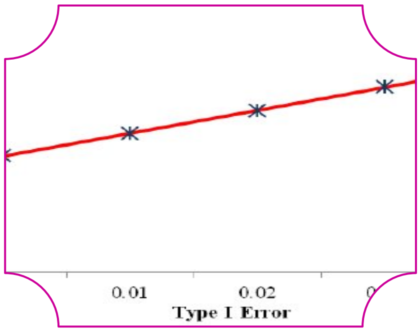
REVIEW OF RESEARCH
IMPACT FACTOR : 5.7631(UIF) UGC APPROVED JOURNAL NO. 48514 ISSN: 2249-894X
VOLUME - 8 | ISSUE - 3 | DECEMBER - 2018

A VENDOR-BUYER INVENTORY POLICY FOR DEFECTIVE ITEM WITH STOCHASTIC DEMAND AND VARIABLE LEAD TIME



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ABSTRACT
This study deals with single-vendor, single-buyer inventory control system with imper-fect production under individual management. The defective rate is a random variable. The model is developed for imperfect production process with perfect and partial infor-mation about the lead-time demand distribution. Here we first assume that the lead-time demand follows a normal distribution and then this conjecture is remove by the assump-tion that the first and second moments of the probability distribution of lead-time demand are known. These models are mixture production inventory models with backorder and lost sale in which the lead-time, order quantity are viewed as decision variables. For each case an algorithm is developed to find the pareto optimal ordering policy for the buyer and hence the production policy for the vendor. Some mathematical properties of the profit functions are analytically derived. Some numerical examples are also presented to illustrate the results of the proposed models.

KEYWORDS: *Inventory, Lead-time, Backorder rate, Minimax distribution-free procedure, Proportion of defectives.*

1.INTRODUCTION
1.1 Background

In global market, vendor-buyer production inventory model is more useful than one side optimal strategy (e.g., a buyer or a vendor). In this area Goyal [1] give an idea about joint optimization for vendor and buyer. Goyal [2] further suggest that the vendor economic production quantity should be an integer multiple of buyer purchase quantity. Ha and Kim [3] extended the concept and proposed an integrated lot-splitting model of facilitating multiple shipments in small lots. Huang [4] developed an inventory model for items with imperfect quantity.

When demand during the cycle period is not deterministic but is stochastic, then lead time takes an important role about optimization of an inventory model. According to Tersine [5] lead time usually consists of the following components: order preparation, order transit, supplier lead time, delivery time and setup time. Most of the authors deal with inventory lead time as a control variable. Liao and Shyu [6] first considered lead time as a variable and controlled it by paying extra crashing cost. This model has been extended by Ben-Daya and Raouf [7], Ouyang et al. [8], Ouyang and Wu [9]. Although, all of them considered the lead time reduction cost as a function of the number of order only but it is not so because Pan and Hsiao [10] proposed that the transportation cost, the overtime wages and extra inventory holding cost for expedition of delivery are all proportional to the item order quantities rushed.

Now-a-day multi-objective production inventory problem is a real phenomenon but there is only a few research papers dealing with multi-objective production inventory problem. Roy and Maiti [11] formulated and solved a multi-objective inventory model of deteriorating items in fuzzy environment. Recently Mahapatra and Maiti [12] have developed an inventory model for breakable item with uncertain preparation time. They [13] have also formulated and solved a production inventory model for deteriorating items with imperfect preparation time for pro-duction in finite time horizon. But as far as knowledge none has consider multi-objective production inventory problem in stochastic environment for defective items.

1.2 ORGANIZATION OF THE PAPER

In this paper we are considering the single vendor, single buyer production inventory model in individual management system. Here percentage of defectiveness follows uniform distribution and the poor quantity items detected in screening process of a lot at the buyer are sold at a discount price. We developed two models, at first assuming that lead-time demand follows a normal distribution, and then remove this assumption by only assuming that the first and second moments of the probability distribution of lead-time demand are known. These models are mixture production inventory model with backorder and lost sale in which the lead-time, order quantity are viewed as a decision variables. Here backorder ratio and setup cost for buyer depend on lead-time. For each model, an effective iterative procedure is developed to determine the optimal policy, and numerical examples are used to illustrate the results. Some mathematical properties of the profit functions are also analytically derived.

2.PRELIMINARIES

Multi-objective Optimization Problem:

Definition 1. A multi-objective optimization problem is of the form

maximize $\{f_1(x), f_2(x), \cdots, f_k(x)\}$

subject to $x \in X$;

where $f_i(x)$, i = 1, 2, ..., k are k objective functions and X is the feasible set of constraints, i.e.,

$X = \{x : g_j(x) \leq 0,$ $\quad$ (j = 1, 2, ..., *l)*

$h_r(x) = 0,$ $\quad$ (r = 1, 2, ..., m)

and $x = (x_1, x_2, ..., x_n)^T \}$

It is said to be concave if all the objective functions are concave and the feasible region is convex.

Definition 2. A vector $x^* \in X$ is Pareto optimal solution of the above multi-objective problem if there does not exist another decision vector $x \in X$ such that $f_i(x) \geq f_i(x^*)$ for all $i = 1,2,\ldots\ldots..,k$ and $f_j(x) < f_j(x^*)$ for at least one index j.

Theorem 1. Let the multi-objective optimization problem be concave. Then every locally Pareto optimal solution is also globally Pareto optimal. [The proof is similar to Miettinen [14] for minimization problem].

3.NOTATION AND ASSUMPTIONS

To develop the proposed models, adopting the following notation and assumptions

3.1Notations

m number of shipment from vendor to buyer.

x+ maximum value of x and 0, i.e., $x^+ = \max\{x,0\}$.

E(.) mathematical expectation.

ϕ(.) probability density function of the standard normal distribution.

Φ(.) cumulative distribution function of standard normal distribution.

c_s per unit maximum retail price (M.R.P.) of non-defective items.

For Vendor

K production rate per unit time.

A_{v0} Set-up cost.
c_p per unit production cost, $c_p = \mu c_s$, where $0 < \mu < 1$.
h_v holding cost per unit per unit time.

