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A STUDY ON NORMED LINEAR SPACE AND ITS APPLICATIONS

$$\|x\|_1 = |x_1| + |x_2| + \ldots + |x_n|,$$

$$\|x\|_2 = \left(x_1^2 + x_2^2 + \ldots + x_n^2\right)^{1/2},$$

$$\|x\|_\infty = \max_{1 \le k \le n} |x_k|.$$

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ABSTRACT

This work is concerned with normed space, which is the subspace of a vector space. In linear algebra, functional analysis and related areas of mathematics, a norm is a function that assigns a strictly positive length or size to all vectors in a vector space, other than the zero vectors. A simple example is the 2-dimensional Euclidean space R^2 equipped with the Euclidean norm. With a norm it is possible to measure the distance between elements, but it is not possible to look at the position of two different elements, with respect to each other.

KEYWORDS: *Normed Space, Euclidean Norm, 2-Dimensional Euclidean Space, Strictly Positive Length or Size of Vectors.*

1. PRILIMINARIES

In this we discuss some basic definitions and some theorems which are required in the succeeding chapters.

Definition 1.1

A Banach space is a complete normed space. A metric space in which each Cauchy sequence converges is called complete. A vector space on which a norm is defined is then called a normed vector space

Definition 1.2
Linear Combination

A vectorβ in v is said to be a linear combination of the vector $\propto_1 + \propto_2 \ldots . \propto_n$In v if there exists scalars$c_1, c_2, \ldots., c_n$in f such that,

$\beta = c_1 \propto_1 + c_2 \propto_2 + \cdots . + c_n \propto_n = \sum_{i=1}^{n} c_i \alpha_i$

Definition 1.3

If a normed space x contains a sequence (e_n) with the property that for every $\times \epsilon$X there is a unique sequence of scalars $(\propto_n)$ such that,

$||x-(\propto_1 e_{1,} + \cdots . + \propto_n e_n)||$ **0**as n $\rightarrow \infty$

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Then(e_n) is called a **basis** for X. The series $\sum_{k=1}^{\infty} \alpha_k e_k$ which has the sum x is then called **the expansion of x** with respect to (e_n), and we write x= $\sum_{k=1}^{\infty} \propto_k e_k$

Definition 1.4

On a vector space X there can be defined an infinitely number of different norms. Between some of these different norms there is almost no difference in the topology they generate on the Vector space X. if some different norms are not to be distinguished of each other; these norms are called **equivalent norms.**

Let X be a vector space with norms $\|\ \|_0$ and $|||_1$. Thenorms$\|\ \|_0$ and$\|\ \|_1$ are said to be equivalent if there exit numbers m $>$ 0 and M $>$ 0 such that for every x ϵ X

$m\| x \|_0 \leq \| x \|_1 \leq M \| x \|_0$

The constants m and M are independent of x.

Definition 1.5

A subspace Y of a normed space X is a subspace of X considered as a vector space, with the norm obtained by restricting, the norm on X to the subset of Y. This norm on X is said to be induced by the norm on X. If Y is closed in X, then Y is called **the closed subspace of X**. A subspace Y of a Banach space X is a subspace of X considered as a normed space.

Definition 1.6

Suppose that V is a normed space over F. For any subset A⊆V, the linear **span space (A**) of A is defined to be the set of all linear combinations of elements of A more precisely,
Span(A) $= \{c_1X_1 + \dots + c_r x_r;\ \ r \in N, c_1 \dots c_r \in F$ and $x_{1,}x_2. \dots x_r$ ∈A}

Definition 1.7

A subset M in a normed space X is bounded if and only if there is a positive number c such that
$||x|| \leq c,\ \ x \in$ M

Definition 1.8

Isometric mapping, Isometric spaces.

Let X =(X,d) and $\overline{x} = \{\overline{X}, \overline{d}\}$ be metric spaces. Then:

a) A mapping T of X onto $\overline{X}$ is said to be isometry or an isometriy if
T preserves, distances, that is if for all x.y ∈ X

$\overline{d}$ (Tx,Ty) =d(x,y)

Where Tx and Ty are the images of x and y, respectively.

b) The space X is said to be isometric with the space $\overline{X}$. If there exists a bijective in connecting of X onto $\overline{X}$. The spaces X and $\overline{X}$ are there called **isometric spaces.**

Definition 1.9

Compactness

A normed space X is said to be **compact** if every sequence in X has a convergent subsequence. A subset M of X is said to be compact if M is compact considered as a subspace of X, that is, if every sequence in M has a convergent subsequence whose limit is an element of M.

