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A GOAL PROGRAMMING APPROACH TO SOLVE GENERAL MULTILEVEL PROGRAMMING PROBLEMS

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ABSTRACT:
In this paper an algorithm is given to solve a multi level programming problem using a linear preemptive goal programming model. A goal programming problem is formulated which is equivalent to given multi-level programming problem.

KEYWORDS: *multilevel programming, Goal programming, optimal.*

INTRODUCTION

The general multi level programming problem has been an area of active research for many years and there have been a number of successful practical applications of the problem in areas like government, autonomous institutions, agriculture, military, maintenance, management, networks, schools, hospitals, banks etc. A multi-level programming problem (MLLP) is characterized by presence of multiple linear objective functions subject to the usual linear constraints. One of the important characteristics of multi-level programming problem is that a planner at a certain level of hierarchy may have their objective function and decision space is determined partially by other levels. As a class of MLPP, most of the developments focus on bi-level linear programming. (Bard, 1982; 1984; Lee, 1972; Steur, 1986). A fuzzy goal programming approach for solving multilevel programming problems with fuzzy parameters was presented which was based on D-cut and fuzzy goal programming.(Pramanik, 2015). A solution of single objective linear goal programming problem with neutrosophic numbers was given in which the neutrosophic numbers are transformed into interval numbers. (Banerjee and Pramanik, 2018). The basic concept of the MLPP technique that the first level decision maker (DM) sets his goal, then asks each subordinate level of the organization for their optima. The lower level decision makers are then submitted and modified first DM in consideration of the over all benefit for the organization, the process continues until a compromise solution is reached.

General linear multi-level programming problem in which we have leader problem with n follower problems is defined as:

MLPP

$$\underset{x_1}{Max}\, F(x_1, x_2, \ldots, x_n) = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n$$

$$\underset{x_2}{Max}\, F(x_1, x_2, \ldots, x_n) = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n$$

$$\cdots \qquad \cdots \qquad \cdots$$

$$\cdots \qquad \cdots \qquad \cdots$$

$$\underset{x_n}{Max}\, F(x_1, x_2, \ldots, x_n) = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n$$

Subject to $C_1x_1 + C_2x_2 + \cdots + C_nx_n \le r$

$x_1, x_2, \ldots, x_n \ge 0$

Where

$a_{11}, a_{21}, \ldots, a_{n1}, x_1 \in R^{n_1}, a_{21}, a_{22}, \ldots, a_{n2}, x_2 \in R^{n_2}, \ldots . a_{n1}, a_{n2}, \ldots, a_{nn}, x_n \in R^{nn}$

$C_1, C_2, \ldots, C_n$ are the matrices $m \times n_1, m \times n_2, \ldots, m \times n_n$ respectively.

Let $S = \{(x_1, x_2, \ldots, x_n) : C_1x_1 + C_2x_2 + \cdots + C_nx_n \le r\}$ denote the constraint region of MLPP. For given $\overline{x_n}$,

let $S(\overline{x_n}) = \{(x_1, x_2, \ldots x_{n-1}) : C_1x_1 + C_2x_2 + \cdots + C_{n-1}x_{n-1} \le r - C_nx_n\}$ denote the (n-1)th follower's solution space.

Let $X(\overline{x_n})$ denote the set of optimal solutions to the (n-1)th follower's problem $Max\, \{Cx_i : (x_1, x_2, \ldots, x_{n-1}) \in S(\overline{x_i})\},\ i = 1, 2 \ldots, n$.

The leader's solution space is defined as

$P(x_1, x_2, \ldots, x_n) : \{(x_1, x_2, \ldots, x_n) : (x_1, x_2, \ldots, x_n) \in S,\ x_1 \in X(\overline{x_1}, \overline{x_2}, \ldots, \overline{x_n})\}$ where $X(\overline{x_1}, \overline{x_2}, \ldots, \overline{x_n})$ is the set of optimal solution to the first follower's problem..