For buyer

D average demand per unit time of non-defective items.
L length of lead time (decision variable).
$A_b(L)$ ordering cost per order which is a linear function of L and $0 < A_b(L) \le A_{b0}$, A_{b0} is the upper bound of ordering cost.
$\beta(L)$ backorder ratio, is a non-linear function of L.
q stock-out probability.
h_b holding cost per unit per unit time, which is same for defective and non-defective item.
P random variable representing the percentage rate of defective items received.
r_s the screening rate $r_s > D$.
Q order quantity (decision variable).
R reorder point.
Π fixed penalty cost per unit.
Π_0 marginal profit per unit.
c_{pr} per unit purchasing cost, $c_{pr} = \lambda c_s$, where $0 < \mu < \lambda < 1$.
c_{sc} the screening cost per unit.
c_{ds} the selling price of defective items per unit, $c_{ds} < c_p$.

3.2Assumptions

1.There is single-vendor and single-buyer for single product.
2. Shortages allowed and backlogged partially.
3. The buyer's demand X during the lead is normally distributed with mean D_L = DL and standard deviation$\sigma_L = \sigma\sqrt{L}$, then the probability density function (p.d.f.) of X is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma_L} exp\left[-\frac{(x-D_L)^2}{2\sigma_L{}^2}\right], -\infty < x < \infty \qquad (1)$$

where σ denote the standard deviation of the demand per unit time for buyer's.
4.Each lot contains percentage defectiveness, P, which is uniformly distributed with p.d.f.

$$g(p) = \frac{1}{b-a},\ a \le p \le b \qquad (2)$$
$$= 0 \quad , \quad \text{elsewhere,}$$

having the interval [a,b] as the spectrum, where $0 \le a < b < 1$.
5.Buyer's inventory system is continuously reviewed. Replenishment are made whenever the buyer's inventory level falls to the reorder point R.
6.The reorder point of the buyer's , R is the expected demand during lead-time plus safety stock (SS), and SS = k×(standard deviation of lead-time demand), i.e., $R = D_L + k\sigma\sqrt{L}$, where k is the safety factor which is treated as known/unknown parameter and satisfying P(X > R) = P (Z > k) = q, Z represents the standard normal random variable, and q represents the allowable stock-out probability during lead-time L.
7.The lead time L has n mutually independent components each having a different crashing cost for reducing lead time. The j-th component has a minimum duration a_j and normal duration b_j and a crashing cost per unit time c_j. Furthermore, we assume that $c_1 \le c_2 \le .... \le c_n$.
8.The components of lead time are crashed one at a time starting with the component of least c_i and so on.
9.Let $L_0 = \sum_{j=1}^{n} b_j$ and L_r be the length of lead-time for buyer's with components 1, 2, 3,……, r crashed to their minimum duration, then L_r can be expressed as $L_r = \sum_{j=1}^{n} b_j - \sum_{j=1}^{r}(b_j - a_j)$, r=1,2,…..,n. The lead-time crashing cost per cycle for buyer's is C(L) for a given $L \in [L_r, L_{r-1}]$ is given by

$$C(L) = c_r[L_{r-1} - L] + \sum_{j=1}^{r-1} c_j(b_j - a_j)\ and\ C(L_0) = 0 \qquad (3)$$

10. The backorder ratio β(L) for the buyer's is a non-linear function of L through $E(X - R)^+$ defined as

$\beta(L) = 1/[1 + \tau\, E(X - R)^+]$ (4)

which is smaller for larger expected shortage quantity, where τ is positive constant.

11. Ordering cost reduction and lead time are related directly as

$$\frac{L_0 - L}{L_0} = \eta \frac{A_{b0} - A_b}{A_{b0}}$$

where η(> 0) is a constant scaling parameter to describe the linear relationship between percentage reduction in lead time and ordering cost. Then ordering cost A_b is linear function of lead time L, i.e., $A_b(L) = u + vL$ (5)

where $u = \left(1 - \frac{1}{\eta}\right) A_{b0}$ and $v = \frac{A_{b0}}{\eta L_0}$. This function has been utilized in many researches (e.g., Chiu [15], Chen et al. [16] and Chang et al. [17]).

4.MODEL OF INDIVIDUAL MANAGEMENT

4.1Model when Lead-time demand is normally distributed

In this vendor-buyer problem of defective items, when buyer orders a lot of size Q then vendor produces mQ at one set-up in order to reduce its set-up cost and as soon as buyers lot size Q produced, the lot is delivered to buyer to reduce the inventory holding cost. The vendor do not conduct any screening process. On the other hand, after purchasing a lot from vendor at price (c_{pr}) buyer start screening and as soon as possible 100% inspection process of the lot is conducted at a fixed cost per unit (c_{sc}) and at a rate (r_s) greater than that of the demand rate. When screening process is completed the defective items (PQ units) are sold as a single batch at a discount price. To avoid shortages within the screening period, P is restricted as $P \leq 1 - \frac{D}{r_s}$. Then the length of one cycle for vendor and buyer are respectively$\frac{m(1-P)Q}{D}$ and $\frac{(1-P)Q}{D}$. Since the buyer's inventory system is continuously reviewed, the buyer's place his order when his on hand inventory reaches a reorder point R. The inventory profile for the vendor and buyer is depicted in Figure-1.

