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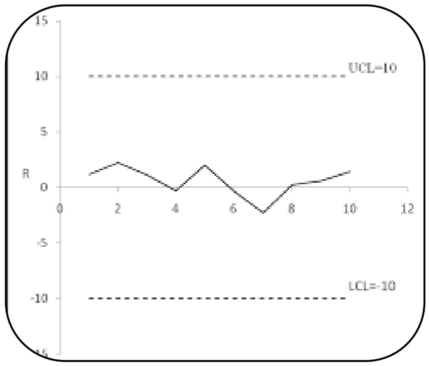

ROBUSTNESS OF THE NON-PARAMETRIC RUN (NP-R) CHART

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ABSTRACT :

In this paper, Non-parametric control chart based on run statistic is proposed for monitoring the location of a continuous symmetric process distribution. The proposed chart is distribution-free so that its application does not require the assumption of parametric model. The performance of the proposed chart is evaluated by simulating data from normal and non-normal distributions and compared with the existing nonparametric chart based on Sign chart. It is observed that the performance of proposed Run chart is similar to that of the Sign chart in terms of detecting shifts in process location under normal and non-normal process distributions.

KEYWORDS : *Quartiles of the run length distribution, Run Statistic, Control chart.*

INTRODUCTION

Shewhart-type control charts for process location are widely used for purposes of determining whether a process is in control, for bringing an out-of-control process into control, and for monitoring a process to make sure that it stays in control. Control charts are used to monitor processes that manufacture products with a single quality characteristic of interest. Most of the control charts are based on the assumption that the underlying distribution of the process is normal. In reality this assumption may not hold in all the situations. Therefore, it is necessary to suggest nonparametric control charts for monitoring processes which do not depend on the assumption of normality.

The location parameter often monitored in nonparametric control charts. The problem of monitoring the location of a process is important in many applications. The location parameter could be the mean or the median or some percentiles of the distribution. In literature, several nonparametric control charts are proposed for monitoring location parameter of a univariate process. Amin et al. (1995) developed Shewhart and CUSUM control charts based on sign test statistic. Chakraborti et al. (2001) presented an extension overview of the literature on univariate nonparametric control charts. Many authors have developed nonparametric control charts to monitor location parameter of the process. Bakir (2004) developed a nonparametric Shewhart control chart for monitoring process center based on the signed-ranks of grouped observations. Chakraborti and Van de Wiel (2008) developed the control chart based on Mann-Whitney statistic for detecting location shifts. Das (2009) presented a comparison study of three non-parametric control charts to detect shift in location parameters. Khilare and Shirke (2010) developed a nonparametric synthetic control chart based on sign statistic to monitor shifts in process location. Yang and Cheng (2011) have proposed a nonparametric CUSUM chart to monitor the possible small shifts in the process mean. Liu et al. (2015) proposed a sequential rank based nonparametric CUSUM control chart for detecting arbitrary magnitude of shifts in the location parameter. Zombade and Ghute (2018) presented

Shewhart-type nonparametric control charts for process location. Zombade and Ghute (2014) proposed nonparametric control charts for monitoring the process variability using runs rules. Zombade and Ghute (2014) proposed nonparametric control charts based on mood statistic for controlling the process location and process variability.

The purpose of this paper is to test the robustness of nonparametric Run chart for monitoring the location of a symmetric process. If underlying process distribution is non-normal, then we consider nonparametric control chart based on appropriate nonparametric test. Many nonparametric tests like sign, signed-rank has been proposed in the literature. In this paper, we use the run test statistic as a charting statistic for detecting location shifts of a symmetric process. The proposed nonparametric chart for monitoring the process location is based on runs computed within samples.

The rest of the paper is organized as follows. The nonparametric control chart for monitoring process location based on run test statistic is presented in Section 2. The performance of nonparametric run chart (NP-R) chart is evaluated and compared in terms of quartiles of the run length distribution with nonparametric sign (NP-S) chart in Section 3. In Section 4, an illustrative example is provided which shows the application of the NP-R chart on simulated data set and some conclusions are given in Section 5.

2. NON-PARAMETRIC CONTROL CHARTS

2.1: Control Chart based on Sign Statistic (NP-S)

Procedure based on sign test statistic require that, at any time t, each observation from the sample be compared with target value θ_0 and number of observations above and below θ_0 be recorded for each sample.

Define,

$$\mathrm{Sign}\,(\mathrm{X}_{ij}) = \begin{cases} 1, & \text{if } \mathrm{X}_{ij} > \theta_0 \\ 0, & \text{if } \mathrm{X}_{ij} = \theta_0 \\ -1, & \text{if } \mathrm{X}_{ij} < \theta_0 \end{cases} \tag{2.1.1}$$

where X_{ij} is the jth observation in the ith sample.