Definition 1.10 (Bounded linear operator)

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Let X and Y be normed spaces and T: D(T) → Y a linear operator, Where D (T) c X. The operator T is said to be **bounded** if there is real number c such that for all x∈ D (T),

|| T(x)|| ≤ c || x ||

Definition 1.11

Two operators T_1 and T_2 are defined to be equal, Written T_1=T_2 if they have the same domain D(T_1) and if D (T_2) and T_1 x = T_2x for all

X ∈ (T_1)=D(T_2)

The restriction of an operator T: D(T) →Y to a subset B C D(T) is denoted by, T|B and is on operator defined by T|B: B →Y, T|B x= Tx, for all x∈ B,

An extension of T to a set M D (T) is an operator $\overline{T}$: M→ T such that $\overline{T}$ $|_{D(T)}$=T that is $\overline{T}$ x=Tx for all x∈D(T)

Definition 1.12

Let X be a normed space. Then the set of all bounded linear functional on X constitutes a normed space with norm defined by

||f|| =sup x∈x |f(x)|=sup x∈X |f(x)|
x≠ 0 $||x||$$||X||$=1

Which is called **the dual space of X** and is denoted by X^1

Definition 1.13

An isomorphism of a normed space X onto a normed space is a bijective linear operator T: X →X which preserves the norm, that is for all x∈X,

||Tx|| = ||x|| (Hence T is isometric) X is then called **isomorphic** with $\overline{X}$ and X and $\overline{X}$ are called isomorphic normed spaces.

Definition 1.14

A linear functional f is a linear operator with domain in a vector space X and range in the scalar field K of X: thus, f: **D**(f) → K Where K=R if X is real and K =C if Xis complete.

Definition 1.15

A bounded linear functional f is a bounded linear operator with range in the scalar field of the normed space X in which the domain **D**(f) lies .Thus there exists **a** real number C such that for **all X**∈**D**(f),

|f(x)|≤ C||x||

Furthermore, the norm of f is||f|| =supx∈D(f)|f(x)| or||f|| = supx∈D(f)|f(x)|

x≠0 ||x||||x||=1

Definition 1.16

A real valued functional P on a vector space X is said to be **subadditive,** if

P(x+y)≤P(x)+P(y),for all x,y∈ X
Also, a real valued functional P on a vector space X is said to be

Positive homogeneous if,
p(∝x) =∝p(x), for all ∝≥ 0 in R and x ∈ X

Definition 1.17 (Strong convergence)

A sequence (x_n) in a normed space X is said to be **strongly convergent** (or convergent in the norm) if there is an x ∈ X such the

$$lim_{n\to\infty}||xn - x|| = 0$$

X is called the strong limit of (x_n) and we say that, (x_n) converges strongly to x.

Definition 1.18 (Weak convergence)

A sequence (x_n) in a normed space X is said to be weakly convergent if there is an x X such that for every fX^1

The element x is called the weak limit of (x_n) and we say that (x_n) converges weakly to x.

Definition 1.19

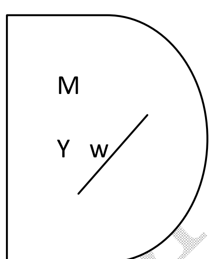
To every x X there corresponds a unique bounded
linear functional g_x X" defined byg_x(f) = f(x) This defines a mapping C: X → X", x→ $g_{x,}$ C is called **the canonical mapping** of X into X"

Definition 1.20 (Reflexivity)

A normed space X is said to be **reflexive** if R(c) = X", When C: X→ X" is canonical mapping.

Definition 1.21

A subset M of a vector space X is said to be **convex** if y, z M implies that the set.
W={v=y+(l-)z:0≤≤1} is a subset of M. This element W is called closed segment. If y and z are called the boundary points of the segment W. Any other point of W is an interior point of W.