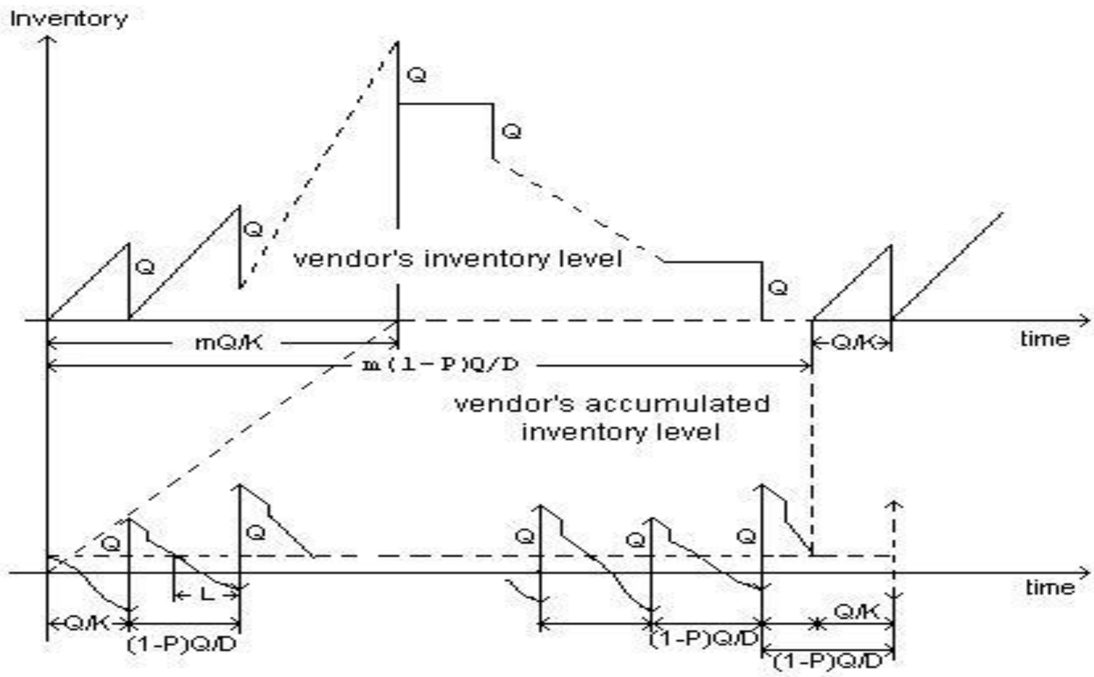

Figure: 1 The inventory situation for vendor's and buyer.

When order is placed the lead time count is started, during this period the demand X is a random variable which is normally distributed and whose expected value is E(X) = D_L and variance var(X) = E[X − E(X)]^2 = σ_L^2. So the accumulated shortage amount just before the arrival of next lot is also a random variable X − R (denoting it as Y) whose expected value is

$$E(Y)^{+} = E(X-R)^{+} = \int_R^{\infty}(x-R)f(x)dx = \sigma\sqrt{L}\psi(k), \quad (6)$$

where ψ(k) = ϕ(k) − k[1 − Φ(k)] > 0. Therefore, the backorder and lost sale amount in each cycle are respectively βY and (1 − β)Y with expected value βE(Y) and (1 − β)E(Y). In this model the average inventory for buyers is calculated by summing the average inventory for non defective and defective item. For the buyer's model the total cost per unit time is as follows:

TCU_b(Q, L, R) = ordering cost + holding cost + shortage cost + screening cost + purchasing cost + lead time crashing cost,

$$= \frac{A_b(L)D}{(1-P)Q} + h_b\left[\left\{\frac{(1-P)Q}{2} + R - D_L + (1-\beta)Y\right\} + \frac{DQP}{r_s(1-P)}\right] + \frac{D}{(1-P)Q}\{\Pi + (1-\beta)\Pi_0\}Y$$
$$+ \frac{c_{sc}D}{(1-P)} + \frac{c_{pr}D}{(1-P)} + \frac{D}{(1-P)Q}C(L),$$

where C(L), β [= β(L)], A_b(L) are calculated form (3), (4), (5) respectively and $R = D_L + k\sigma\sqrt{L}$. Since the random variables P and X are mutually independent, therefore, the expected value of T CU_b(Q, L; k) is

$$TECU_b(Q, L; k) = D\left[\frac{A_b(L)}{Q} + \{\Pi + (1-\beta)\Pi_0\}\frac{E(Y)^{+}}{Q} + c_{sc} + \lambda c_s + \frac{C(L)}{Q}\right]E[1/(1-P)]$$

$$+h_b\left[\frac{Q}{2}E[1-P] + k\sigma\sqrt{L} + (1-\beta)E(Y)^{+}\right] + \frac{h_bDQ}{r_s}E[1/(1-P)] \quad (7)$$

The total expected revenue collection from sales of good quantity and defective quantity items per unit time is calculated as

$$TERU_b = c_{ds}DE\,[P/(1-P)] + c_sD \quad (8)$$

The buyer's total expected profit per unit time, $TEPU_b$(Q, L; k), is determined by the total expected revenue per unit time, $TERU_b$, less the total expected cost per unit time, $TECU_b$(Q, L; k). The equation can be formulated as

$TEPU_b$(Q, L; k) = $TERU_b$ − $TECU_b$(Q, L; k),

$$= c_sD + D\left(c_{ds} - \frac{h_bQ}{r_s}\right)E_4(a,b) - D\left[\frac{A_b(L)}{Q} + \frac{\Pi F(L)}{\tau Q} + c_{sc} + \lambda c_s + \frac{C(L)}{Q}\right]E_3(a,b)$$

$$-\frac{F(L)^2}{\tau[1+F(L)]}\left[\frac{\Pi_0 D}{Q}E_3(a,b) + h_b\right] - h_b\left[\frac{Q}{2}E_2(a,b) + k\sigma\sqrt{L}\right] \quad (9)$$

where, F (L) = τ E(Y)$^+$ = τ σ $\sqrt{L}$ψ(k) and the calculations to evaluate E_2(a, b), E_3(a, b) and E_4(a, b) are given in Appendix-A.

In the production period, when the first Q units have been produced, the vendor delivers them to buyer, after that the vendor will make the delivery on an average of every $\frac{(1-P)Q}{D}$

unit time until the inventory level diminishes to zero. So the total inventory of the vendor for a single cycle can be calculated as follows:

$$mQ\left[\left\{\frac{Q}{K}+(m-1)\frac{(1-P)Q}{D}\right\}-\frac{m^2Q^2}{2K}\right]-\left\{\frac{(1-P)Q}{D}(1+2+\cdots\dots\dots+(m-1))\right\}$$

$$=\frac{m(1-P)Q^2}{2D}\left[\frac{D}{(1-P)K}(2-m)+m-1\right].$$

Hence the vendor's total cost per unit time is as follows:
$TCU_v(Q; m)$ = setup cost + holding cost + production cost,

$$=\frac{A_{v0}D}{m(1-P)Q}+h_v\frac{Q}{2}\left[\frac{D}{(1-P)K}(2-m)+m-1\right]+\frac{c_pD}{(1-P)}.$$

The expected value of $TCU_v(Q)$ is

$$TECU_v(Q; m) = D\left[\frac{A_{v0}}{mQ}+\frac{(2-m)h_vQ}{2K}+\mu c_s\right]E[1/(1-P)]+\frac{h_vQ}{2}(m-1). \quad (10)$$

