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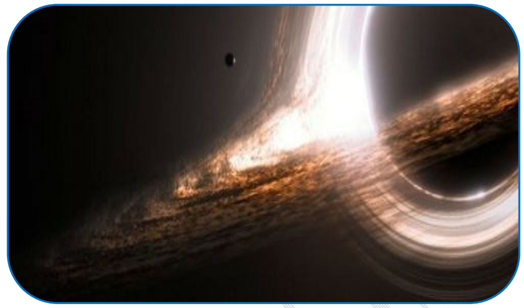
STUDY OF THERMODYNAMIC ENTROPY OF BLACK HOLES WITH APPLICATION TO FLUCTUATION THEORY



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ABSTRACT:

In recent years there has been enormous progress in understanding the entropy and other thermodynamic properties of black holes within string theory going well beyond the thermodynamic limit. It has become possible to begin exploring finite size effects in perturbation theory in inverse size with highly nontrivial agreements between thermodynamics and statistical mechanics. Thermodynamic fluctuation theory also has applications to a wide variety of thermodynamic parameters of black holes supported by other methods. In this research paper some thermodynamic parameters of black-holes such as the entropy has been supposed to be determined by application of fluctuation theory applied in thermodynamic equilibrium state of the system. The decisive results have been found to agree to a good approximation with the reported results in the literatures.

KEYWORDS: *Black holes, Thermodynamic properties, Thermodynamic fluctuations.*

1. INTRODUCTION:

Black holes are one of the simplest as well as the most complex astronomical bodies in the solar system. It is simplest because it is completely specified by its mass, charge and spin. It can be said that a black hole is very much like a structure-less elementary particle such as an electron [1,2,4]. For an astrophysical object like the earth, the gravitational field around it depends not only on its mass but also on how the mass is distributed and on the details of the oblations of the earth and on the shapes of the valleys and mountains. It is not so for a black hole. Once a star collapses to form a black hole, the gravitational field around it forgets all details about the star that disappears behind the event horizon except for its mass , spin, and charge [5,13,]. And yet it is the most complex in the sense that it possesses huge entropy. In fact the entropy of a solar mass black hole is enormously bigger than the thermal entropy of the star that might have collapsed to form it. Entropy gives an account of the number of microscopic states of a system. Hence, the entropy of a black hole signifies an incredibly complex microstructure. In this respect, a black hole is very unlike an elementary particle. A black hole may be supposed to be as an asymptotically flat space time that contents a region which is not in the backward light cone of future time like infinity. The boundary of such a region is a stationary null surface which has been is called the event horizon.

2. BLACK HOLES PARAMETERS:

There are a number of important parameters of the black holes. For general black holes their actual values are different but for all black holes these parameters govern the thermodynamics of black holes [7].

[1] The radius of the event horizon r_H is the radius of the two spheres. For a Schwarzschild black hole we have Rh = 2GM.
[2] The area of the event horizon A_H is given as:
$A_H = 4 \pi Rh^2$. For a Schwarzschild black hole we have $A_H = 16 \pi G^2 M^2$.
[3] The surface gravity is the parameter k. For a Schwarzschild black hole we have k = 1/ 4GM.

3. BLACK HOLE MECHANICS:

One of the remarkable properties of black holes is that we can derive a set of laws of black hole mechanics which bear a very close resemblance to the laws of thermodynamics. This is quite surprising because a priori there is no reason to expect that the space time geometry of black holes has anything to do with thermal physics [8, 9].

(0). Zeroth Law: In thermal physics, the zeroth law states that the temperature T of body at thermal equilibrium is constant throughout the body. Otherwise heat will flow from hot spots to the cold spots. Correspondingly for stationary black holes, we can show that surface gravity κ is constant on the event horizon. This is obvious for spherically symmetric horizons but is true also more generally for non-spherical horizons of spinning black holes [10].

(1) First Law: Energy is conserved, $dE = T\,dS + \mu dQ + \Omega dJ$, where E is the energy, Q is the charge with chemical potential μ and J is the spin with chemical potential Ω. Correspondingly for black holes, we may have:

$dM = \frac{k}{8\pi G}\, dA + \mu dQ + \Omega dJ$. ---[1]

For a Schwarzschild black hole we have $\mu = \Omega = 0$ because there is no charge or spin.

