ENIGMA OF SUPER MASSIVE BLACK HOLES

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ABSTRACT:
The entire galaxies and solar systems are full of enigmatic elements with their complex behaviours. Super massive black holes are one of the central bodies around which the galaxies exist. The tremendous gravitational attraction of the black hole does not allow any particle to pass through the black holes. It is engulfed by the black hole. Even light particles are not allowed to pass through the black holes. It is also documented that in each galaxy there are so many black holes, which seems to be contradictory statements to each other. It has become a matter of interest how it is possible. This puzzle may be allowed to hold in the universe if explained on the basis of Carnot's ideal heat engine.

KEYWORDS: Super massive black holes, galaxies, Carnot’s ideal heat engine.

1. INTRODUCTION
The formation of stellar-mass black holes is well documented. But it is not clear how black holes having masses of a million to a billion times larger the mass of the Sun could have been formed. It is also investigated that the Super massive black hole cannot be formed through only the collapse of a single star. There must be some other processes for the formation and growth of super massive black holes. The investigation of such processes has become the interest of intense study in the present era. One possible scenario for the formation of super massive black holes is that the black holes of intermediate masses merged to form a more massive black hole. But this type of interaction must be allowed under certain constraints, otherwise due to persistence of such attractive interaction the whole universe will converge into a single entity of black hole. The mass of a black hole can be determined by studying the material that orbits around it. This is one of the most common techniques for measuring the masses of black holes. The best evidence for the existence of a super massive black hole is provided by the centre of our own galaxy. The movements of the individual star at the Galactic Centre have been interests of study by a number of astronomer’s for more than two decades. To present such model of orbits of the stars there will be need of an unseen mass of four million times the mass of the Sun located near the centre of the orbits. We have discussed here a method to produce work by running a heat engine between two black holes which serve as thermal reservoirs. In accordance with the Carnot's heat engine it can be concluded that the two black holes can ultimately merge into one bigger black hole keeping the total entropy unchanged. The resulting black hole thus must have a smaller mass than the total mass of the input black holes. The
difference corresponds to the extracted work. It can be shown easily that by merging two black holes with masses \( M_1 \) and \( M_2 \), we can extract up the useful work. Black hole thermodynamics has been discussed extensively since the seminal papers by Bekenstein [1] and Hawking [2,3]. However, not much attention has been paid to heat engine methods. A model suggested by Kaburaki and Okamoto [4] uses a Kerr black hole as the working medium in a Carnot-like engine running between two reservoirs formed by boxes with radiation. Recently Deng and Gao proposed a Carnot engine with radiation as the working fluid and a black hole as the cold reservoir [5]. In this paper, we have presented a simplified model of heat engine that allows the bigger black hole to absorb the light black hole which is responsible for the formation of super massive black holes by applying the principle of ideal Carnot’s ideal heat engine.

2. BLACK HOLE RADIATION IN A BLACK BOX:

A non-rotating and uncharged black hole packed into a box is considered which is filled with radiation [10]. The inner wall of the box is assumed to be perfectly reflecting. Thus the box is in isolation from the outside world. Therefore, the total energy in the box is constant. In this way, the black hole is taken to be in thermodynamic equilibrium with the radiation. The mechanism of black hole radiation has been described by Hawking [1-3]. In this model, the radiation is assumed to be entirely due to photons. The entropy of the black hole may be given as [1]:

\[
S_{BH} = 4\pi K \left( \frac{M}{M_P} \right)^2
\]

Where \( k \) is the Boltzmann constant, \( M \) is the mass of the black hole, and \( M_P \) is called the Planck’s mass: \( M_P = \frac{\hbar c}{2\pi G} \approx 2.2 \times 10^{-8} \text{Kg} \). \( G \) is the Gravitational constant, \( c \) is the velocity of light, \( h \) is the Planck’s constant.

The real composition of the radiation would influence the numerical values of the parameters. The total energy of the system is given as:

\[
E_{total} = M c^2 + aV T^4
\]

where the first term is the black hole energy and second term is the energy of the radiation:

\[
a = \frac{\pi^2 k^4}{15 c^3 h^3}
\]

\( V \) is the volume of the box and \( T \) is the radiation temperature. \( k \) is Boltzmann constant, \( c \) is velocity of light, and \( h \) is the Planck’s constant. The black hole temperature is given as [7]:

\[
T_{BH} = \frac{\partial E_{BH}}{\partial S_{BH}} = \frac{\hbar c^3}{8\pi k G M}
\]

The entropy of the combined system is given as:

\[
S_{total} = S_{BH} + S_{rad}
\]

The radiation entropy is given as [2]
By using Eq. (2), we can express the radiation temperature in terms of the total energy and the black hole mass such that the total entropy is a function of M. The entropy of the system will be composed of a black hole and radiation inside a box with insulating walls on the mass of the black hole.