Also the vendor's total expected revenue collection per unit time from selling of the quantity mQ to buyer is given by

$$TERU_v = \lambda c_s DE\,[1/(1-P)] \quad (11)$$

Therefore, the expected profit of the vendor per unit time, $TEPU_v(Q, m)$, is formulated as follows

$TEPU_v(Q; m) = TERU_v - TECU_v(Q; m)$

$$=D\left[(\lambda-\mu)c_s-\frac{A_{v0}}{mQ}-\frac{(2-m)h_vQ}{2K}\right]E_3(a,b)-\frac{h_vQ}{2}(m-1). \quad (12)$$

So, if the system is treated as a individual management then the above model is formulated as a multi-objective problem, which is given by

$$\text{Maximize } \{TEPU_b(Q, L; k), TEPU_v(Q; m)\} \quad (13)$$

subject to, $0 < A_b \le A_{b0}$,

Several methods have been developed for solving this type of multi objective optimization problem. Here, Weighted Sum Method given in Miettinen [14] is used to solve the multi objective problem (13).

$$\text{Maximize } F^1\{(Q, L; k, m)\} \quad (14)$$

$$subject\ to\ 0 < A_b \le A_{b0}.$$

and $w_1 + w_2 = 1$, $w_1, w_2 \ge 0$,

where $F^1(Q, L; k, m) = w_1TEPU_b(Q, L; k) + w_2TEPU_v(Q; m)$.

Theorem 2. The solution of the weighted sum problem (14) is weakly Pareto optimal. **Theorem 3.** The solution of the weighted sum problem (14) is Pareto optimal if the weighting coefficients are positive, that is $w_i > 0$ for all i = 1, 2.

For the **proof** of the **Theorems 2** and **Theorems 3**, refer to Miettinen [14].

Proposition 1. For fixed Q, and known k, the maximum value of $TEPU_b(Q, L; k)$ will occur at the end points of the interval $[L_r, L_{r-1}]$.

Proof. The first and second order partial derivative with respect to L of $TEPU_b(Q, L; k)$ for fixed Q and for known k are respectively

$$\frac{\partial TEPU_b(Q,L;k)}{\partial L} = \frac{D}{Q}\left(c_r - \frac{A_{b0}}{\eta L_0}\right)E_3(a,b) - \frac{[2+F(L)]F(L)^2}{2\tau L[1+F(L)]^2}\left[\frac{\Pi_0 D}{Q}E_3(a,b) + h_b\right]$$

$$-\frac{\Pi DF(L)}{2\tau LQ}E_3(a,b) - \frac{1}{2}h_b k\sigma L^{\frac{-1}{2}}.$$

and $\frac{\partial^2 TEPU_b(Q,L;k)}{\partial L^2} = \frac{[3+F(L)][F(L)]^3}{\tau[1+F(L)]^3}\left[\frac{\Pi_0 D}{Q}E_3(a,b) + h_b\right] + \frac{\Pi DF(L)}{4\tau L^2 Q}E_3(a,b) + \frac{1}{4}k\sigma h_b L^{\frac{-3}{2}}.$

Since, $\frac{\partial^2 TEPU_b(Q,L;k)}{\partial L^2} > 0$ for fixed Q, and known k, therefore, $TEPU_b(Q, L; k)$ convex in $L \in [L_r, L_{r-1}]$. Hence for fixed Q, and for known k, the maximum value of T $EPU_b(Q, L; k)$ occur at the end points of the interval $[L_r, L_{r-1}]$.

Proposition 2. For each $L_r(r = 1, 2, ..., n)$, fixed m and known parametric value of k, the multi objective problem given in (13) is concave in Q.

Proof. For each $L_r(r = 1, 2, ..., n)$, fixed m and known k the second order partial derivative of $TEPU_b(Q, L_r; k)$ and $TEPU_v(Q; m)$ with respect to Q are respectively

$$\frac{\partial^2 TEPU_b(Q,L_r;k)}{\partial Q^2} = -\frac{2D}{Q^3}\left[A_b(L_r) + \frac{\Pi F(L_r)}{\tau} + C(L_r)\right]E_3(a,b) - \frac{F(L_r)^2}{\tau[1+F(L_r)]}\left[\frac{2\Pi_0 D}{Q^3}E_3(a,b)\right], \quad (15)$$

and $\frac{\partial^2 TEPU_v(Q;m)}{\partial Q^2} = -\frac{2DA_{v0}}{mQ^3}E_3(a,b).$ (16)

From (15) and (16) it is clear that both the $\frac{\partial^2 TEPU_b(Q,L_r;k)}{\partial Q^2}$ and $\frac{\partial^2 TEPU_v(Q;m)}{\partial Q^2}$ are negative. So $TEPU_b(Q, L_r; k)$ and $TEPU_v(Q; m)$ both are concave in Q for known k and m and hence the the multi-objective problem given in (13) is concave in Q for each $L_r(r = 1, 2, ..., n)$, fixed m and known k (From the Definition-1 of multi-objective optimization problem given in Section-2).

Proposition 3. The multi-objective problem given in (13) possesses global Pareto-optimal solutions.

Proof. By **Proposition-2**, we have, the multi objective maximization problem (13) is concave. So it possesses global pareto-optimal solutions (following **Theorem-1** of **Section-2**).

Proposition 4. Global pareto-optimal solutions are obtained from $\frac{\partial F^1(Q,L_r;k,m)}{\partial Q} = 0$ for fixed m and known k.

Proof. If Q_{r1} and Q_{r2} be the solution of the equations $\frac{\partial TEPU_b(Q,L_r;k)}{\partial Q} = 0$ *and* $\frac{\partial TEPU_v(Q;m)}{\partial Q} = 0$ respectively for each L_r(r = 1, 2, ..., n), fixed m and known k, then the solutions Q_{r1} and Q_{r2} gives separately the maximum value of $TEPU_b(Q, L_r; k)$ and $TEPU_v(Q; m)$ but they are not the pareto-optimal solution of the multi-objective problem (13). Because of the contradiction and possible in-commensurability of the objective functions, it is not possible to find a single solution that would be for all the objectives simultaneously.