The MLPP can be expressed as

$$\underset{x_i}{Max}\{a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n : (x_1, x_2, \ldots, x_n) \in S,\ x_i \in X(x_i)\} \quad \forall\, i = 1, 2, \ldots, n.$$

With notions of feasibility and optimality defined as follows:

Definition 1. A point $(x_1, x_2, \ldots, x_n)$ is said to be feasible to the MLPP if $(x_1, x_2, \ldots, x_n) \in P$.

Definition 2. A point $(x_1^*, x_2^*, \ldots, x_n^*)$ is said to be an optimal solution of the MLPP if

$$\sum_{i=1}^{n} a_{1i}x_i^* \ge \sum_{i=1}^{n} a_{1i}x_i \qquad \forall\, i = 1, 2, \ldots, n.$$

It is assumed that S and P are bounded and non empty as it generates the existence of optimal solution of the MLPP.

This problem then becomes maximizing the degree of attainment of these goals called goal programming (GP). GP was introduced by Charnes and Cooper and developed (Candler and Townsley, 1982; Ijiri, 1965). The main idea behind GP is to minimize the distance between the objective function Z and aspiration level $\overline{Z}$. The aspiration level $\overline{Z}$ is determined by the decision maker or the decision analyst.

Now consider the linear multi-objective model

(MP) $\quad Max\, Z = \sum Ax_i$

Subject to $Cx \le q,\quad x \ge 0$

Where $x \in R^N, Z = (z_1, z_2, \ldots, z_k)^T$ is the vector of objectives A is a $K \times N$ matrix of objectives and C is $M \times N$ matrix and $q \in R^M$.

One of the approaches to solve the multiobjective programming is GP approach. This approach minimizes the distance between the objective function vector Z and an aspiration level vector Z^*. The aspiration level is either determined by the decision maker or its taken as $Z^* = (z_1^*, z_2^*, \ldots, z_k^*)$, where z_k^* is the optimal value of z_k subject to the set of constraints in MP.

General preemptive GP model to solve MP is given by

(GP) $$Min\, Z = \left\{ \sum_{k \in p_i} w_k g_k(n_k, p_k),\ i = 1, 2, \ldots, n \right\}$$

Subject to $$Cx \le q$$

$$A_k x + n_k - p_k = Z_k^*$$

$$x \ge 0$$

$n_k,\ p_k \ge 0, n_k p_k = 0,\ k = 1, 2, \ldots, K$.

$n_k,\ p_k$ are deviational variables and w_k are their weights and $g_k(n_k, p_k) = p_k$ in case of minimizing z_k and $g_k(n_k, p_k) = n_k + p_k$ when $z_k = z_k^*$ is required. A_k is the k^th^ row vector of matrix A. it the number of priority levels, $k \in P_i$ means that the k^th^ goal is in the i^th^ priority level.

Since MLPP is NP hard, it is not so easy to solve it. Here an algorithm to solve MLP problem using preemptive goal programming model is presented.

FORMULATION OF GOAL PROGRAMMING PROBLEM EQUIVALENT TO MULTI-LEVEL PROGRAMMING PROBLEM

Phase I. In this phase the MLPP is converted into an equivalent GP problem. We consider the problem P1 given by

(P1)
$$\begin{aligned}
&\underset{x_1}{Max}\, F(x_1, x_2, \ldots, x_n) = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\
&\underset{x_2}{Max}\, F(x_1, x_2, \ldots, x_n) = a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\
&\cdots \qquad \cdots \qquad \cdots \\
&\cdots \qquad \cdots \qquad \cdots \\
&\underset{x_n}{Max}\, F(x_1, x_2, \ldots, x_n) = a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n
\end{aligned}$$

Subject to $$C_1x_1 + C_2x_2 + \cdots + C_nx_n \le r$$

$$x_1,\ x_2, \ldots,\ x_n \ge 0$$

Let $F_1^{(1)}$ be the maximum value of F corresponding to the points $(x_{1i}^{(1)},\ x_{2i}^{(1)}, \ldots, x_{ni}^{(1)}) \in S,\quad i = 1, 2, \ldots, n^1$. Therefore

