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REFLECTION AND TRANSMISSION COEFFICIENT FOR DILATON-AXION BLACK HOLES

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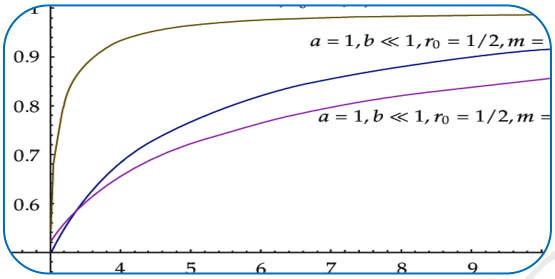
ABSTRACT :

We derive the expression of reflection & transmission coefficient of dilaton-axion black hole in low frequency limit. Both terms get modified by the expression $\left(1-\frac{r_0^2}{r_+^2}\right)^2$.

KEYWORDS : *transmission coefficient , low frequency limit , astrophysical objects.*

INTRODUCTION:



Black holes being an astrophysical objects has become very challenging to study – due to its strong observational evidence[1,2]. Recent experimental works suggest SGrA[*] as well as M87 as supermassive black-holes[3-8]. Other experimental team having very high resolving power telescope highlights gravitational waves emanating from black-holes as well as its formation of shadow[9-13]. Recent works on scattering and absorption of particles and waves in the black-holes background spacetime has also reveal the relevance of experimental observations. In this context, the absorption of massive or massless scalar field, fermions, electromagnetic and gravitational wave in Schwarzschild back ground spacetime was studied[14-20]. In the background of Reissner-Nordstrom, rotating and regular black hole spacetime in this context considerable number of works exist in the literature[21-46]. Our main objective is to analysis the expression of reflection and transmission coefficient for a monochromatic massive charged scalar particles in the background of dilation-axion black-holes in low frequency regime.

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Reflection & Transmission Coefficient at Low Frequency Regime :

We begin with the action describing dilaton-axion black hole as[47] :

$$S=\int d^4x\sqrt{-g}\left[R-\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi-\frac{1}{2}e^{2a\varphi}\partial_\mu\xi\partial^\mu\xi-e^{-a\varphi}F_{\mu\nu}F^{\mu\nu}-b\zeta F_{\mu\nu}*F^{\mu\nu}\right] \quad ...(1)$$

where R is the scalar curvature, $F^2=F_{\mu\nu}F^{\mu\nu}$ represents Maxwell field, φ and ζ are the massless scalar dilaton and the massless pseudoscalar axion respectively. a and b represent the coupling of dilaton and axion field with electromagnetic field. We begin with the spherically symmetric metric,

$$ds^2=U(r)dt^2+\frac{1}{U(r)}dr^2+p(r)^2d\Omega^2 \quad ...(2)$$

Solving Einstein's equation, one can get different classes of dilaton-axion black hole solutions for different choices of a and b. we write the metric function as,[47]

$$U(r) = \left[\frac{(r + r_{+})(r - r_{-})}{r^2 - r_0^2}\right] \quad ...(3)$$

with

$$p(r)^2 = (r^2 - r_0^2) \quad ...(4)$$

r_{+} and r_{-} are the two horizons and r_0 is the dilaton-axion charge[47]. r_0 and $r_{\pm}$ are related to electric charge Q_e, magnetic charge Q_m, dilaton field at infinity φ_0 and black hole mass m as $r_0 = \frac{(Q_e^2 - Q_m^2)\, e^{-\varphi_0}}{2m}$, and $r_{\pm} = m_{\pm}\sqrt{m^2 + r_0^2 - (Q_e^2 + Q_m^2)\, e^{-\varphi_0}}$[47].

In the background of our black-hole, the dynamics of a charged massive perturbing scalar field can be described by the Klein-Gordon equation as follows:

$$[(\nabla^{\mathrm{v}} - iqA^{\nu})(\nabla_{\mathrm{v}} - iqA_{\nu}) - m_1^2]\Phi = 0 \quad ...(5)$$

where q and m_1 are the charge and mass of the scalar field and vector potential $A_0 = \frac{Q}{(r + r_0)}$. Using the ansatz $\Phi = e^{-i\omega t} R(r) Y_{lm'}(\theta, \phi)$, the radial wave equation can be written as,

$$\Delta \frac{d}{dr}\left(\Delta \frac{dR}{dr}\right) + U_1 R = 0 \quad ...(6)$$

where ω, l and m' are the conserved energy, spherical harmonic index and azimuthal harmonic index respectively. In our case $\Delta = (r - r_{+})(r - r_{-})$ and $U_1 = \frac{(r^2 - r_0^2)^2}{r^2}\left(\omega r - qQ\frac{r}{r + r_0}\right)^2 - \Delta(r^2 - r_0^2)m_1^2 + l(l + 1)$. Where $Q^2 = Q_e^2$. Using the tortoise coordinate $\frac{dr_*}{dr} = \left(\frac{r^2 - r_0^2}{\Delta}\right)$ and changing the radial function as $\tilde{R} = rR(r)$, we get an wave equation for the radial part of the equation as,

$$\frac{d^2\tilde{R}}{dr_*^2} + \tilde{U}_1\tilde{R} = 0 \quad ...(7)$$

where the domain for r_* is $(-\infty, +\infty)$ and

$$\tilde{U}_1 = \frac{U_1}{r^4} - \frac{\Delta}{r^3}\frac{d}{dr}\left(\frac{\Delta}{r^2}\right) \quad ...(8)$$