There exist a compromise solution $Q_r = \sqrt{\frac{K_1(L_r,k,m)}{K_2(m)}}$ which is determined by solving

$$\frac{\partial F^1(Q,L_r;k,m)}{\partial Q} = 0$$

for Q, the expression for $K_1(L_r, k, m)$ and $K_2(m)$ are given in **Appendix-A**. By Theorem-3, Q_r is a pareto-optimal solution of the multi objective problem (13) for each L_r(r = 1, 2, ..., n), fixed m and known k. The Proposition-3 says that this pareto-optimal solution must be Global pareto-optimal.

4.2 Model when Lead-time demand is distribution free

It is not necessary that the lead-time demand must be a normally distributed. We relax the assumption about the normal distribution of the lead time demand and only assume that the lead time demand has a d.f. F of X belong to the class Γ of d.f.'s with finite mean DL and standard deviation $\sigma\sqrt{L}$. Then the problems (13) is reduces to

$$Maximize\{\text{minimize}_{F\in\Gamma}\, TEPU_b(Q,L;k),\ \text{minimize}_{F\in\Gamma}\, TEPU_v(Q;m)\} \qquad (17)$$

$$subject\ to\ 0 < A_b \le A_{b0}.$$

Moreover, Gallego and Moon [18] already proved that for any $F \in \Gamma$

$$E(X) = E(X-R)^+ \le \frac{1}{2}\left[\sqrt{\sigma^2 L + (R-D_L)^2} - (R-D_L)\right].$$
$$= \frac{1}{2}\sigma\sqrt{L}\left(\sqrt{1+k^2} - k\right), \qquad (18)$$

because $R = D_L + k\sigma\sqrt{L}$. Then, as per assumption (10) and inequality (18), we have

$$\beta \ge \left[1 + \frac{1}{2}\tau\sigma\sqrt{L}\left(\sqrt{1+k^2} - k\right)\right]^{-1} \qquad (19)$$

Therefore, the problem (17) is reduces to

$$\text{maximize}_{Q,L}\{\overline{TEPU_b}(Q,L;k), \overline{TEPU_v}(Q;m)\} \qquad (20)$$

$$subject\ to\ 0 < A_b \le A_{b0}\ ,$$

where, $\overline{TEPU_b}(Q,L;k) = \text{minimize}_{F\in\Gamma}\, TEPU_b(Q,L;k)$

$$= c_s D + D\left(c_{ds} - \frac{h_b Q}{r_s}\right)E_4(a,b) - D\left[\frac{A_b(L)}{Q} + \frac{\Pi G(L)}{\tau Q} + c_{sc} + \lambda c_s + \frac{C(L)}{Q}\right]E_3(a,b)$$

$$-\frac{G(L)^2}{\tau[1+G(L)]}\left[\frac{\Pi_0 D}{Q}E_3(a,b)+h_b\right]-h_b\left[\frac{Q}{2}E_2(a,b)+k\sigma\sqrt{L}\right] \quad (21)$$

$$\overline{TEPU_v}(Q;m)=\text{minimize}_{F\in\Gamma}\, TEPU_v(Q;m).$$

$$=D\left[(\lambda-\mu)c_s-\frac{A_{v0}}{mQ}-\frac{(2-m)h_vQ}{2K}\right]E_3(a,b)-\frac{h_vQ}{2}(m-1), \quad (22)$$

and $G(L)=\frac{\tau}{2}\sigma\sqrt{L}\left(\sqrt{1+k^2}-k\right)$.

The corresponding single objective problem is

Maximize $\{F^2(Q, L; k, m)\}$ (23)

$subject\ to\ 0 < A_b \le A_{b0}.$
$and\ v_1+v_2=1,\ v_1,v_2\ge 0.$
where $F^2(Q, L; k, m) = v_1 TEPU_b(Q, L; k) + v_2 TEPU_v(Q; m)$

Remark 1. The multi-objective problem (20) satisfy all the Proposition (Proposition-1, Proposition-2, Proposition-3) of the multi-objective problem (13).
Proposition 5. (Ouyang and Wu [19]) If X is the demand during lead time which has a p.d.f $f_X(x)$ with finite mean D_L and standard deviation $\sigma\sqrt{L}(>0)$, then for any real number d > 0

$$P(X>d)\le\frac{\sigma^2 L}{\sigma^2 L+(d-D_L)^2}. \quad (24)$$

Taking $d=R=D_L+k\sigma\sqrt{L}$ in inequality (20), we get

$$P(X>R)\le\frac{1}{1+k^2}. \quad (25)$$

Further, it is assume that the allowable stock out probability q[= P (X > R)] during lead time is given, then inequations (25) gives $0\le k\le\sqrt{(1/q)-1}$.

The following algorithmic procedures was developed to identify global pareto-optimal solutions for (Q, L, m) when the demand during lead time is normally distributed (Model-4.1.1) and for (Q, L, k, m) when demand during lead time is distribution free (Model-4.1.2).

Algorithm 1. (For Model 4.1.1)
Step 1: Set m = 1.

Step 2: For each L_r perform (a) and (b), r=1, 2, ..., n.

(a). For known k find ψ(k) from normal distribution table, using this value of ψ(k) compute $Q_r=\sqrt{\frac{K_1(L_r,k,m)}{K_2(m)}}$, where $K_1(L_r,k,m)$ and $K_2(m)$ are given in (26) and (27) in **Appendix-A** respectively.
(b). Find corresponding $F^1(Q_r, L_r; k, m)$, where $F^1(Q_r, L_r; k, m) = w_1 TEPU_b(Q_r, L_r; k) + w_2 TEPU_v(Q_r; m)$, $w_1 + w_2 = 1$ and $w_1, w_2 \ge 0$.

Step 3: Find $F^1(Q^*, L^*; k, m) = \text{maximize}_{r=1,2,....,n} F^1(Q_r, L_r; k, m)$,then (Q^*, L^*) is the pareto-optimal solution and $TEPU_b(Q^*, L^*; k), TEPU_v(Q^*; m)$ are compromise maximum value of the objective functions for fixed m and known k.

Step 4: Set m = m + 1.

Step 5: If $F^1(Q^*, L^*; k, m+1) \geq F^1(Q^*, L^*; k, m)$ then go to Step-2, otherwise go to Step-6.