Let $\mathrm{S_i} = \sum_{j=1}^{n} \mathrm{Sign}\,(\mathrm{X}_{ij} - \theta_0), i = 1, 2, 3, \ldots$ where $\mathrm{S_i}$ is the just difference between the number of observations above θ_0 and number of observations below θ_0 in the ith sample. A random variable $\mathrm{T_i} = \dfrac{\mathrm{S_i} + n}{2}$ gives the number of positive signs in the sample of size n and has binomial distribution with parameters *n* and $\mathrm{p} = \mathrm{P}[\mathrm{X}_{ij} > \theta_0]$, when $\mathrm{p} = 0.5$ in the in-control case and θ_0 is in-control location of process.

For sign control chart, $\mathrm{ARL}\,(\theta)$ for two-sided control chart is

$$\mathrm{ARL}\,(\theta) = \frac{1}{\mathrm{P}}$$

where P is the detecting power of sign chart,

$$\mathrm{P} = \mathrm{P}\,[|\mathrm{S_i}\,| > \mathrm{UCL}\,|\,\theta\,]$$

The ARL for one-sided control limit in the positive direction is,

$$\mathrm{ARL}\,(\theta) = \frac{1}{\mathrm{P}\,(\mathrm{S_i}\ \geq\ \mathrm{UCL}\ \ |\,\theta\,)} \qquad (2.1.2)$$

We consider S as the control statistic for the nonparametric control chart for monitoring process location and the chart is referred as nonparametric S (denoted NP-S) chart. The chart operates by plotting S values on the chart with *UCL*. When the plotted point lies above the *UCL,* it indicates that there has been a change in location θ, and the process is considered to be out-of-control.

2.2 Control Chart based on Run Statistic (NP-R)

In this Section, nonparametric control chart based on Run statistic is proposed for monitoring the location of a continuous symmetric process distribution. The proposed chart is distribution-free or nonparametric so that its application does not require the assumption of parametric model *i.e.* normality.

A run is defined as a succession of two or more identical symbols which are followed and preceded by different symbols or no symbol at all. At each inspection point, a nonparametric run statistic is computed using *n* observations $\mathrm{X}_1, \mathrm{X}_2, \ldots, \mathrm{X}_n$.

Define $\eta_{\mathrm{j}} = \mathrm{S}(\mathrm{X_{Dj}}) = \begin{cases} 1\ , & \text{if } \mathrm{X_{Dj}} > 0\ ,\ \mathrm{j} = 1,2,3,\ldots,n \\ 0\ , & \text{otherwise} \end{cases}$, (2.2.1)

where $\mathrm{D_j}$ is the antirank of $|\,\mathrm{X}\,|_{(\mathrm{j})}$. Hence $\mathrm{D_j}$ labels the X which corresponds to the j^{th} order absolute value.

Define $\mathrm{I_1} = 1$

$$\mathrm{I_j} = \begin{cases} 1\ , & \text{if } \eta_{\mathrm{j-1}} \neq \eta_{\mathrm{j}},\ \mathrm{j} = 2,3,\ldots,n. \\ 0\ , & \text{if } \eta_{\mathrm{j-1}} \neq \eta_{\mathrm{j}} \end{cases} \qquad (2.2.2)$$

Define the partial sums,

$$\mathrm{r_i} = \sum_{\mathrm{j}=1}^{\mathrm{i}} \mathrm{I_j}\ ,\mathrm{i} = 1,2,\ldots,n. \qquad (2.2.3)$$

Naturally $r_i \le r_j$ for $i < j$ and r_n is the total number of runs in the sequence. Test statistic based on runs (Corzo (1989)) is given as

$$R = \frac{1}{r_n}\sum_{j=1}^{n}\delta_j r_j \qquad (2.2.4)$$

where $$\delta_j = \begin{cases} 1, & \text{if } \eta_j = 1, \\ & \qquad j = 1, 2, 3, \ldots, n. \\ -1, & \text{if } \eta_j = 0 \end{cases} \qquad (2.2.5)$$

Note that R includes the number of runs until every element of the dichotomized succession, increasing their value when $\eta_j = 1\ (\delta_j = 1, \text{runs of ones})$ and decreasing when $\eta_j = 0\ (\delta_j = -1, \text{runs of zeros})$ the large value of R indicate greater number of runs of ones and it is an indication that $\theta > 0$. Additionally the inverse of total number of runs $\frac{1}{r_n}$ is used as a factor of standardization.

