X_{i2}, \ldots, X_{in}),\ i = 1, 2, \ldots, n$ of size *n* from a process.

Step 2. Compute run statistic R_i from the subgroup sample X_i, *i* = 1, 2, … , *n*.

Step 3. Construct the CUSUM statistic as $S_i^+ = Max\ \{0,\ S_{i-1}^+ + (R_i - \theta_0) - k\}$, where ($k \geq 0$) is reference parameter of CUSUM scheme and S_i^+ will detect upward location shift.

Step 4. Plot S_i^+ against control limit *h*.

Step 5. If S_i^+ exceeds *h*, process is declared to be out-of-control at the i^{th} sample otherwise the process is considered to be in-control and monitoring continues to the next sample.

5. PERFORMANCE COMPARISONS

The performance of a control chart is usually measured by ARL, which is the average number of samples required to signal an out-of-control case. The in-control ARL is denoted by ARL_0 and out-of-control ARL is denoted by ARL_1. The performance of the proposed CUSUM-type chart is compared under normal, uniform and Laplace. A computer programme developed in C language is used to simulate ARL. Three process distributions are considered in simulation study namely, normal distribution with location θ and scale σ, Laplace distribution with location θ and scale λ, which is symmetric and having heavier tails than normal distribution and the uniform distribution with location θ and scale λ, which is symmetric and having lighter tails than the normal distribution. Equation (10), (11) and (12) respectively gives probability density functions of normal distribution with location θ and scale σ, Laplace distribution with location θ and scale λ and uniform distribution with location θ and scale λ.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{\frac{-1}{2}\left(\frac{x-\theta}{\sigma}\right)^2\right\},\ -\infty < x < \infty \text{ and } \sigma > 0 \tag{10}$$

$$f(x) = \frac{1}{2\lambda} \exp\left(-\frac{|x-\theta|}{\lambda}\right),\ -\infty < x < \infty \text{ and } \lambda > 0 \tag{11}$$

$$f(x) = \frac{1}{2\lambda}\ ,\ \theta - \lambda < x < \theta + \lambda \text{ and } \lambda > 0 \tag{12}$$

The distributions have been shifted and scaled such that they all have an expected value of 0 and standard deviation 1, so simulation results are easily comparable. To achieve standard deviation of 1, we choose $\sigma = 1$ for normal distribution, $\lambda = \frac{1}{\sqrt{2}}$ for Laplace distribution and $\lambda = \sqrt{3}$ for uniform distribution.

Consider a process where quality characteristic of interest X is distributed with location θ and standard deviation σ. Let θ_0 and σ_0 be the in-control values of θ and σ respectively. When a shift in process location occurs, we have change from the in-control value θ_0 to the out-of-control value $\theta_1 = \theta_0 + \delta\,\sigma_0$, $(\delta > 0)$. Therefore, when control chart for location is employed, the process shifts are measured through $\delta = \frac{|\theta_1 - \theta_0|}{\sigma_0}$, where θ_1 is the shifted location and θ_0 is in-control location. The amount of a shift in the location is taken over the range $\delta = 0\ (0.2)1.2$. When $\delta = 0$, the process is in-control. The ARL values of the proposed CUSUM-type chart computed using 10000 simulations. The optimal reference value *k* is taken to be $\frac{\delta}{2}$ if a CUSUM-type chart is to be able to detect a standardized location shift of size δ. In simulation study we choose k = 0.5., h=4.72 and achieve ARL_0=31.00 for sample sizes n=5 and k=5, h=21.0 and achieve ARL_0=987.00 for sample sizes n=10.

Table 1 and Table 2 provide the ARL and SDRL performance of the proposed CUSUM-type chart when underlying process distributions are normal, uniform and Laplace distributions of sample sizes *n* = 5 and 10.

Table1: ARL and SDRL of CUSUM-type chart under normal, uniform and Laplace distribution for n=5, h=4.72, k=0.5.