Convex

Not Convex

Definition 1.22

A strictly convex is a norm such that for all x, y of norm l
||x+y|| < 2(x≠y)

A normed space with such a norm is called **a strictly convex normed space**

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Definition 1.23

An inner product space is a vector space with an inner product defined on X. A Hilbert space is a complete inner product space. Also an inner product on X is a mapping X:XxX→ K and is denoted by $< x, y >$ for every pair of vectors x and y.

Definition 1.24

Let X be a normed space .Let Y C X and x ∈ X then, distance of x from y is, dist (x,y) = inf{||x-y||:y∈ Y}

Definition 1.25

A functionγ:[a,b]→C where [a,b] CR is of bounded variation, if there exist a constant M>0 such that,
V (γ, p)=$\sum_{k=1}^{m}|\gamma(t_k) - \gamma(t_{k-1})| < M$ for some M>0 and for every partition P
P ={a=t_o <……< t_1=b},
Sup {V(γ,p): P partition of [a,b]} called total variation of γ denoted by V(γ)
Clearly, V(γ)≤ M < ∞

Definition 1.26

A scalar valued function is said to be **P-integrable**if $\int_E |x|^p$ dm< ∞ where m be the Lebesgue measure on R and E be a measurable subset of R.

Theorem 1.27

Suppose that V is a normed vector space over F_1 and that W is a linear subspace of V. Then the closure $\overline{W}$ of W is a closed subspace of V.

Theorem 1.28

For a metric space X = (X,d) there exists a complete metric space $\overline{X} = \left(\overline{X}, \overline{d}\right)$ Which has a subspace W that is isometric with X and is close in $\overline{X}$ .This space $\overline{X}$ is unique except for isometrics, that is if $\overline{X}$ is any complete metric space having a dense subspace $\overline{W}$ isometric with X, then X and $\overline{X}$ are isometric.

Theorem 1.29

Hahn - Banach generalized theorem

Let X be a real or complex vector space and P be a real valued functional on X which is subadditive, that is for all x,y∈X

P(x+y) ≤ p(x)+p(y) and for every scalar ∝ satisfies p(∝x)= |∝| p(x).

Furthermore, let f be a linear functional which is defined on a subspace Z of X and satisfies |f(x)| < p(x) for all x∈Z. Then f has a linear extension $\overline{f}$ from Z to X satisfying | $\overline{f}$ (x)| ≤p(x) for all x ∈X.

2. NORMED SPACE

Vector Spaces play a role in many branches of mathematics and its applications. In fact, in various practical problems we have a set X whose elements may be vectors in three dimensional space, or sequence or numbers, or functions, and these elements can be added and multiplied by constants (numbers) in a natural way, the result being again an element of X such concrete situations suggest the concept of a vector space. The definition will involve a general field K, but in functional analysis, K will be R or C. The elements of K are called scalars; hence in one case they will be real or complex numbers.

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A vector space (or linear space) over a field K is a nonempty set X of elements x,y….(called vectors) together with two algebraic operations. These operations are called vector addition and multiplication of vectors by scalars that are by elements of K.

Normed space

Let X be a linear space. A function $\|\|:X\to R$ that satisfies the following properties is called a norm on X, for $x, y \in X$

$i)$ $\|x\| \geq 0$ and $\|x\| = 0$ if and only if $x=0$

$ii)$ $\|\propto x\| = |\propto|\|x\|$, for all $\propto \in R$ (Absolute Homogeneity)

$iii)$ $\|x+y\| \leq \|x\|+\|y\|$(Triangle Inequality). If $\|\|$ is a norm on X, then we say that $(X,\|\|)$ is a normed linear space. If $\|\|$ only satisfies the requirements (ii) and (iii), then $(X,\| \|)$ is called a seminormed linear space.

Here a norm on a (real on complex) vector space X is a real-valued function on X.

A norm on X defines a metric d on x which is given by,

$d(x,y) = \|x-y\|(x, y\ \ X)$

Be assigned positive length, and hence properly (i). Third, the norm of a vector –x should equal the norm of x, because multiplying a vector by – 1 should change only the direction of the vector, not its length. Fourth, doubling a vector should double the norm of that vector, simply because the intuitive notion of 'length' behaves in this manner. Property (ii) means that when a vector is multiplied by a scalar, its length is multiplied by the absolute value of the scalar. Property (iii) is illustrated in Fig (1.1) It means that the length of one side of a triangle cannot exceed the sum of the lengths of the two other sides.