3. CARNOT PROCESS BLACK BOX:

The model is shown in Fig. (1) There are two boxes, each with radiation and a black hole in stable equilibrium. Each box serves as a heat reservoir, with its temperature determined by the mass of the black hole. Between the boxes there is a connecting cylinder with a movable piston. The volume of the cylinder is much smaller than the volume of the reservoirs. The cylinder walls as well as the piston are assumed to be perfectly reflecting. The cylinder can either be completely isolated or it can be open to one of the reservoirs.

A is big black hole in the box A’ in which there is cold radiation and B is small black hole in the box B’ in which there is hot radiation. (a) Radiation from the hot reservoir enters the cylinder and pushes the piston, so that the radiation in the cylinder expands isothermally. (b) The cylinder is isolated and the radiation expands adiabatically by pushing the piston and cooling to the temperature of the cold reservoir. (c) The cold radiation is pushed out of the cylinder isothermally into the cold reservoir.

4. WORKING OF THE BLACKHOLE HEAT ENGINE:

The P-V diagram of the cycle is shown in Fig. 2. The cycle starts with the piston close to the hot reservoir, and the volume of the working medium is zero. (a) In the first step, the cylinder is open to the hot reservoir and the piston moves to increase the volume of the working medium to Va. The process is isothermal and isobaric. The radiation pressure is given by:

\[ P = \frac{4aT^4}{3} \]

Thus the radiation pressure is uniquely determined by the temperature. The work done by the system during this step is given as:

\[ W_a = P_a V_a = \frac{4aT^4}{3} V_a \]
where \( p_1 \) and \( T_1 \) are the pressure and temperature of the hot reservoir. The energy extracted from the reservoir is given as:

\[
Q_a = T_1 \Delta S_a = \frac{4\alpha V_a r_1^4}{3}
\]

\[\text{Fig. 2. A P-V diagram of the cycle.}\]

The process consists of (a) isothermal and isobaric expansion, (b) adiabatic expansion, and (c) isothermal compression.

(b) In the second step, the cylinder is isolated and the radiation expands adiabatically by pushing the piston to volume \( V_b \), cooling to the temperature \( T_2 \). Because the radiation entropy given by Eq. (7) remains constant, the resulting volume is \( V_b = \left[ V_a \left( \frac{T_1}{T_2} \right)^3 \right] \) and the work done by the system in this step is given by:

\[
W_b = a T_1^4 V_a \left( 1 - \frac{T_2}{T_1} \right)
\]

(c) In the third step, the cylinder is opened to the cold reservoir and the radiation is isothermally pushed out. Because the radiation is pushed to the region with non-zero pressure, work must be performed on the system. The work is given by:

\[
W_c = -P_2 V_b = -\frac{1}{3} a T_1^3 T_2 V_a
\]

And the energy transported to the cold reservoir is given as:

\[
Q_c = T_2 \Delta S_c = \frac{4}{3} a V_b T_2^4 = \frac{4}{3} a V_a T_1^3 T_2
\]

(d) In the last step, the piston is carried through the empty cylinder to its initial position. No working medium is involved, so that no energy exchange or work occurs. The net work in one cycle is therefore,

\[
W = W_a + W_b + W_c
\]

\[
W = \frac{4}{3} a V_a T_1^2 (T_1 - T_2)
\]

\[
W = Q_a - Q_c
\]
In this way it is found that Carnot’s Equation is satisfied.

Therefore, \( \frac{Q_a}{Q_c} = \frac{T_1}{T_2} \)  

Therefore the efficiency of the engine may be written as:

\[ \eta = \frac{W}{Q_a} = 1 - \frac{T_2}{T_1} \]

5. ENERGY EXTRACTION:

In each Carnot’s cycle energy is extracted from the hotter reservoir and deposited to the cooler reservoir. The properties of the reservoir will also change with time. This change can be expressed by Eq. (14) with:

\[ Q_a = -dM_1C^2, \quad Q_c = dM_2C^2, \]

which means that the energy transport is compensated by the decrease or increase of the black hole mass to find a new equilibrium with the radiation. Thus, we obtain

\[ \frac{dM_1}{dM_2} = -\frac{T_1}{T_2} \]