In this work we make use of a dimensionless parameter v corresponding to the scalarfield as,

$$v = \sqrt{1 - \frac{m^2}{\omega^2}} \quad ...(9)$$

For unbound modes $0 < v < 1$ and $\omega > m$. Being $\omega > m$, in low frequency region, $m\omega \ll 1$ and thus $mm_1 \ll 1$. Region near the event horizon $(r \to r_{+})$, we have from equation (8),

$$\lim_{r\to r_+} \widetilde{U}_1 = \left(1-\frac{r_0^2}{r_+^2}\right)^2 \left(\omega - \frac{qQ}{(r_+ + r_0)}\right)^2 = \sigma^2 \qquad \dots(10)$$

Thus the transmitted part of the radial wave equation has solution as,

$$\tilde{R} = A_{trans}\, e^{-i\sigma r_*} \qquad \dots(11)$$

where the tortoise coordinate is represented as,

$$r_* \approx \frac{1}{U^1(r=r_+)} \ln(r-r_+) + r_*(0) \qquad \dots(12)$$

and $r_*(0)$ is a constant. Substituting the expressions of r_* and σ, the solution of equation (6) for $r \to r_+$ will be,

$$\tilde{R}_I = A_I\left(1 - \frac{i\left(\omega - \frac{qQ}{(r_+ + r_0)}\right)\left(1-\frac{r_0^2}{r_+^2}\right)}{U'(r_+)} \log(r - r_+)\right) \qquad \dots(13)$$

We take the limit $\omega \to 0$ as well as $m \to 0$ in eqn(6). For $m\omega, mm_1 \ll 1$, we want to concentrate on the dominating mode $1 = 0$. Now the differential equation describing the intermediate region can be written as,

$$\frac{d^2\tilde{R}}{dr^2} - \left[\frac{(r_+ + r_- - 2r)}{(r-r_+)(r-r_-)} + \frac{2r_0^2}{r(r^2-r_0^2)}\right]\frac{d\tilde{R}}{dr} = 0 \qquad \dots(14)$$

The solution of equation (14) in the limit $r \to r_+$ can be written as,

$$\tilde{R}_{II} = \zeta_1\left(1-\frac{r_0^2}{r_+^2}\right)\ln(r-r_+) - \zeta_1\left(1-\frac{r_0^2}{r_+^2}\right)\ln(r_+ - r_-) + \tau \qquad \dots(15)$$

where ζ_1 and τ are constants to be determined. Comparing the equation (13) and (15) we get,

$$\zeta_1 = -A_I i\alpha\left(\omega - \frac{qQ}{(r_+ + r_0)}\right) \qquad \dots(16)$$

where $\alpha = \frac{1}{U'(r_+)}$, and

$$\tau = A_I\left[1 - i\left(\omega - \frac{qQ}{(r_+ + r_0)}\right)\left(1-\frac{r_0^2}{r_+^2}\right)\beta\right] \qquad \dots(17)$$

Where

$$\beta = \alpha \ln(r_+ - r_-) \qquad \dots(18)$$

For the asymptotic region $(r \gg r_+)$ the differential equation (6) can be written as,

$$\left(\frac{d^2}{dr^2}+\left[(\omega^2-m_1^2)+\frac{2m(2\omega^2-m_1^2)}{r}-\frac{l(l+1)}{r^2}\right]\right)r\,f^{\frac{1}{2}}\,\tilde{R}=0 \qquad ...(19)$$

We neglect the terms $O\left(\frac{1}{r^2}\right)$ that are proportional to $\omega^2, m_1^2, r_0^2 m_1^2$ and terms of order $\frac{1}{r^3}$. In the asymptotic limit for $\omega r \ll 1$ and $1=0$, the solution of equation (19) can be written as,

$$\tilde{R}_{III}=a_1\rho\omega v+\frac{b_1}{\rho r} \qquad ...(20)$$

where $\rho=\frac{-m\omega\,(1+v^2)/v}{\left(e^{-m\omega\,(1+v^2)/v}-1\right)}$

In the asymptotic limit we can write the equation (15) as,

$$\tilde{R}_{II}=-\zeta_1\frac{(r_+-r_-)\left(1-\frac{r_0^2}{r_+^2}\right)}{r}+\tau \qquad ...(21)$$