It is not difficult to conclude from (i) to (iii) properties that does define a metric. Hence normed space is a metric space.

Recall that the basic viewpoint of vector calculus is regard a "vector" x in a linear space as a directed line segment that begins at zero and ends at x. This allows on to think of the "length" (or the "magnitude") of a vector in a natural way. For instance, we think of the magnitude of a positive real number x as the length of the interval (0, x] and that of –x as the length of [-x, 0). Indeed, it is easily verified that the absolute value function defines a norm on R. Similarly, it is conventional to think of the length of a vector in R^n as the distance between this vector and the origin, and as you would expect, $x\to d_2(x,0)$ defines a norm on R^n .Just as a notion of a metric generalizes the geometric notion of "distance" therefore, the notion of a norm generalizes that of "length" or "magnitude" of a vector.

This interpretation also motivates the properties that a "norm" must satisfy. First a norm must be non negative, because "length" is an inherently non negative notion. Second, a non zero vector should when the norm under consideration is apparent from the context, it is customary to dispense with the notion $(x,\|\|)$ and refer to the set X itself as a normed linear space. We shall frequently adopt these conversions in what follows. That is, when we say that X is a normed linear space, you should understand that X is a linear space with norm $\|\|$ lurking in the background.

Let X be a normed linear space. Take any x, y $\in$ X Note that, by triangle inequality.

||x|| = ||x-y+y||
$\leq$ ||x-y||+||y|| so that
||x|| - ||y|| $\leq$ ||x-y||

Moreover, if we change the roles of x and y in this inequality, we get ||y||-||x|| < ||y-x||=||x-y||where the last equality follows from the absolute homogeneity of || || thus,
| ||x||-||y||| $\leq$ ||x-y|| for any x, y ϵ X

Example 2.1
Let C denote the set of all convergent sequences of scalars
ie, C={x $\in l^{\infty}$:x(j) converges in k as j→ ∞} Clearly, C is a subspace ofl^{∞}
ie , C $\subset l^{\infty}$
∴ C is normed space with norm defined by,
$||x||_{\infty} = \text{Sup}_{j=1,2,\ldots}\{|x(j)|\}$

Example 2.2
For $1 \leq p < \infty$ consider the following sequence in K
L^p= {(x(1),x(2),……):x(j) ϵ K and $\sum_{j=1}^{\infty} |x(j)|^p < \infty$}
For x =(x(1),x(2),-.) in l^p, let $||x||_p = (\sum_{j=1}^{\infty} |x(j)|^p)\ 1/p$, `for j=1,2,….
We show that, $||\ ||_p$ satisfies there conditions of a norm,

i. $||x||_p \geq 0$ and

$||x||_p$**= 0** $\Leftrightarrow (\sum_{j=1}^{\infty} |x(j)|^p)\ 1/p$ = 0
$\Leftrightarrow (\sum_{j=1}^{\infty} |x(j)|^p)$ = 0, for all j
$\Leftrightarrow |x(j)|^p$ =0 , for all j
$\Leftrightarrow |x(j)|$ =0 , for all j
$\Leftrightarrow x(j)$ =0 , for all j
ie, x(1)=x(2)….=0
ie, x=(0,0,…..)
$\Leftrightarrow$ x=0

ii. $||x||_p = (\sum_{j=1}^{\infty} |x(j)| + y(j)|^p)\ 1/p$ = 0
BY Minkowski's inequality

$(\sum_{j=1}^{n} |x(j)| + y(j)|^p)\ 1/p \leq (\sum_{j=1}^{n} |x(j)|^p)\ 1/p\ \ (\sum_{j=1}^{n} |y(j)|^p)\ 1/p$

$(\sum_{j=1}^{n} |x(j)| + y(j)|^p)\ 1/p \leq ||x||_{p+}||\mathbf{y}||_p$

$lim_{n\to\infty}(\sum_{j=1}^{n} |x(j)| + y(j)|^p)\ 1/p \leq lim_{n\to\infty}||x||p + ||y||p$