For a black hole \( T \propto \frac{1}{M} \) Therefore, we can write \( M_1dM_1 = M_2dM_2 \)

\[ M_1^2 + M_2^2 = M_{1,0}^2 + M_{2,0}^2 \]

Where \( M_{1,0} \) and \( M_{2,0} \) are the initial masses of the black holes. We can find this result directly by observing that during the reversible processes the total entropy does not change. During this process the hotter reservoir loses mass and becomes hotter and the colder reservoir gains mass and becomes colder. This behaviour is counter intuitive. The temperatures of two bodies in thermal contact converge. Since a black hole has a negative heat capacity, therefore, its temperature increases when energy is taken from it. In theory the black hole of the hot reservoir will ultimately disappear. At the end point the cold black hole will survive with total mass of the system.

\[ M_f = \sqrt{M_{1,0}^2 + M_{2,0}^2} \]

The total work extracted during the entire process is equivalent to the decrease of the mass, We illustrate this situation on the energy-entropy diagram in Fig. 3. The shaded area represents the available states of the system consisting of black holes. The boundary \( S \propto E_2 \) represents states with a single black hole. In the equilibrium states for the given energy the entropy is maximum. If we start the initial state in which the state is not in equilibrium state as Ao of two or more than two black hole then, equilibrium can be reached by different paths. Path AB_1 represents the merging of the two black holes when energy is kept fix. Therefore through this path no work is being done. In this state entropy is maximized with the available mass. Path AB_2 represents a collision of two black holes: a single black hole is produced and part of the energy is carried away by the gravitational waves. This energy is in the range of \( 10^{-3} - 10^{-2} \) of the rest energy of the input black holes [8]. Path AB_4 corresponds to the reversible process described in this section for which the entropy is unchanged and the maximum possible work is extracted. Path AB_3 represents a slightly more realistic version of this process. Because due to imperfections the entropy will increase and less work will be gained.
The shaded area represents no equilibrium states with several black holes in the system, and the boundary $S \propto E^2$ represents equilibrium states with maximum entropy. When system A is in an equilibrium state, it can attain equilibrium by several paths. A difference in the $E$-coordinate represents the work gained from the system.

6. POWER:

The Carnot cycle is idealized because it assumes only the reversible processes, which are infinitely slow. In real processes, a compromise is made between the requirements for the maximum efficiency and the maximum power. In the black hole process we have discussed, the power of the engine will be limited by the rate at which the source of heat can supply the heat energy and the sink can absorb energy, which is given by the radiation power of the black hole. The power of the unused amount of heat, the difference of the two heats, is converted into work. The amount of work done per second is called the power. To maintain the thermal equilibrium energy would be released at a slower rate from the hot reservoir than that of the absorbed by the black hole of larger mass at lower temperature.

7. BLACK HOLE ECONOMY:

If there are two black holes, the first possibility is that they would combine together and there will be gain of useful work and finally a bigger black hole will be created. The second possibility is that the two black holes would be evaporated, some work will be accomplished and some radiation would be produced. The latter option would perform the work faster, but in the end there would be no black hole left in this phenomenon. If we assume that thermal radiation is the ultimate waste, the former option is environmentally friendlier. In this sense, there will be gain of energy by cleaning the mess out of the universe. While growing the holes, we can also extract useful work, but it is possible at much slower pace than that of with evaporating the holes. But, provided that the expansion of the universe continues, and the background radiation cools down, all the black holes will ultimately be hotter than the background radiation and no more work can be obtained by feeding the background radiation to the holes. Therefore by merging two black holes and getting energy out of the radiating the waste energy to empty space would become the only source of useful energy available.

8. DISCUSSION AND CONCLUSION:

In this proposition, we have not considered the role of external radiation outside the boxes and the cylinder. It may be possible only when it is assumed that the universe outside the system is empty. Again in this process, we shall have to assume that this process will never come to a stop point. Because in the long run, the bigger black hole will absorb the smaller one, then what will happen to the...
smaller black hole? Or what will happen to the black hole engines? But if we assume that the process is natural and sustainable then such question will not arise. If it can be found that the larger black hole is continuously increasing in mass the theory of black hole engine will get support. The universe will sustain till the super massive black hole is increasing its mass. For this it is also necessary that in the influence of supper massive black there must also be black holes of smaller masses. Thus in each solar system there will be several black holes having their masses abnormally different. The heat exchange between the reservoir and the engines must be sufficiently slow so that the reservoir has time to reach thermal equilibrium via the interaction of the black hole with the field.

REFERENCES