Review Of Research
Vol. 3 | Issue. 9 | June. 2014
Impact Factor : 2.1002 (UIF)
ISSN:-2249-894X
Available online at www.ror.isrj.net

ORIGINAL ARTICLE

N- DIMENSIONAL BIANCHI TYPE-I (KASNER FORM) COSMOLOGICAL MODEL IN $F(R)$ GRAVITY

L. S. Ladke

Department of Mathematics, Sarvodaya Mahavidyalaya,
Sindewahi, Dist:Chansrapur, INDIA.

Abstract:

N-dimensional Bianchi types-I (Kasner form) cosmological model in f(R) theory of gravity has been considered. The general solution of the field equations of Bianchi type-I in Kasner form with N-dimensions has been obtained by using the concept of special form of deceleration parameter. The physical properties of the model are also studied.

KEYWARDS:

N-dimensions, Bianchi type-I in Kasner form, $f(R)$ theory of gravity, special form of deceleration parameter.

1. Introduction:

We know that the $f(R)$ theory of gravity is the modification of the general theory of relativity proposed by Einstein. The $f(R)$ theory of gravity provides a very natural unification of the early-time inflation and late-time acceleration. As per Nojiri and Odintsov (2007,2008) a unification of the early time inflation and late time acceleration is allowed in $f(R)$ gravity. Non-vacuum solutions in Bianchi type-I & type-V have been obtained by Sharif & Shamir (2010) by using perfect fluid in $f(R)$ gravity. Sharif & Kausar (2011) studied Bianchi type-III space time with anisotropic fluid in $f(R)$

gravitation. Adhav (2012) discussed the Kantowski–Sachs string cosmological model in $f(R)$ gravity. Anisotropic models in $f(R)$ theory of gravity have been studied by Aktas *et al.* (2012). Singha *et al.* (2013) studied functional form of $f(R)$ with power-law expansion in anisotropic model. Recently, Reddy *et al.* (2014) studied vacuum solution of Bianchi type-I and V models in $f(R)$ theory of gravitation using a special form of deceleration parameter.

The study of higher dimensional space-time is important because of the idea that the cosmos at early stage of evolution might have had a higher dimensional era. The possibility that the space-time has more than four dimensions has attracted many researcher to the field of higher dimensions. Wesson (1983, 1984) has studied several aspects of five dimensional space-time in variable mass theory and bimetric theory of relativity. Lorentz and Petzold (1985), Ibanez and Verdaguer (1986), Reddy and Venkateswara (2001), Adhav *et. al.* (2007) have studied the multi dimensional cosmological model in general relativity and in other alternative theories of gravitation. Gomkar *et al.*(2012) have studied N-dimensional static plane symmetric vacuum solution in $f(R)$ gravity. Ladke (2014a,b) has obtained the vacuum solution of Bianchi type-I (Kasner form) cosmological model in $f(R)$ theory of gravity in four and five dimensions and also discuss the physical aspects of the model using special form of deceleration parameter.

Inspiring with the above research work in $f(R)$ gravity, N-dimensional Bianchi type-I (Kasner form) cosmological model is considered. Assuming the special form of deceleration parameter the general solution of the field equations in Bianchi type-I space-time in kasner form with N- dimensions have been obtained. The physical aspects of the model are also discussed.

2. $f(R)$ Theory of Gravity and Deceleration Parameter:

The $f(R)$ theory of gravity is the modification of general theory of relativity

The corresponding field equations of $f(R)$ theory of gravity in V_n are given by

$$F(R)R_{ij} - \frac{1}{2} f(R) g_{ij} - \nabla_i \nabla_j \, F(R) + g_{ij} \,\Box\, F(R) = \kappa T_{ij} \;, \quad (i, j = 1,2,......n),$$

(2.1)

where $F(R) = {df(R)}/{dR}$, $\Box \equiv \nabla^i \nabla_i$,

(2.2)

with ∇_i the covariant derivative, T_{ij} is the standard matter energy-momentum tensor.

After contraction of the field equations (2.1), we get

$$F(R)R - \frac{n}{2} f(R) + (n-1) \,\Box\, F(R) = \kappa T \,,$$

(2.3)

In vacuum this field equation (2.3) reduced to

$$F(R)R - \frac{n}{2} f(R) + (n-1) \,\Box\, F(R) = 0 \,,$$

(2.4)

This yields a relationship between $f(R)$ and $F(R)$.

Inserting this value of $f(R)$ from equation (2.4) in the vacuum field equations (2.1), we get

$$\frac{1}{n}[F(R)R - \Box\, F(R)] = \frac{F(R)R_{ij} - \nabla_i \nabla_j F(R)}{g_{ij}} .$$

(2.5)

Since the left side does not depend on the index i , so the field equation can be expressed as

$$K_i \equiv \frac{F(R)R_{ii} - \nabla_i \nabla_i F(R)}{g_{ii}}$$

(2.6)

is independent of the index i and hence $K_i - K_j = 0$ for all i and j.