(2) Second Law: In a physical process the total entropy S never decreases,

Thus $\Delta S \geq 0$. ---[2]

Correspondingly for black holes we can prove the area theorem that the net area in any process never decreases $\Delta A \geq 0$. For example, two Schwarzschild black holes with masses M_1 and M_2 can coalesce to form a bigger black hole of mass M. This is consistent with the area theorem since the area is proportional to the square of the mass and $(M_1 + M_2)^2 \geq (M_1 + M_2)$. The opposite process where a bigger black hole fragments is however disallowed by this law. Thus the laws of black hole mechanics, crystallized by Bardeen, Carter, Hawking, and other bears a striking resemblance with the three laws of thermodynamics for a body in thermal equilibrium [11,12,13].

4. FLUCTUATION THEORY:

Physical quantities which describe a macroscopic body in equilibrium are almost always close to their mean value. However there are always certain deviations from the mean value which is the natural behaviour of the system. These deviations are called the thermodynamic fluctuations. The main problem is to find the probability distribution of these distributions [10]. The entropy of the system may be written as:

$S = \log_e w$

This gives $w = e^s$ ---[3]

However, we denote p as the probability distribution as $p \propto e^s$.

Applying the Tailor theory, we can write the fluctuation in x as:

$S(x) = s(0) - ½ \beta x^2$ --[4]

Where $\beta = \frac{\partial^2 s}{\partial x^2} \Big|_0$ and $s(0) = 0$

Since the entropy s has a maximum For x= 0 as

$\frac{ds}{dx}$= 0. From equation (1) and (2), it can be written as :

$P(x) = A\ e - 1/2\beta\ x^2$ --[5]

In differential form it can be written as:

$P(x)\ dx = (A\ e - 1/2\beta\ x^2)\ dx$ --[6]

The constant A is given by the normalization condition which gives

$\int p(x)dx= 1$ --[7]

The integration limit is over all space ie from -∞ to + ∞ . This constant is found to be equal to $\sqrt{\frac{\beta}{2\pi}}$ by Gaussian integration formula. Thus the probability distribution of the various values of the Fluctuation is given as :

$$P(x) = \sqrt{\beta \frac{e}{2\pi}} - \sqrt{\frac{\beta}{2\pi} - \frac{1}{2}\beta x^2} \quad \text{---(8)}$$

This probability distribution is categorized as Gaussian distribution. It reaches a maximum value when x= 0. It decreases rapidly and symmetrically as modulus of x increases. The mean square fluctuation is defined as :

$$\langle x^2 \rangle = \int p(x) x^2 dx = \frac{1}{\beta} \quad \text{--[9]}$$

Therefore, the Gaussian distribution may be written as

$$P(x)\ dx = \frac{1}{\sqrt{2\pi\langle x^2 \rangle}} \exp\left(-\frac{x^2}{2\langle x^2 \rangle}\right) dx \quad \text{--[10]}$$

It is readily seen that the smaller the $\langle x^2 \rangle$ the sharper the maximum of p(x) , which is the characteristics of the Gaussian distribution for more than one variable. The simultaneous deviation of several thermodynamic quantities can be determined from their mean values. We define the entropy S (x 1 x 2 x 3 .....x n) as a function of the quantities of a simultaneous deviations and the Tailor expansion S in the same manner as done before.

$$S\text{-}S0 = -\frac{1}{2}\sum_{ij=1}^{p} \beta ij\ xi\ xj \text{ --}[11]$$

Since Bij =Bji For simplicity the summation sign can be omitted.

Thus we can write S- S 0 = 1/2Bij xi xj ----------------------[12]

And the probability takes the form

$$P = A \exp\left(-\tfrac{1}{2}\beta_{ij}x_ix_j\right) \text{ --}[13]$$

Here A is the normalization constant. Its value is determined by the Condition given below:

$$\int dx1 \int dx2 \int dx3 \int dx4\ p(x_1x_2x_3x_4......x_n) = 1 \text{ --------------------}[14]$$

After some algebraic manipulation the value of A can be found to be:

$$A = \frac{\sqrt{\beta}}{(2\pi)^{\frac{n}{2}}} \text{ ---}[15]$$

Therefore the required form of the Gaussian distribution for more than one variable may be written as:

$$P = \frac{\sqrt{\beta}}{(2\pi)^{\frac{n}{2}}} \exp\left(-\tfrac{1}{2}\beta ij\ xixj\right) \text{ --}[16]$$

$$\text{Where } \beta\ ij = -\frac{\partial 2\ s}{\partial xi \partial xj} \text{ and } \beta = |\beta ij| \text{ --------------------------}[17]$$