δ	Normal	Uniform	Laplace
0.0	31.22	30.77	30.47
	(27.53)	(26.28)	(26.77)
0.2	11.31	12.45	14.50
	(10.80)	(11.94)	(13.99)
0.4	6.36	7.15	8.90
	(5.84)	(6.63)	(8.39)
0.6	4.60	5.15	6.50
	(4.07)	(4.62)	(5.98)
0.8	3.78	4.14	5.22
	(3.24)	(3.61)	(4.69)
1.0	3.39	3.57	4.50

	(2.85)	(3.03)	(3.97)
1.5	3.04	3.03	3.62
	(2.49)	(2.48)	(3.08)
2.0	3.00	3.00	3.26
	(2.45)	(2.45)	(2.71)

Table 2: ARL and SDRL of CUSUM-type chart under normal, uniform and Laplace distribution for n=10, h=21.0, k=0.5

δ	Normal	Uniform	Laplace
0.0	987.29	976.89	989.00
	(961.16)	(956.60)	(969.18)
0.2	26.05	28.61	47.74
	(25.55)	(28.11)	(47.24)
0.4	11.49	12.70	19.16
	(10.98)	(12.19)	(18.65)
0.6	7.65	8.42	12.36
	(7.13)	(7.90)	(11.85)
0.8	5.99	6.50	9.40
	(5.47)	(5.98)	(8.89)
1.0	5.06	5.39	7.75
	(4.53)	(4.86)	(7.23)
1.5	4.12	4.05	5.75
	(3.59)	(3.51)	(5.23)
2.0	4.00	4.00	4.86
	(3.46)	(3.46)	(4.33)

Examination of Table 1 and Table 2 leads to the following findings:

- For monitoring a process operating under normal distribution, it is observed that, the proposed CUSUM-type chart is more efficient for detecting small shifts in the process location under normal distribution than uniform and Laplace distributions for sample sizes n = 5 and n=10.
- Nonparametric CUSUM-type chart for normal process distribution performs better for detecting small shifts in the process location under normal, uniform and Laplace distributions.

6. CONCLUSIONS

In this paper, nonparametric CUSUM-type control chart based on run statistic for location of a continuous process distribution. The performance of the CUSUM-type chart is studied by simulation study under normal, uniform and Laplace distributions. The proposed CUSUM-type chart is more efficient to detect small shifts under normal than uniform and Laplace distributions.

REFERENCES

1. R. W. Amin, M. R. Reynolds Jr., S. T. Bakir (1995), Nonparametric quality control charts based on the sign statistic, Communications in Statistics-Theory and Methods, 24(6), 1597-1623.

2. S.T. Bakir, M. R. Reynolds Jr. (1979), A nonparametric procedure for process control based on within group ranking, Technimetrics, 21, 175-183.

3. S. Chakraborti, M. A. Graham (2007), Nonparametric control charts, Encyclopedia of Statistics in Quality and Reliability, John Wiley, New York,.

4. S. Chakraborti, P. Van der Lann, M. Van de Wiel (2004), A class of distribution-free control charts, Applied Statistics, 53, 443-462.

5. S. Chakraborti, P. Van der Lann, S. T. Bakir (2001), Nonparametric control charts: An overview and some results, Journal of Quality Technology, 33, 304-315.

6. N. Das (2009), A multivariate nonparametric control chart based on sign test, Quality Technology and Quantitative Management 6(2), 155-169.

7. V. B. Ghute and D. T. Shirke (2012a), A nonparametric signed-rank control chart for bivariate process location, Quality Technology and Quantitative Management 9(4), 317-328.

8. Y. Li, L. C. Tang, S. H. Ng (2010), Nonparametric CUSUM and EWMA control charts for detecting mean shifts, Journal of Quality Technology, 42(2), 209-226.

9. L. Liu, J. Zhang, X. Zi; (2015), Dual nonparametric CUSUM control chart based on ranks, Communications in Statistics- Theory and Methods, 44, 756-772.

10. D. McDonald (1990), CUSUM procedure based on sequential ranks, Naval Research Logistics, 37(5), 627-646.

11. C. A. Mc Gilchrist, K. D. Woodyer (1975), A note on a distribution-free CUSUM technique, Technimetrics, 17(3), 321-325.

12. M. J. R. Varon (2010), Ph. D. Dissertation on Nonparametric test based on runs for a single sample location problem. L: http://kops.ub.uni-konstanz.de/volltexte/2010/11634.

13. S. Yang, S. W. Cheng (2011), A new nonparametric CUSUM mean chart, Quality and Reliability Engineering International, 27, 867-875.