$(\sum_{j=1}^{n} |x(j)| + y(j)|^p)\ 1/p \leq ||x||_p$+$||\mathbf{y}||_{\boldsymbol{p}}$

$||x+y||_p = ||x||_p$ **+**$||y||_p$

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iii. $||x+y||_p = (\sum_{j=1}^{\infty} |kx(j)|^p)\ 1/p$

$= (\sum_{j=1}^{\infty} |k|^p)\ 1/p \quad (|x(j)|^p)\ 1/p$

$=|k(\sum_{j=1}^{\infty} |x(j)|^p)\ 1/p$

$=|k|\,||x||_p$

Thus, $||\mathbf{x}||_p$ is a norm on k^n

Example 2.3

L^p Spaces: Let $L^p(E)$ denote the set of all equivalence classes of p- integrable functions on E for $1 \leq p < \infty$
$L^p(E)=\{x: \int E|x|^p\, dm < \infty\}$.For define the norm, $||\mathbf{x}||_p=$
, $||\mathbf{x}||_p=(\int_E |x|^p dm)^{1/p} \geq 0$
$||\mathbf{x}||_p=0 \Leftrightarrow (\int_E |x|^p dm)^{1/p} = 0$
$\Leftrightarrow |\mathbf{x}||_p=0$, almost everywhere
$\Leftrightarrow \mathbf{x}=0$, almost everywhere
ii. $||x+y||_p = (\int_E |x+y|^p dm)^{1/p} \leq (\int_E |x|^p dm)^{1/p} + (\int_E |y|^p dm)^{1/p}$
$\leq ||x||_p + ||y||_p$
Iii $||kx||_p = (\int_E |kx|^p dm)^{1/p} = (\int_E |k|^p |x|^p dm)^{1/p}$
$= (|k|^p)^{1/p} (\int_E |x|^p dm)^{1/p} = |k|\,||x||_p$
Thus $L^p(E)$ the space of equivalence classes of P-integrable function and E is a normed space with norm,
$||\mathbf{x}||_p=(\int_E |x|^p dm)^{1/p}$

3. FUNDAMENTAL THEOREMS FOR NORMED SPACE

Hahn-Banach theorem

The Hahn-Banach theorem is an extension for linear functional. The Hahn-Banach theorem becomes one of the most important theorems in connection with bounded linear operators. Furthermore, our discussion will show that the theorem also characterizes the extent to which values of a linear functional can be preassigned. The theorem was discovered by H.Hahn (1927), rediscovered in its present more general form by S. Banach and generalized to complex vector spaces by H.F. Bohnenblust and A. Sobczyk (1938)

Hahn-Banachtheorem

Let f be a bounded linear functional on a subspace Z of a normed space X. Then there exists a bounded linear functional… On X which is an extension of f to X and has the same norm.

$||\bar{f}||_x = ||f||_z$
Where, $||\bar{f}||_x = =\sup x \in X$, $||f||_z = \sup x \in Z \quad |f(x)|$
$||x||_{=1} \quad ||X||=1$
(and $||f||_z = 0$ in the trivial case $Z = \{0\}$)

Theorem (Bounded linear functionals)

Let X be a normed space and let $x_0 \neq 0$ be any element of X. Then there exists a bounded linear functional $\bar{f}$ on X such that, $||\bar{f}|| = 1$, $\bar{f}(x_0) = ||x_0||$

Proof

We consider the subspace Z of X consisting of all elements $x = \propto x_0$ where $\propto$ is a scalar.

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On Z we define a linear functional f by,
f(x) = f($\propto x_0$) = $\propto ||x_0||$.......... (1)
f is bounded and has ||f||=1 because
|f(x)| = |f($\propto x_0$)| = $|\propto| ||x_0||$ = || $\propto x_0$|| = ||x||
Hahn-Banach theorem for normed space implies f has a linear extension $\bar{f}$ Form Z to X of norm $||\bar{f}||$ = 1=||f|| from (1) we see that
$\bar{f}(x_0)$ = $||x_0||$ =$f(x_0)$

4. APPLICATIONS OF NORMED SPACE

Application to Bounded linear functionals on C [a, b]

The Hahn-Banach theorem (Normed space) has many important applications. In fact, we shall use the Hahn-Banachtheorem for obtaining a general representation formula for bounded linear functional on C [a, b], where [a, b] is a fixed compact interval. In the present case the representation will be in terms of a Riemann-Stieltjes intergral. So let us recall the definition and a few properties of their integral, which is a generalization of the familiar Riemann integral. We begin with the following concept.