Step 6: $TEPU(Q^*, L^*; k, m^*)$= $TEPU(Q^*, L^*; k, m)$, then (Q^*, L^*, m^*) is pareto-optimal solution and $TEPU_b(Q^*, L^*; k), TEPU_v(Q^*; m^*)$ are the compromise maximum values of objective functions for known k.

Step 7: Find corresponding $A_b(L^*) = u + vL^*$ where $u = \left(1 - \frac{1}{\eta}\right) A_{b0}$ and $v = \frac{A_{b0}}{\eta L_0}$,
$\beta(L^*, k) = \frac{1}{[1+\tau\sigma\sqrt{L^*}\psi(k)]}$ and $R(L^*, k) = DL^* + k\sigma\sqrt{L^*}$.

Step 8: End.

Algorithm 2. (For Model 4.1.2)
Step 1: Set m = 1.

Step 2: For a given q let $k_0 = 0, k_s = \sqrt{\left(\frac{1}{q}\right) - 1}$ and $k_j = k_{j-1} + \frac{(k_s - k_0)}{s}, j = 1,2, \dots, s-1$.

Step 3: For each L_r perform (a) and (b), r=1, 2, ..., n.

(a). Find the value of Q_{rj} for each $k_j \in \{k_1, k_2, ..., k_{s-1}\}$ using the formula $Q_{rj} = \sqrt{\frac{K_3(L_r, k_j, m)}{K_2(m)}}$, where $K_3(L_r, k_j, m)$ and $K_2(m)$ are given in (28) and (27) in **Appendix-A** respectively.

(b). Find $F^2(Q_r, L_r; k^{**}, m) = \text{maximize}_{j=1,2,....,s-1} F^2(Q_{rj}, L_r; k_j, m)$,where

$$F^2(Q_{rj}, L_r; k_j, m) = v_1 \overline{TEPU_b}(Q_{rj}, L_r; k_j) + v_2 \overline{TEPU_v}(Q_{rj}; m),\ v_1 + v_2 = 1.$$

$and v_1, v_2 \geq 0.$

Step 4: Find $F^2(Q^{**}, L^{**}; k^{**}, m) = \text{maximize}_{r=1,2,....,n} F^2(Q_r, L_r; k^{**}, m)$, then (Q^{**}, L^{**}, k^{**}) is the pareto-optimal solution and the values $\overline{TEPU_b}(Q^{**}, L^{**}; k^{**}, m)$ and $\overline{TEPU_v}(Q^{**}; m)$ are compromise maximum values of the objective functions for fixed m.

Step 5: Set m = m + 1.

Step 6: If $F^2(Q^{**}, L^{**}; k^{**}, m+1) \geq F^2(Q^{**}, L^{**}; k^{**}, m)$, then go to Step-2, otherwise go to Step-7.

Step 7: $F^2(Q^{**}, L^{**}; k^{**}, m^{**}) = F^2(Q^{**}, L^{**}; k^{**}, m)$, then $(Q^{**}, L^{**}, k^{**}, m^{**})$ is the pareto-optimal solution and the values $\overline{TEPU_b}(Q^{**}, L^{**}, k^{**})$, $\overline{TEPU_v}(Q^{**}; m^{**})$ are compromise maxi-mum values of the objective functions.
Step 8: Find corresponding $A_b(L^{**}) = u + vL^{**}$ [where $u = \left(1 - \frac{1}{\eta}\right) A_{b0}$ and $v = \frac{A_{b0}}{\eta L_0}$], $\beta(L^{**}, k^{**}) = \left[1 + \tau\sigma\sqrt{L^{**}}\left(\sqrt{1 + (k^{**})^2} - k^{**}\right)\right]^{-1}$ and $R(L^{**}, k^{**}) = DL^{**} + k^{**}\sigma\sqrt{L^{**}}$.

Step 9: End.

5.NUMERICAL EXAMPLE

Illustration 1: To illustrate the present model by an example, following data have been used. D = 600 units/year, K = 1500 units/year, r_s = 1000 units/year, A_{b0} = \$75 per order, A_{v0} = \$250 per setup, h_b = \$2 per unit per year, h_v = \$1.4 per unit per year, c_s = \$50 per unit, c_{ds} = \$15 per unit, c_{sc} = \$1 per unit, σ = 4 units/week, λ = 0.7, μ = 0.4, τ = 0.5, η = 45, $\Pi_0 = \$8, \Pi = \$2$ and per unit. By our earlier assumption $P \leq 1 - \frac{D}{r_s} = 0.4$, due to avoid shortages within the screening period, so we take [a, b] = [0.1, 0.25]. The lead time have three components (i.e, n = 3) with data shown in the Table-1.

Table 1: Lead time data for first retailer.

Lead time component (r)	1	2	3
Normal duration b_r(days)	20	20	16
Minimum duration a_r(days)	6	6	9
Unit crashing cost c_r(\$/day)	0.4	1.2	6.0

Using the data given in Table-1, we have L_0 = 56 days (8 week), L_1 = 56 − 14 = 42 days (6 weeks) L_2 = 42 − 14 = 28 days (4 weeks) L_3 = 28 − 7 = 21 days (3 weeks). Hence $L_3 = \min_{0 \leq r \leq n} L_r = 3\, weeks$, $L_0 = \max_{0 \leq r \leq n} L_r = 8\, weeks.$

Assuming that the lead time demand follows a normal distribution with stock-out probability q = 0.1 (the value of the safety factor k can be found directly from the standard normal table, which is 1.28) and value of ψ(1.28) is 0.0069567. Using Algorithm-I, for different values of the pair (w_1, w_2), the corresponding results of multi-objective model for normally distributed lead-time demand are shown in Table-2.

Table-2 : Pareto optimal solutions of the Model-4.1.1 for different weights.

