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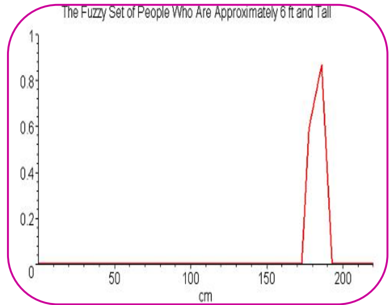
WEAK FUZZY CUT POINT AND WEAK FUZZY BRIDGE



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ABSTRACT

Let $x \in V$. Then x is called weak fuzzy cut point if there exist a $t \in (0, h(\mu)]$ such that x is a cut node for G^t and Let $xy \in \mu^*$ .Then xy is called a weak fuzzy bridge if there exist a $t \in (0, h(\mu)]$ such that xy is a bridge for G^t.

KEYWORDS: *Height and Depth of μ,Bridge, Cut node, Connected graph.*

1. INTRODUCTION:

Kaufman gave the first definition of a fuzzy graph. But it was Rosenfield and Yeh and Bang who laid the foundations for fuzzy graph theory in 1975. Fuzzy graph theory is now finding numerous applications in modern science and technology especially in the fields of neural networks, expert systems, information theory, cluster analysis, medical diagnosis, control theory, etc. Rosenfeld has obtained the fuzzy graph-theoretic concepts like bridges, paths, cycles, trees and connectedness and established some of their properties. Sunil Mathew, John N. Mordeson, Davender S.Malik discussed weak fuzzy bridge in “Fuzzy graph theory” and K.R.Bhutani, J.Mordesonand A.Rosenfeld discussed weak fuzzy cut node in “On degrees of end nodes and cut nodes”. In this paper we give necessary and sufficient condition to weak fuzzy bridge and weak fuzzy cut vertex.

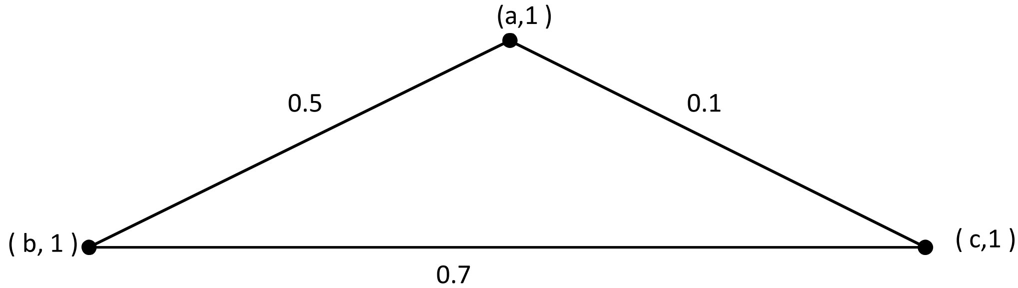
2. PRELIMINARIES:

Definition: 2.1

A ***fuzzy graph*** G= *(V,* σ, μ*)* is a triple consisting of a non-empty set *V* together with a pair of functions $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ such that for all $x, y \in V$, $\mu(x,y) \le \sigma(x) \wedge \sigma(y)$.

The fuzzy set σ is called the ***fuzzy vertex set*** of *G* and *μ* the ***fuzzy edge set*** of G. Clearl yμ is a fuzzy relation on σ. We consider *V* as a finite set, unless otherwise specified. For notational convenience, we use simply *G* or *(*σ, μ*)* represent the fuzzy graph G = (V, σ, μ). Also, we denote the underlying crisp graph by $G:(\sigma^*, \mu^*)$ where $\sigma^* = \{u \in V: \sigma(u) > 0\}$ and $\mu^* = \{(u, v) \in V \times V: \mu(u, v) > 0\}$

Example 2.2



Definition 2.3

Let $d(\mu) = \wedge\{\mu(xy) \mid xy \in \mu^*\}$ and $h(\mu) = \vee\{\mu(xy) \mid xy \in \mu^*\}$. Then $d(\mu)$ is called the **depth** of μ and $h(\mu)$ is called the **height** of μ. Note that $d(\mu)$ and $h(\mu)$ are undefined in if $\mu^* = \emptyset$.

Definition 2.4

Let $xy \in \mu^*$.Then

(i) xy is called a **bridge** if xy is a bridge of (σ^*, μ^*).

(ii) xy is called a **weak fuzzy bridge** if there exist $t \in (0, h(\mu)]$ such that xy is a bridge for G^t.

Definition 2.5

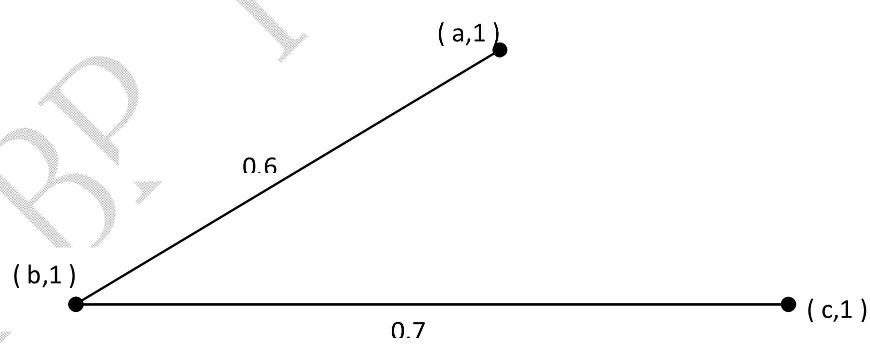
Let $x \in V$. Then

(i) x is called a **cut node** if x is a cut node of G^*.