Comparing the equations (16),(17),(20) and (21) the expressions of a_1 and b_1 can be written as,

$$a_1=\frac{A_I\left[1-i\left(\omega-\frac{qQ}{(r_++r_0)}\right)\left(1-\frac{r_0^2}{r_+^2}\right)\beta\right]}{\rho\omega v} \qquad ...(22)$$

And

$$b_1=A_I\rho(r_+-r_-)i\alpha\left(\omega-\frac{qQ}{r_+}\right)\left(1-\frac{r_0^2}{r_+^2}\right) \qquad ...(23)$$

Thus the expression of amplitude for incident wave can be written as,

$$A_{our}^{inc}=\frac{-A_I\left[1+\rho^2\omega v(r_+-r_-)\alpha\left(\omega-\frac{qQ}{(r_++r_0)}\right)\left(1-\frac{r_0^2}{r_+^2}\right)-i\beta\left(\omega-\frac{qQ}{(r_++r)}\right)\left(1-\frac{r_0^2}{r_+^2}\right)\right]}{2i\rho\omega v}$$

...(24)

and the expression for reflected wave can be written as,

$$A_{our}^{ref}=\frac{A_1\left[1-i\left(\omega-\frac{qQ}{(r_++r_0)}\right)\left(1-\frac{r_0^2}{r_+^2}\right)\beta-v\rho^2\omega\alpha(r_+-r_-)\left(\omega-\frac{qQ}{(r_++r_0)}\right)\left(1-\frac{r_0^2}{r_+^2}\right)\right]}{2i\rho\omega v}$$

...(25)

From the expressions of A_{our}^{inc} and A_{our}^{ref}, the reflection and transmission coefficient of the incident wave in the back ground of dilaton-axion black hole can be written respectively as,

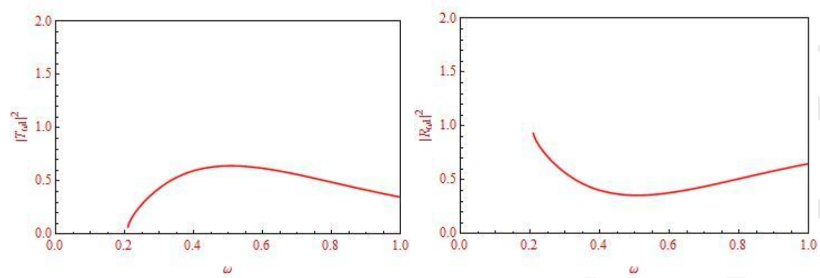
$$|R_{\omega l}|_{our}^2=\left|\frac{A_{our}^{ref}}{A_{our}^{inc}}\right|^2=1-\frac{4\rho^2\omega^2 v(r_+-r_-)\alpha\left(\omega-\frac{qQ}{r_+}\right)\left(1-\frac{r_0^2}{r_+^2}\right)}{\left[\left(1+\rho^2\omega^2 v\alpha(r_+-r_-)\left(\omega-\frac{qQ}{(r_++r_0)}\right)\left(1-\frac{r_0^2}{r_+^2}\right)\right)^2+\left(\left(\omega-\frac{qQ}{r_+}\right)\left(1-\frac{r_0^2}{r_+^2}\right)\beta\right)^2\right]}$$

...(26)

and

$$|T_{\omega l}|^2_{our} = \frac{4\rho^2\omega\upsilon(r_+ - r_-)\alpha\left(\omega - \frac{qQ}{(r_+ + r_0)}\right)\left(1 - \frac{r_0^2}{r_+^2}\right)}{\left[\left(1 + \rho^2\omega\upsilon\alpha(r_+ - r_-)\left(\omega - \frac{qQ}{(r_+ + r_0)}\right)\left(1 - \frac{r_0^2}{r_+^2}\right)\right)^2 + \left(\left(\omega - \frac{qQ}{(r_+ + r_0)}\right)\left(1 - \frac{r_0^2}{r_+^2}\right)\beta\right)^2\right]} \quad ...(27)$$

Substituting the expressions of α and β, we find for $r_0 = 0, q = 0, |R_{\omega l}|^2$ & $|T_{\omega l}|^2$ will reduce to the expressions of Reissner – Nordstrom black-hole.



The behaviour of transmission & reflection coefficients with respect to frequency has been depicted in the figures. As transmission coefficient increases , reflection coefficient decreases with respect to ω.

CONCLUSION :

We study the reflection & transmission coefficient of dilaton-axion black hole under charged massive scalar field perturbation in low frequency limit. We observed that the presence of $\left(1 - \frac{r_0^2}{r_+^2}\right)^2$ modifies the expressions of reflection & transmission coefficient of our black hole in comparison to Reissner-Nordstrom black hole in low frequency limit.

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