(w_1, w_2)	m	F^1	($TEPU_b$, $TEPU_v$)	Q^*	$R(L^*, k)$	$A_b(L^*)$	$\beta(L^*, k)$
(0.3, 0.7)	1	8786.10	(5004.00, 10406.99)	506.62	56.39(4, 1.28)	74.17	0.9729
	2	8843.71	(5143.89, 10429.35)	314.79	56.39(4, 1.28)	74.17	0.9729
	3	8853.42	(5170.82, 10431.67)	235.30	81.77(6, 1.28)	74.58	0.9670
	4	**8850.05**	(5164.79, 10429.45)	191.65	81.77(6, 1.28)	74.58	0.9670
	5	8841.52	(5145.94, 10425.35)	163.64	81.77(6, 1.28)	74.58	0.9670
(0.6, 0.4)	1	7194.76	(5111.78, 10319.23)	368.61	56.39(4, 1.28)	74.17	0.9729
	2	7263.09	(5162.16, 10414.47)	273.38	81.77(6, 1.28)	74.58	0.9670
	3	**7275.24**	(5171.11, 10431.43)	231.11	81.77(6, 1.28)	74.58	0.9670
	4	7271.82	(5169.09, 10425.92)	205.21	81.77(6, 1.28)	74.58	0.9670
	5	7262.26	(5162.70, 10411.60)	187.04	81.77(6, 1.28)	74.58	0.9670
(0.9, 0.1)	1	5664.62	(5166.50, 10147.70)	259.21	81.77(6, 1.28)	74.58	0.9670
	2	5692.46	(5170.66, 10388.72)	236.97	81.77(6, 1.28)	74.58	0.9670
	3	**5697.23**	(5171.25, 10431.01)	226.89	81.77(6, 1.28)	74.58	0.9670
	4	5695.99	(5171.12, 10419.79)	220.14	81.77(6, 1.28)	74.58	0.9670
	5	5692.36	(5170.69, 10387.36)	214.84	81.77(6, 1.28)	74.58	0.9670

Illustration 2: In this illustration, we use same data as in Illustration-1. Assuming here that the demand during lead time is distribution free and also considering q = 0.1 (stock-out probability), then from Proposition-4 it is clear that 0 ≤ k ≤ 3. Let $k_j = k_{j-1} + (k_s - k_0)/s$, j=1, 2,..., s-1 where k_0 = 0, k_s = 3 and take s = 300. Applying Algorithm-2 , the pareto optimal solutions of Model-4.1.2 for different v_1 and v_2 are obtained and presented in Table-3.

Table-3 : Pareto optimal solutions of the Model-4.1.2 for different weights.

(v_1, v_2)	m	F^2	$(\overline{TEPU_b},\ \overline{TEPU_v})$	Q^{**}	$R(L^{**}, k^{**})$	$A_b(L^{**})$	$\beta(L^{**}, k^{**})$
	1	8777.70	(4971.44, 10408.95)	512.01	59.05(4, 1.6123)	74.17	0.4674
	2	8832.12	(5103.06, 10430.29)	319.36	62.38(4, 2.0286)	74.17	0.5175
	3	**8839.14**	(5123.05, 10431.75)	240.09	64.71(4, 2.3198)	74.17	0.5478
	4	8833.58	(5111.97, 10428.56)	195.96	66.53(4, 2.5475)	74.17	0.5692
	5	8823.20	(5088.10, 10423.58)	167.58	68.04(4, 2.7356)	74.17	0.5854
	1	7173.92	(5071.30, 10327.83)	377.49	61.13(4, 1.872)	74.17	0.4997
	2	7237.63	(5116.50, 10419.31)	283.55	63.32(4, 2.1461)	74.17	0.5302
	3	**7246.53**	(5123.06, 10431.74)	240.70	64.69(4, 2.3170)	74.17	0.5475
	4	7240.64	(5119.41, 10422.47)	214.21	65.72(4, 2.4454)	74.17	0.5598
	5	7229.02	(5111.72, 10404.99)	195.49	66.56(4, 2.5503)	74.17	0.5694
	1	5625.17	(5119.09, 10179.90)	273.65	63.61(4, 2.1823)	74.17	0.5340
	2	5650.46	(5122.64, 10400.83)	251.57	64.31(4, 2.2699)	74.17	0.5429
	3	**5653.93**	(5123.06, 10431.74)	241.32	64.67(6, 2.3143)	74.17	0.5473
	4	5651.76	(5122.83, 10412.19)	234.32	64.92(6, 2.3460)	74.17	0.5504
	5	5647.37	(5122.31, 10372.93)	228.76	65.13(6, 2.3723)	74.17	0.5529

For each pair of values (w_1, w_2) [or (v_1, v_2)] the Model-4.1.1 [or Model-4.1.2] gives pareto optimal solutions, which are also shown in the Table-2 [or Table-3].

From the above results it is seen that the models for normally distributed lead time demand give the better results than the corresponding models for distribution free lead time demand. This excess quantity can be regarded as the expected value of penalty due to prefer the distribution free demand during lead-time.

6.CONCLUDING REMARKS

In this study, we consider a single-vendor and single-buyer production inventory problem. Previous works on this problem mostly focused on stochastic demand, controllable lead-time and the effect of lead-time on joint expected cost through ordering cost, lead-time crash cost and back order ratio [e.g., Ouyang et al. [20], Chang et.al. [17]]. Here we extend those models by considering imperfect production process in individual management system. We investigate to maximize the joint total expected cost and individual total expected cost by simultaneously optimizing order quantity, lead time, reorder point and the numbers of lots deliver from vendor to buyer for joint and individual management system respectively.

In addition develop some algorithmic procedure to find optimal order quantity, optimal lead time, optimal reorder point and optimal numbers of lots deliver from vendor to buyer. Further we get the significant results in the total expected annual profit for different models. To do this firstly we assume that the lead-time demand follows a normal distribution and secondly we assume that only the first and second moments of the probability distribution of lead-time demand are known.

In further research on this problem, it would be interesting to deal with the inventory model with a service level constrained in fuzzy stochastic environment.