$$\begin{aligned}
F_1^{(1)} &= F_1\left(x_{1i}^{(1)}, x_{2i}^{(1)}, \ldots, x_{ni}^{(1)}\right) = a_{11}x_{1i}^{(1)}, a_{12}x_{2i}^{(1)}, \ldots, a_{1n}x_{ni}^{(1)} \\
F_2^{(1)} &= F_2\left(x_{1i}^{(1)}, x_{2i}^{(1)}, \ldots, x_{ni}^{(1)}\right) = a_{21}x_{1i}^{(1)}, a_{22}x_{2i}^{(1)}, \ldots, a_{2n}x_{ni}^{(1)} \\
&\cdots \qquad \cdots \qquad \cdots \\
&\cdots \qquad \cdots \qquad \cdots \\
F_n^{(1)} &= F_n\left(x_{1i}^{(1)}, x_{2i}^{(1)}, \ldots, x_{ni}^{(1)}\right) = a_{n1}x_{1i}^{(1)}, a_{n2}x_{2i}^{(1)}, \ldots, a_{nn}x_{ni}^{(1)} \\
&\qquad \forall i = 1, 2, \ldots, n^{(1)}
\end{aligned}$$

Calculate the value of lower level objective function $f(x_1, x_2, \ldots, x_n)$ at these points $\left(x_1^{(1)}, x_2^{(1)}, \ldots, x_n^{(1)}\right)$.

Let $f_i^{(1)} = f(x_{1i}^{(1)}, x_{2i}^{(1)}, \ldots, x_{ni}^{(1)});\quad i = 1, 2, \ldots, n^{(1)}$.

Arrange $f_i^{(1)}, \quad i=1,2,...,n^{(1)}$ in decreasing order. Let

$$f(x_{11}^{(1)}, x_{21}^{(1)},...,x_{n1}^{(1)}) \geq f(x_{12}^{(1)}, x_{22}^{(1)},...,x_{n2}^{(1)}) \geq \cdots\cdots\cdots \geq f(x_{1n}^{(1)}, x_{2n}^{(1)},...,x_{nn}^{(1)})$$

$$i.e., \qquad f_1^{(1)} \geq f_2^{(1)} \geq \cdots\cdots\cdots \geq f_n^{(1)}$$

Then the preemptive GP model of MLPP at the point $\left(x_1^{(1)}, x_2^{(1)},...,x_n^{(1)}\right)$ can be formulated (P2) given by

(P2) $$Min\, P_1(d_1^- + d_1^+)$$

$$Min\, P_2(d_2^- + d_2^+)$$

... ...

$$Min\, P_n(d_n^-)$$

Subject to $$x_1 + d_1^- - d_1^+ = f_1^{(1)}$$

$$x_2 + d_2^- - d_2^+ = f_2^{(1)}$$

... ...

$$x_n + d_n^- = f_n^{(1)}$$

$$C_1x_1 + C_2x_2 + \cdots + C_nx_n \leq r$$

$$x_1, x_2,..., x_n \leq 0$$

$$d_q^-, d_q^+ = 0; \qquad d_q^-.d_q^+ = 0, \ q=1,2,....,n$$

The first (n-1) objectives are considered absolute.

Phase II. In this phase, on solving problem (P_2) the iteration methodology for GP problem can be used. (Candler and Townsley, 1982). The solution of problem (P_2) is either feasible or infeasible where feasible means objective function values of goals at priority P_1 and P_2 are zero.

In case (i) $d_n^- = 0,\ d_n^+ = 0,$ by the choice of f as $f_n^{(1)}$ and hence $(x_{1n}^{(1)},\ x_{2n}^{(1)},...,x_{nn}^{(1)})$ is the solution of MLPP.

In case (ii) $d_{n-1}^+ > 0$ and $d_n^- \geq 0,\ d_n^+ > 0.$

If $d_n^- > 0$, then $(x_{1n}^{(1)},\ x_{2n}^{(1)},...,x_{nn}^{(1)})$ is a solution of MLPP.

If $d_n^- = 0 \ and \ d_n^+ = 0$, even then $(x_{1n}^{(1)},\ x_{2n}^{(1)},...,x_{nn}^{(1)})$ is a solution of MLPP.

If $d_n^+ > 0$, then $(x_{1n}^{(1)},\ x_{2n}^{(1)},...,x_{nn}^{(1)})$ is not a solution of MLPP.