Here K_i is just a notation for the traced quantity.

By assuming various physical or mathematical conditions, many authors obtain the exact solution of the modified Einstein's field equations. Many authors use condition of deceleration parameter. In 1983 Berman proposed a law of variation for

Hubble parameter which yields constant deceleration parameter models of the universe. Akarsu and Kilinc (2010), Yadav *et al*. (2011) and Adhav (2011a, 2011b) have extended this law for Bianchi types models. Recently, Adhav (2011c, 2011d) extended this law for Bianchi type-I , Bianchi type-V anisotropic cosmological models respectively. Cunha & Lima (2008) favours recent acceleration and past deceleration. Singha and Debnath (2009) has defined a special form of deceleration parameter for FRW metric as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{\alpha}{1+a^\alpha}, \tag{2.7}$$

where $\alpha > 0$ is a constant and a is mean scale factor of the universe.

After solving equation (2.7) one can obtain the mean Hubble parameter H as

$$H = \frac{\dot{a}}{a} = k\left(1 + \frac{1}{a^\alpha}\right), \tag{2.8}$$

where k is a constant of integration.

On integrating equation (2.8), we obtain the mean scale factor as

$$a = (e^{k\alpha t} - 1)^{1/\alpha}. \tag{2.9}$$

The physical parameters that are of cosmological importance are

The mean anisotropy parameter: $$\Delta = \frac{1}{(n-1)}\sum_{i=1}^{(n-1)}\left(\frac{H_i - H}{H}\right)^2. \tag{2.10}$$

The shear scalar: $$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{(n-1)} H_i^2 - (n-1)H^2\right). \tag{2.11}$$

The expansion scalar : $$\theta = (n-1)H. \tag{2.12}$$

3. Bianchi type-I (Kasner form) Cosmological Model:

The line element for Bianchi type-I space-time in Kasner form in *n*- dimensions is

$$ds^2 = dt^2 - \sum_{i=1}^{(n-1)} \left(t^{p_i} dx^i\right)^2 ,$$

(3.1)

where p_i , $i = 1,2,....(n-1)$ are constant.

The corresponding Ricci scalar becomes

$$R = 2\left[\sum_{i=1}^{(n-1)} \frac{p_i}{t^2}(p_i - 1) + \sum_{i=1}^{(n-2)} \frac{p_i}{t}\left(\sum_{j=i+1}^{(n-1)} \frac{p_j}{t}\right)\right],$$

(3.2)

The corresponding field equations for the metric (3.1) using vacuum equation (2.1), we get

$$\sum_{\substack{i=1 \\ i\neq k}}^{(n-1)} \frac{p_i}{t^2}(p_i - 1) + \sum_{\substack{i=1 \\ i\neq k}}^{(n-2)} \frac{p_i}{t}\left(\sum_{\substack{j=i+1 \\ j\neq k}}^{(n-1)} \frac{p_j}{t}\right) + \sum_{\substack{i=1 \\ i\neq k}}^{(n-1)} \frac{\dot{p}_i}{t} \frac{\dot{F}}{F} - \frac{\ddot{F}}{F} = 0 ,$$

(3.3)

We define the spatial volume V as

$$V = a^{(n-1)} = t^S .$$

(3.4)

where $S = p_1 + p_2 + p_3 + ... + p_{n-1}$.

(3.5)

and $V = t^S \Rightarrow a^3 = t^{S/n-1}$.

(3.6)

Subtracting equation (3.3) for $k = 1$ from equation (3.3) for $k = 2$ and solving, we get

$$t^{p_1} = d_1 t^{p_2} \exp\left(x_1 \int \frac{dt}{t^S F}\right) ,$$

(3.7)

where $S = p_1 + p_2 + p_3 + ... + p_{n-1}$.

Subtracting equation (3.3) for $k = 2$ from equation (3.3) for $k = 3$ and solving, we get

$$t^{p_2} = d_2 t^{p_3} \exp\left(x_2 \int \frac{dt}{t^S F} \right) \quad ,$$

(3.8)

Subtracting equation (3.3) for $k=3$ from equation (3.3) for $k=4$ and solving, we get

$$t^{p_3} = d_3 t^{p_4} \exp\left(x_3 \int \frac{dt}{t^S F} \right) \quad ,$$

(3.9).