Appendix-A

The values of $E_1(a, b)$, $E_2(a, b)$, $E_3(a, b)$ and $E_4(a, b)$ are from the following formulas:-

$$E_1(a,b) = E[P] = \int_{-\infty}^{\infty} p g(p) dp = \int_a^b \frac{p}{b-a} dp = \frac{1}{2}(b+a),$$

$$E_2(a,b) = E[1-P] = \int_{-\infty}^{\infty} (1-p) g(p) dp = 1 - \frac{1}{2}(b+a),$$

$$E_3(a,b) = E\left[\frac{1}{1-p}\right] = \int_{-\infty}^{\infty}\frac{1}{1-p}g(p)dp = -\frac{1}{b-a}[\ln(1-p)]_a^b = \frac{1}{b-a}\ln\frac{(1-a)}{(1-b)},$$

$$E_4(a,b) = E\left[\frac{P}{1-P}\right] = \int_{-\infty}^{\infty}\frac{p}{1-p}g(p)dp = \int_{-\infty}^{\infty}\frac{1}{1-p}g(p)dp - \int_{-\infty}^{\infty}g(p)dp = \frac{1}{b-a}\ln\left(\frac{1-a}{1-b}\right) - 1.$$

The expressions for $K_1(L_r, k, m)$, $K_2(m)$, $K_3(L_r, k, m)$, $K_4(L_r, k, m)$, $K_5(m)$ and $K_6(L_r, k, m)$ are respectively:-

$$K_1(L_r,k,m) = w_1 D\left[A_b(L_r) + \frac{\Pi F(L_r)}{\tau} + C(L_r) + \frac{\Pi_0 F(L_r)^2}{\tau[1+F(L_r)]}\right]E_3(a,b) + \frac{w_2 D A_{v0}}{m}E_3(a,b), \quad (26)$$

$$K_2(m) = w_1 h_b\left[\frac{1}{2}E_2(a,b) + \frac{D}{r_s}E_4(a,b)\right] + w_2 h_v\left[\frac{1}{2}(m-1) + \frac{D(2-m)}{2K}E_3(a,b)\right], \quad (27)$$

$$K_3(L_r,k,m) = w_1 D\left[A_b(L_r) + \frac{\Pi G(L_r)}{\tau} + C(L_r) + \frac{\Pi_0 F(G_r)^2}{\tau[1+G(L_r)]}\right]E_3(a,b) + \frac{w_2 D A_{v0}}{m}E_3(a,b), \quad (28)$$

$$K_4(L_r,k,m) = D\left[A_b(L_r) + \frac{\Pi F(L_r)}{\tau} + C(L_r) + \frac{A_{v0}}{m} + \frac{\Pi_0 F(L_r)^2}{\tau[1+F(L_r)]}\right]E_3(a,b), \quad (29)$$

$$K_5(m) = h_b\left[\frac{1}{2}E_2(a,b) + \frac{D}{r_s}E_4(a,b)\right] + h_v\left[\frac{1}{2}(m-1) + \frac{D(2-m)}{2K}E_3(a,b)\right], \quad (30)$$

$$K_6(L_r,k,m) = D\left[A_b(L_r) + \frac{\Pi G(L_r)}{\tau} + C(L_r) + \frac{A_{v0}}{m} + \frac{\Pi_0 G(L_r)^2}{\tau[1+G(L_r)]}\right]E_3(a,b), \quad (31)$$

Where $F(L) = \tau E(Y) = \tau\sigma\sqrt{L_r}\psi(k)$ *and* $G(L_r) = \frac{\tau}{2}\sigma\sqrt{L_r}\left(\sqrt{1+k^2} - k\right)$.

REFERENCES

(1). Goyal,S.K. An integrated inventory model for a single supplier-single customer problem. International Journal of Production Research 1976; 15(1): 107-111.
(2). Goyal,S.K. A joint economic-lot-size model for purchaser and vendor: A comment. Decision Sciences 1988; 19: 236-241.
(3). Ha,D., Kim S.L. Implimentation of JIT purchasing: An intregated appoach. Production planning and Control 1997; 8(2): 152-157.
(4). Huang, C-K. An integrated vendor-buyer cooperative inventory model for items with imper-fect quantity. Production Planning & conttol 2002; 13(4): 355-361.
(5). Tersine,R.J. Principles of inventory and materials management. New York: North-Holland, 1982.
(6). Liao, C-J., Shyu,C-H. An analytical determination of lead time with normal demand. Inter-national Journal of Operations and Production Management 1991; 11: 72-78.
(7). Ben-Daya, M., Raouf, A. Inventory models involving lead time as a decision variable. Journal of the Operational Research Society 1994; 45(5): 579-582.
(8). Ouyang, L-Y., Yen, N-C., Wu, K-S. Mixture inventory model with backorders and lost sales for variable lead time. Journal of the OperationalResearch Society 1996; 47: 829-832.
(9). Ouyang, L-Y., Wu, K-S. A minimax distribution free procedure for mixed inventory model with variable lead time. International Journal of Production Economics 1998; 56: 511-516.
(10). Pan, J-C., Hsiao, Y-C. Inventory models with backorder discounts and variable lead time. International Journal of Systems Science 2001; 32: 925-929.
(11). Roy,T.K., Maiti, M. Multi-objective inventory models of deteriorating items with some constraints in a fuzzy environment. Computers & Operations Research 1998; 25: 1085-1095.

(12). Mahapatra, N.K., Maiti, M. Inventory model for deteriorating items with uncertain setup time. Tamsui Oxford Journal of Management Science, Taiwan, ROC 2004; 20(2): 83-102.
(13). Mahapatra, N.K., Maiti, M. Production-Inventory Model for a Deteriorating Item with Im-precise Preparation Time for Production in a Finite Time Horizon. Asia-Pacific Journal of Operational Research 2006; 23(2): 171-192.
(14). Miettinen, K.M. Non-linear Multi-objective Optimization, Kluwer's International Series, London, 1999.
(15). Chiu, P-P. Economic production quantity models involving lead time as a decision variable. Master Thesis: National Taiwan University of Science and Technology 1998.
(16). Chen, C-K., Chang, H-C., Ouyang, L-Y. A continuous review inventory model with ordering cost dependent on lead time. International Journal of Information and Management Science 2001; 12(3): 1-13.
(17). Chang, H-C., Ouyang, L-Y., We, K-S., Ho, C-H. Integrated vendor-buyer cooperative inven-tory models with controllable lead time and ordering cost reduction. Europian Journal of Operation Research 2006; 170: 481-495.
(18). Gallego, G., Moon, I. The distribution free Newsboy problem: review and extensions. Journal of the Operational Research Society 1993; 44: 825-834.
(19). Ouyang, L-Y., Wu, K-S. Mixture inventory model inventory model involving variable lead time with a service level constraint. Computers and Operation Research 1997; 24: 875-882.
(20). Ouyang, L-Y., Wu, K-S., Ho, C-H. Integrated vendor-buyer cooperative models with stochastic demand in controllable lead time. International Journal of Production Economics 2004; 92: 255-266.

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