3. PERFORMANCE COMPARISONS

Zombade and Ghute (2018) developed Shewhart-type nonparametric control chart based on run statistic, for monitoring process location of a continuous symmetric distribution. The result shows that in terms of the average run length (ARL), the standard deviation of run length (SDRL), median run length (MRL) the performance NP-R chart under normal, double exponential and Cauchy process distributions is more efficient than the traditional Shewhart X chart under heavy tailed distributions. That is NP-R chart is efficient than the NP-S chart for detecting shift in process median for considered process distributions.

However Chakraborti and Eryilmaz (2007) suggested that the entire run length distribution is more robust to compare the performance along with the study of ARL and SDRL performance of the control charts. Accordingly in this study the robustness of NP-R chart is tested in terms of quartile run length distribution having fixed in-control values.

These results are presented in Table 1 to Table 6 with sample subgroup size *n* =5 and 10 for normal, double exponential and uniform process distributions.

Table 1. Quartiles of the run length distribution of charts under normal distribution (*n* = 5).

Shift δ	NP-S chart				NP-R chart			
	Q1	Q2	Q3	IQR	Q1	Q2	Q3	IQR
0.0	9	22	44	35	9	22	43	34
0.2	5	11	21	16	5	11	21	16
0.4	3	6	11	8	3	6	11	8
0.6	2	4	7	5	2	4	7	5
0.8	1	2	4	3	1	2	4	3
1.0	1	2	3	2	1	2	3	2
1.5	1	1	2	1	1	1	2	1
2.0	1	1	1	0	1	1	1	0

Table 2. Quartiles of the run length distribution of charts under normal distribution (*n* = 10).

Shift δ	NP-S chart				NP-R chart			
	Q1	Q2	Q3	IQR	Q1	Q2	Q3	IQR
0.0	298	720	1430	1140	301	713	1400	1099
0.2	68	163	332	264	71	163	324	253
0.4	19	46	94	75	20	46	95	75
0.6	7	17	34	27	7	17	34	27
0.8	3	8	15	12	4	8	15	11
1.0	2	4	8	6	2	4	8	6
1.5	1	2	3	2	1	2	3	2
2.0	1	1	1	0	1	1	1	0

Table 3. Quartiles of the run length distribution of charts under double exponential distribution (*n* = 5).

Shift δ	NP-S chart				NP-R chart			
	Q1	Q2	Q3	IQR	Q1	Q2	Q3	IQR
0.0	10	23	45	35	9	22	44	35
0.2	4	10	19	15	5	12	23	18
0.4	3	5	10	7	3	7	15	12
0.6	2	4	7	5	2	5	10	8
0.8	1	3	5	4	2	4	7	5
1.0	1	2	4	3	2	3	6	4
1.5	1	1	2	1	1	2	3	2
2.0	1	1	2	1	1	1	2	1

Table 4. Quartiles of the run length distribution under double exponential distribution (*n* = 10).

Shift δ	NP-S chart				NP-R chart			
	Q1	Q2	Q3	IQR	Q1	Q2	Q3	IQR
0.0	301	714	1402	1101	294	706	1412	1118
0.2	57	138	268	211	56	135	263	207
0.4	17	40	80	63	17	41	82	65
0.6	7	17	34	27	7	17	34	27
0.8	4	9	17	13	4	9	17	13
1.0	3	5	10	7	3	5	10	7
1.5	1	2	4	3	1	2	4	3
2.0	1	2	3	2	1	2	2	1

Table 5. Quartiles of the run length distribution of charts under uniform distribution (*n* = 5).

Shift δ	NP-S chart				NP-R chart			
	Q1	Q2	Q3	IQR	Q1	Q2	Q3	IQR
0.0	10	22	44	34	10	23	45	35
0.2	6	13	26	20	6	13	26	20
0.4	4	8	16	12	4	8	15	11
0.6	2	5	10	8	2	5	10	8
0.8	2	3	6	4	2	3	6	4
1.0	1	2	4	3	1	2	4	3
1.5	1	1	2	1	1	1	2	1
2.0	1	1	1	0	1	1	1	0

Table 6. Quartiles of the run length distribution of charts under uniform distribution (*n* = 10).

Shift δ	NP-S chart				NP-R chart			
	Q1	Q2	Q3	IQR	Q1	Q2	Q3	IQR
0.0	281	705	1429	1148	295	709	1403	1108
0.2	100	239	478	378	97	237	476	379
0.4	37	87	174	137	38	90	180	142
0.6	16	37	72	56	15	37	72	57
0.8	7	16	32	25	7	16	32	25
1.0	3	8	15	12	3	7	15	12
1.5	1	1	2	1	1	2	3	2
2.0	1	1	1	0	1	1	1	0

In above tables, it is experiential that the quartiles and IQR values of the NP-R and NP-S charts are almost similar. Therefore, NP-R chart has similar performance to NP-S chart for detecting location shifts when the underlying process distributions are normal, double exponential and uniform.