Subtracting equation (3.3) for $k=n-2$ from equation (3.3) for $k=n-1$ and solving, we get

$$t^{p_{n-2}} = d_{n-2} t^{p_{n-1}} \exp\left(x_{n-2} \int \frac{dt}{t^S F} \right) \quad ,$$

(3.10)

Subtracting equation (3.3) for $k=1$ from equation (3.3) for $k=n-1$ and solving, we get

$$t^{p_1} = d_{n-1} t^{p_{n-1}} \exp\left(x_{n-1} \int \frac{dt}{t^S F} \right) \quad ,$$

(3.11)

where $d_{n-1} = d_1 d_2 .... d_{n-2}$, $x_{n-1} = x_1 + x_2 + ... + x_{n-2}$.

Equations (3.7), (3.8),…., (3.11) can be written as

$$t^{p_1} = D_1 t^{S/n-1} \exp\left(Y_1 \int \frac{dt}{t^S F} \right) ,$$

(3.12)

$$t^{p_2} = D_2 t^{S/n-1} \exp\left(Y_2 \int \frac{dt}{t^S F} \right) ,$$

(3.13)

$$t^{p_3} = D_3 t^{S/n-1} \exp\left(Y_3 \int \frac{dt}{t^S F} \right) ,$$

(3.14)

.
.

$$t^{p_{n-1}} = D_{n-1} t^{S/_{n-1}} \exp\left(Y_{n-1} \int \frac{dt}{t^S F} \right) ,$$

(3.15)

In general each t^{p_i} can be written as

$$t^{p_i} = D_i t^{S/_{n-1}} \exp\left(Y_i \int \frac{dt}{t^S F} \right) ,$$

(3.16)

where $\prod_{i=1}^{n-1} D_i = 1$ and $\sum_{i=1}^{n-1} X_i = 0$ are satisfied by D_i and X_i for $i = 1,2,.... n-1$.

(3.17)

To solve the integral part in the above equation we use the power-law. In recent year Kotub Uddin *et al.*(2007) and Sharif & Shamir (2009) have established a result in the context of $f(R)$ gravity which shows that

$$F \propto a^m \quad \text{i.e.} \quad F = h a^m ,$$

(3.18)

where h is the constant of proportionality and m is any integer which is taken as -2.

Case i) When *n* (dimension) is even i.e. $n = 4,6,8,...$

Using equations (2.9),(3.18) for $k = 1$, $\alpha = 2$ and $m = -2$ in the equations and noting (3.4), The value of the scale factor in V_4 becomes

$$t^{p_i} = D_i (e^{2t} - 1)^{\frac{1}{2}} \exp\left\{ \frac{Y_i}{h} \left[\psi_1 + \tan^{-1}\left(e^{2t} - 1\right)^{\frac{1}{2}} \right] \right\} ,$$

(3.19)

Here D_i , Y_i for $i = 1,2,3$ are constants and $\psi_1 = 0$.

The value of the scale factor in V_6 are

$$t^{p_1} = D_i (e^{2t} - 1)^{\frac{1}{2}} \exp\left\{ \frac{Y_i}{h} \left[\psi_2 + \tan^{-1}\left(e^{2t} - 1\right)^{\frac{1}{2}} \right] \right\} ,$$

(3.20)

Here D_i , Y_i for $i = 1,2,3,4,5$ are constants and ψ_2 is a function of *t*.

In V_8, the value of the scale factor are

$$t^{p_1} = D_i (e^{2t} - 1)^{\frac{1}{2}} \exp\left\{ \frac{Y_i}{h} \left[\psi_3 + \tan^{-1}\left(e^{2t} - 1\right)^{\frac{1}{2}} \right] \right\},$$

(3.21)

Here D_i , Y_i for $i = 1,2,3,4,5,6,7$ are constants and ψ_3 is a function of *t*.

and so on…..

From equations (3.19),(3.20),(3.21)…. it is observed that when *n* (dimension) is even then scale factors are given by

$$t^{p_1} = D_i (e^{2t} - 1)^{\frac{1}{2}} \exp\left\{ \frac{Y_i}{h} \left[\psi + \tan^{-1}\left(e^{2t} - 1\right)^{\frac{1}{2}} \right] \right\},$$

(3.22)

where ψ is a function of *t* or zero.

Case ii) When *n* (dimension) is odd i.e. $n = 5,7,9,...$

In V_5 , the value of the scale factor are

$$t^{p_1} = D_i (e^{2t} - 1)^{\frac{1}{2}} \exp\left\{ \frac{Y_i}{h} \left[\phi_1 + \log\left(\frac{e^{2t} - 1}{e^{2t}} \right)^{\frac{1}{2}} \right] \right\},$$

(3.23)

Here D_i , Y_i for $i = 1,2,3,4$ are constant and $\phi_1 = 0$.