5. BLACK HOLE ENTROPY:

Entropy is, in a sense, a measure of the disorder of a system. This quantity was first introduced by R. Clausius in 1850 as the amount of heat reversibly exchanged at a temperature T. Entropy undoubtedly plays a major role in thermodynamics and statistical mechanics. It is also the most extensive parameter in thermodynamics, namely when it is expressed in terms of other extensive parameters- it basically tells us the physics that underlines the system [3]. Entropy is a part of the first and the second law of thermodynamics directly- it enters the first law to complete the differential representation of the internal energy, namely

dU=TdS-pdV+1?2---[18]

We can also express the entropy as a thermodynamic potential as:

$$dU = \frac{1}{T}dU + \frac{P}{T}dV - \frac{\mu}{T}Dn \text{ ---}[19]$$

Which is the differential form of the entropy. In this view, if the dependence of the entropy S(U,V,N) on the variables U, V, N is known, then complete knowledge of all the thermodynamics parameters can be obtained. Furthermore, the entropy tells us that for isolated systems, Where dQ reversible=0, in equilibrium dS=0. The entropy of a black hole makes for a fascinating study in the history of science. It is one of the very rare examples where a scientific idea has gestated and evolved over several decades into an important conceptual and quantitative tool almost entirely on the strength of theoretical considerations. It is therefore worthwhile to place black holes and their entropy in a broader context before coming to the more recent results pertaining to the quantum aspects of black holes within string theory. A black hole is now so much a part of our vocabulary that it can be difficult to appreciate the initial intellectual opposition to the idea of 'gravitational collapse' of a star and of a 'black hole' of nothingness in space time by several leading physicists, including Einstein himself.

The Schwarzschild solution was immediately accepted as the correct description within general relativity of the gravitational field outside a spherical mass. It would be the correct approximate description of the field around a star such as our sun [15]. But something much more bizarre was implied by the solution. For an object of mass M, the solution appeared to become singular at a radius $R = 2GM/c^2$. For our sun, for example, this radius, now known as the Schwarzschild radius, would be about three kilometres. Now, as long the physical radius of the sun is bigger than three kilometres, the 'Schwarzschild's singularity' is of no concern because inside the sun the Schwarzschild solution is not applicable as there is matter present. But what if the entire mass of the sun was concentrated in a sphere of radius smaller than three kilometres. We would then certainly have to face up t o this singularity. Einstein's reaction to the 'Schwarzschild singularity' was to seek arguments that would make such a singularity inadmissible. It is interesting that Einstein's paper on the inadmissibility of the Schwarzschild singularity appeared only two months before Oppenheimer and Snyder published their definitive work on stellar collapse with an abstract that read, "When all thermonuclear sources of energy are exhausted, a sufficiently heavy star will collapse. "Once a sufficiently big star ran out of its nuclear fuel, then there was nothing to stop the inexorable inward pull of gravity. The possibility of stellar collapse meant that a star could b e compressed in a region smaller than its Schwarzschild radius and the 'Schwarzschild singularity' could no longer be wished away as Einstein had desired. Indeed it was essential to understand what it means to understand the final state of the star.

6. BEKENSTEIN-HAWKING ENTROPY:

Even though we have "derived" the entropy in the context of fluctuation theory, this beautiful relation between area and entropy is true quite generally essentially because the near horizon geometry is always Rindler-like. For all black holes with charge, spin and in number of dimensions, the Hawking temperature and the entropy are given in terms of the surface gravity and horizon area by the formulae. This is a remarkable relation between the thermodynamic properties of a black hole on one hand and its geometric properties fluctuation on the other. The fundamental significance of entropy stems from the fact that even though it is a quantity defined in terms of gross thermodynamic fluctuation properties, it contains nontrivial information about the microscopic structure of the theory through Boltzmann relation $S = k \log(d)$, where d is the degeneracy or the total number of microstates of the system of for a given energy, and k is Boltzmann constant. Entropy is not a kinematic quantity like energy or momentum but rather contains information about the total number microscopic degrees of freedom of the system. Because of the Boltzmann relation, one can learn a great deal about the microscopic properties of a system from its thermodynamics properties.

7. CONCLUSION:

Black hole density led to a novel insight into black hole entropy. Black hole entropy may prove to be as malleable as the conservation of energy. Whenever energy conservation was challenged, a way may be found to preserve by applying the fluctuation theory. It is not clear why this simple model works so well. For little black holes this may be due to asymptotic freedom of fluctuation. This model expects the entropy of a black hole directly related to its horizon area, even in the non-equilibrium state. It says that the black hole area reflects the number and entropy of the constituents inside it.

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