Then we repeat the process of phase II with the next alternate solution $(x_{1s}^{(1)},\ x_{2s}^{(1)},...,x_{ns}^{(1)})$ taking $F^{(n)} = F^{(s)}$ and $f_n^{(1)} = f_s^{(1)}$ where $s=(n-1)$ continuing this process till either some $(x_{1i}^{(1)},\ x_{2i}^{(1)},...,x_{ni}^{(1)})$ $\forall i=1,2,...,n$ turns out to be the solution of MLPP or none of $(x_{1i}^{(1)},\ x_{2i}^{(1)},...,x_{ni}^{(1)})$ is the solution of MLPP. In the former case we stop as the solution is attained and in later case find the next best solution of problem P1. Let the next best value of F as $F^{(s)}$ at the points $(x_{1i}^{(2)},\ x_{2i}^{(2)},...,x_{ni}^{(2)}) \in S \quad \forall i=1,2,...,n^{(2)}$

i.e. $F(x_{1i}^{(2)},\ x_{2i}^{(2)},...,x_{ni}^{(2)}) = F^2 \quad \forall i=1,2,...,n$

Find the value of f at these points and arrange them in a decreasing order.

Let $f(x_{1i}^{(2)}, x_{2i}^{(2)},\dots,x_{ni}^{(2)}) = f_2^{(2)} \quad \forall i = 1,2,\dots,n$ and $f_1^{(2)} \geq f_2^{(2)} \geq \cdots \geq f_n^{(2)}$.
Repeat phase II with the point $(x_{1i}^{(2)}, x_{2i}^{(2)},\dots,x_{ni}^{(2)})$, $i = 1,2,\dots,n$.

Continue this procedure till some extreme points turns out to be the solution of the given multi-level programming problem. Since, the set of extreme points is finite, the process converges in a finite steps.

GOAL PROGRAMMING ALGORITHM

Step 1: Solve the linear programming problem P1 with the leader's objective function. Let $(x_{1i}^{(1)}, x_{2i}^{(1)},\dots,x_{ni}^{(1)})$, $\forall i = 1,2,\dots,n$ be its optimal solutions. Let $F(x_{1n}^{(1)}, x_{2n}^{(1)},\dots,x_{nn}^{(1)}) = F^n$ and go step 2.
Step 2: Find the value of the lower level objective function f at these points and arrange than in descending order. Let f_n be the maximum value of f at the point $(x_{1n}^{(1)}, x_{2n}^{(1)},\dots,x_{nn}^{(1)})$. Formulate corresponding goal programming problem P2 and solve it.
Step 3: If $(x_{1n}^{(1)}, x_{2n}^{(1)},\dots,x_{nn}^{(1)})$ is the feasible solution of goal programming problem P2 then $(x_{1n}^{(1)}, x_{2n}^{(1)},\dots,x_{nn}^{(1)})$ is the solution of MLPP otherwise go to next step.
Step 4: Since the problem is infeasible there are arises two cases.
(i) If $d_n^- > 0$, $d_n^+ = 0$ and $d_3^- \geq 0$ and then $(x_{1n}^{(1)}, x_{2n}^{(1)},\dots,x_{nn}^{(1)})$ will be solution of MLPP.
(ii) If $d_{n-1}^- > 0$ and $d_n^+ > 0$ then go to next step.
Step 5: Starting with the point $(x_{1n}^{(1)}, x_{2n}^{(1)},\dots,x_{nn}^{(1)})$ go to step 3. If $(x_{1n}^{(1)}, x_{2n}^{(1)},\dots,x_{nn}^{(1)})$ is not the solution of MLPP, repeat the step with other values of $(x_{1n}^{(1)}, x_{2n}^{(1)},\dots,x_{nn}^{(1)})$. If none of these give the solutions of MLPP then go to step 6.
Step 6: Find the next best solution of problem P1 and process further.
Hence we find the best solution for a multi-level programming problem from this algorithm.

CONCLUSION

We have obtained feasible solution of multi-level programming problem which satisfies the constraints. The GP approach for this MLP problem is simple and practical which gives the best solution of MLPP.

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