In V_7 , the value of the scale factor are

$$t^{p_1} = D_i (e^{2t} - 1)^{\frac{1}{2}} \exp\left\{ \frac{Y_i}{h} \left[\phi_2 + \log\left(\frac{e^{2t} - 1}{e^{2t}} \right)^{\frac{1}{2}} \right] \right\},$$

(3.24)

where D_i , Y_i for $i = 1,2,3,4,5,6$ are constants and ϕ_2 is a function of *t*.

In V_9 , the scale factors are given by

$$t^{p_1} = D_i (e^{2t} - 1)^{\frac{1}{2}} \exp\left\{ \frac{Y_i}{h} \left[\phi_3 + \log\left(\frac{e^{2t} - 1}{e^{2t}} \right)^{\frac{1}{2}} \right] \right\},$$

(3.25)

Here D_i , Y_i for $i = 1,...8$ are constants and ϕ_3 is a function of *t*.

and so on…..

From equations (3.23),(3.24),(3.25), …. it is observed that when *n* (dimension) is odd, the scale factors are given by

$$t^{p_1} = D_i (e^{2t} - 1)^{\frac{1}{2}} \exp\left\{ \frac{Y_i}{h} \left[\phi + \log\left(\frac{e^{2t} - 1}{e^{2t}} \right)^{\frac{1}{2}} \right] \right\} ,$$

(3.26)

where ϕ is a function of *t* or zero.

4. CONCLUSION:

As the scale factor for even and odd dimensions are of similar form to the scale factors obtained in four and five dimension by Ladke (2014a,b) using special form of deceleration parameter therefore we conclude that this N-dimensional Bianchi type-I (Kasner form) cosmological model behaves in the same manner as in four and five dimensions. It is interesting to note that all the results obtained for $n = 4,6,8,...$ even and for $n = 5,7,9,,...$ odd are similar to the results obtained earlier by Ladke (2014a,b) respectively.

REFERENCES:

1. Adhav. K.S. *et al*.: Astrophysics and Space Science, **310,** (3-4), 231, (2007).
2. Adhav. K.S. *et al*.: Astrophysics and Space Science, **332**(2), 497(2011a).
3. Adhav. K.S. *et al*.: Cent. Eur. J. Phys., **9**(4) , 919 (2011b).
4. Adhav. K.S. *et al*.: Eur.Phys. J. Plus, **126** , 122 (2011c).
5. Adhav. K.S. *et al*.: Int. Jou.Math.Arch., **2(11)** , 2149 (2011d).
6. Adhav, K.S.: Can.J.Phys.,**90**,119, (2012).
7. Akarsu O.and Kilinc C.B., Gen. Relat. Gravit. **42**, 763 (2010).
8. Berman, M.S.: Nuovo Cimento B, **74**, 2, 182, (1983)
9. Cunha J.V. and Lima J.A.S.: Mon. Not. R. Astron. Soc.: **390**(1),210 (2008).
10. Gomkar, *et al.* : OIIRJ. **2**, 2,pp.292-299, (2012).
11. Ibanez, Z. and Verdaguer,E.: Phys. Rev. D, **34**, 1202, (1986).

12. Kotub Uddin et al. “Cosmological perturbations in Palatini-modified gravity” Class. uantum Grav. 24 , pp.3951-3962 (2007).
13. Ladke, L.S.: IJIRSET, **3**,6,(2014)
14. Ladke, L.S.: OIIRJ, (2014)
15. Lorentz and Petzold : Gen. Relativ. Grav. **17**, 1189, (1985).
16. Nojiri S and Odintsov S D : J. Phys. Conf.Ser. **66**, 012005,(2007).
17. Nojiri S and Odintsov S D : Phys. Rev. **D, 78**, 046006, (2008).
18. Reddy, D.R.K. and Venkateswara Rao,N.: Astrophys Space Sci, **277**, 461, (2001).
19. Reddy, D.R.K. *et al.*: IJSAT, **4**, 3, (2014).
20. Sharif,M. and Shamir,M.F. : Class. Quantum Grav. **26** , 235020 (2009).
21. Sharif,M. and Shamir,M.F. : Gen. Relativ. Grav. **42**, 1557 (2010).
22. Sharif, M. and Kausar, H.R.: Phys. Lett. B, **697**, 1, (2011).
23. Singh,V . and Singh, C.P.: Astrophys Space Sci, **346**, 285, (2013).
24. Singha A.K. and Debnath U.: Int. J. Theor Phys., **48**(2),351 (2009).
25. Wesson, P.S. : Astro. and Astrophys. **119**, 145 (1983).
26. Wesson, P.S.: Gen. Relativ. Grav. **16**, 193 (1984).
27. Yadav,A.K. *et al.* : Int. J. Theor. Phys. **50**, 871-881, (2011).