4. AN EXAMPLE

In this Section, simulation illustration is presented performance NP-R control chart. The data set in this example is generated from double exponential distribution with mean 0 and standard deviation 1. Table 7 gives ten samples each of size 10 from double exponential distribution. Using section (2.2) the value of the run statistic R for each of the 10 samples are computed and are shown in Table 8.

Table 7. Simulated data from exponential distribution

Sample No.	Sample observations									
1	0.5395	-1.0155	-1.2856	0.1259	1.5060	1.4281	-0.1562	1.2564	-0.5999	0.2541
2	0.5296	0.1727	-2.0451	0.0900	1.7344	1.4107	-0.1735	1.0278	0.5023	-0.2899
3	0.1710	-0.7242	0.4274	0.1606	-0.5665	4.9129	1.0042	1.7433	-1.2108	-0.2286
4	-1.3459	-1.7578	-0.3721	1.7640	-0.5212	-1.0860	1.1580	-0.0799	0.9395	0.6263
5	0.2694	0.1710	1.3393	0.1238	0.2321	-0.6975	1.2601	1.7014	-3.7945	0.0204
6	-2.0526	-1.5252	-3.1896	-1.8396	2.1510	1.4828	-0.4909	0.3905	1.8507	1.9256
7	2.8170	-0.3667	-1.1984	2.8432	-2.1345	-1.7695	-1.6432	0.2900	-0.7101	-0.6257
8	-0.4338	0.3436	4.4573	-1.9470	-1.4813	0.8621	-0.4660	-2.2136	1.1756	2.2403
9	-1.1194	-1.4130	0.3945	-2.3840	-0.5808	4.1810	4.0641	-0.4763	1.4142	0.4039
10	0.0900	1.3131	-2.5451	0.8876	-1.4078	0.1470	1.0479	0.2588	-0.3389	3.2231

Table 8: Computed value of run statistic

Sample No.	1	2	3	4	5	6	7	8	9	10
R	1.20	2.25	1.14	-0.33	2.00	-0.38	-2.33	0.20	0.60	1.40

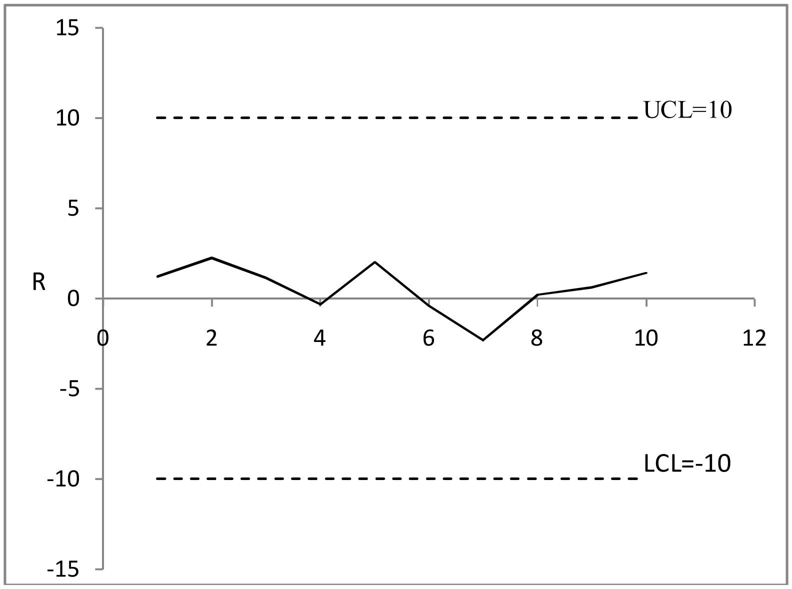

Figure 1. Run control chart for data in Table 8

Figure 1. Shows that, all the values of sample statistic R lie within the control limits of NP-R chart, therefore we consider NP-R chart to be robust in order to maintain process.

5. CONCLUSIONS

In this paper, the robustness of the NP-R control chart is tested by simulation under normal, light tailed and heavy tailed process distributions. Simulation study indicates that the in terms of quartiles of the run length distribution NP-R chart performing similar to that of the sign chart in terms of detecting shifts in process location under normal and non-normal process distributions. It is also observed that quartiles and IQR values of the NP-R and NP-S charts are almost comparable. Therefore, NP-R chart has parallel performance to NP-S chart for detecting location shifts when the underlying process distributions are normal, double exponential and uniform.

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