A function W defined on [a, b] is said to be of bounded variation on [a, b] if its total variation Var (w) on [a, b] is finite, where

Var (w)sup$\sum_{j=1}^{n} |w(t_j) - w(t_j) - w(t_{j-1})|$ ... ... (1)thesupremum

being taken over all partitions.

$a = t_0 <$.......< $t_n = b$...... (2)
of the interval [a, b] here n ∈ N is
Arbitrary and so is the choice of values t_1 ..... t_{n-1} in [a,b] which, however, must satisfy equation (2)

Obviously, all functions of bounded variations on [a,b] from a vector space. A norm on this space is given by,

$||w|| = | w(a)| = +Var(w)$ ... ...(3)

The normed space thus defined is denoted by Bounded Variation [a,b].

We now obtain the concept of a Riemann-Stieltjes integral as follows.
Let **x**ϵC[a,b] and w ϵ Bounded Variation [a,b] Let $p_n$be any partition of [a,b] given by eqn (1) and denote by n(P_n) the length of a largest interval

The length of a largest interval

$[t_{j-1,} t_j]$ that is, n(P_n) =max ($t_1 - t_{0,}$ ... . $t_n - t_{n-1}$)

For every participation $P_n$of [a,b] we consider the sum,

$s(p_n) < \sum_{j=1}^{n} x(t_j)\,[w(t_j) - w(t_{j-1})]$ ... .(4)

There exists a number f with the property that for every f >0 there is a ε >0 there is a δ>0 such that

$n(p_n) < \delta$ ... ... (5)

implies, | f$-s(p_n)$ | $< \varepsilon$ ... . (6)

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f is called the Riemann-Stieltjes integral of x over [a, b]

with respect to w and is denoted by $\int_a^b x(t)dw(t)$ ... ...(7)

Hence we can obtain eqn (7) as the limit of the sums eqn (4) for a sequence (p_n) of partition of [a,b] satisfying n $(P_n) \to 0$ as $n \to \infty$
Note that for w (t) = t, the integral eqn (7) is the familiar Riemann integral of x over [a,b]
Also, if x is continuous on [a,b] and w has a derivative which is integrable on [a,b] then

$$\int_a^b x(t)dw(t) = \int_a^b x(t)w'(t)\ dt$$

Where the prime denote the differentiation with respect to t.
The integral eqn (7) depends linearly on $x \in$ [a,b] that is for all

$x_{1,}x_2 \in$C[a,b] and scalars α andβ we have

$$\int_a^b [ax_1(t) + \beta x_2(t)]\, dw(t) = \alpha \int_a^b x_1(t)dw(t) + \beta \int_a^b x_2(t)dw(t)$$

The integral also depends linearly on w ϵ Bounded Variation [a,b] that is for all $w_{1,}w_2 \in$ Bounded Variation [a,b] and scalars γ, δwe have

$$\int_a^b x(t)\, d(\gamma w_1 + \delta w_2)(t) = \delta \int_a^b x(t)dw_1(t) + \delta \int_a^b x(t)dw_2(t)$$

We shall also need the inequality,

$$|\int_a^b x(t)dw(t)| \leq max_{t\in j}|x(t)|Var(w) \ldots\ldots\ldots (9)$$

Where J= [a,b] we note that this generalize a familiar formula from calculus. In fact, if w(t) = t then Var (w) =b-a and eqn (9) takes the form,

$|\int_a^b x(t)d(t)| \leq max_{t\in j}|x(t)|(b-a)$ The representation theorem for bounded linear functional on C[a,b] by F Riesz (1909) can now be stated.

5. CONCLUSION

In this work we studied about the normed space and its application. We know that, in □□□□□□ Algebra, Functional Analysis and related areas of mathematics, a norm is a function that assigns a strictly positive length to all vectors in a □□□□□□ space, other than the zero vector. In this we studied the basic definitions related to normed space and the examples of normed space and its some applications.

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