(ii) x is called a **weak fuzzy cut node** if there exist a $t \in (0, h(\mu)]$ such that x is a cut node for G^t.

Example 2.6

Let V = {a, b, c}. Define the fuzzy subsets σ of V and μ of E= {ab, bc} as follows: $\sigma(a) = \sigma(b) = \sigma(c) = 1$ and $\mu(ab) = 0.6$, $\mu(bc) = 0.7$. Then $d(\mu) = 0.6$ and $h(\mu) = 0.7$. For $0 < t \leq 0.6$, $G^t = (V, \{ab, bc\})$ and for $0.6 < t \leq 0.7$, $G^t = (V, \{bc\})$. Both ab and bc are bridges and weak fuzzy bridges. Node 'b' is a cut node and weak fuzzy cut node.



Definition 2.7

Two vertices u and v of G are said to be connected if there is a (u, v)-path in G^*. A graph G^* is said to be connected if every two pairs of nodes are connected.

3. MAIN RESULT:

In the following propositions 3.1 and 3.3 the existing G^t should be connected and has 3 or more than 3 vertices

Proposition 3.1

If xy is a weak fuzzy bridge then to prove at least one of x or y is a weak fuzzy cut point.

Proof:

Given xy is a weak fuzzy bridge.

By the definition of weak fuzzy bridge, If there exist $t \in (0, h(\mu)]$ such that xy is a bridge for G^t.

Hence removal of xy disconnects G^t.

G^t has at least one additional vertex other than x and y.

At least one of the x and y is adjacent to a vertex w of G^t.

Without loss of generality say $xw \in \mu^t$.

Then removing x from G^t removes the line xy but leaves the point y and w which are necessarily in different components because they were separated by the bridge.

Thus x is a cut vertex for G^t.

Hence x is a weak fuzzy cut point

Example 3.2

Let V = {p, q, r, s}. Define the fuzzy subsets σ of V and μ of E= {pq, qr,rs,,qs}as follows:

$\sigma(p) = \sigma(q) = \sigma(r) = \sigma(s) = 1$ and $\mu(pq) = 0.6$, $\mu(qr) = 0.7$,$\mu(rs) = 0.8\mu(qs) = 0.7$

Then $d(\mu) = 0.6$ and $h(\mu) = 0.8$.

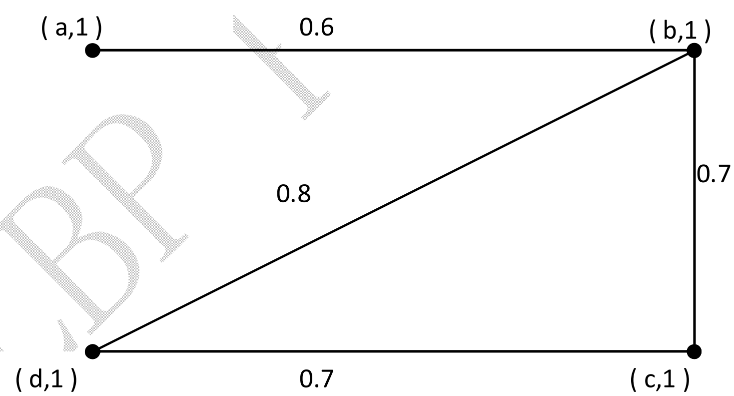
For $0 < t \leq 0.6$, G^t= (V, {pq, qr,rs,qs})

Here pq is weak fuzzy bridge and b is a weak fuzzy cut vertex.

For $0.6 < t \leq 0.7$, G^t= (V, {qr,qs,rs}).

Here all the edges are not a weak fuzzy bridge.

For $0.7 < t \leq 0.8$ G^t= (V, {qs}) Here qs is a weak fuzzy bridge but G^t has two nodes



Proposition 3.3

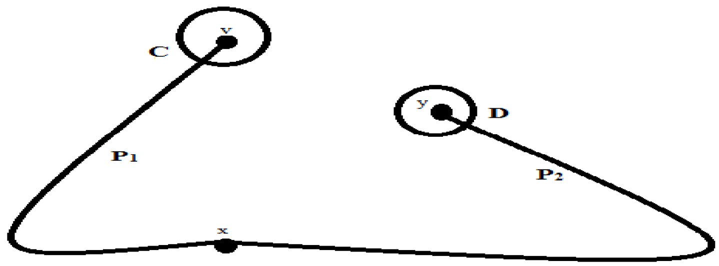
Every fuzzy graph G has two nodes as non- weak fuzzy cut node

Proof:

Let G be a fuzzy graph.

We have to prove that x and y are not weak fuzzy cut node of G.

It is enough to prove that x and y are not a cut node of any G^t.
We take P be a path of maximum length in G^t.
Let P = P (x, y).
Suppose x is a cut vertex for a G^t.
Consider G^t-x.
It contains at least two components say C and D where $y \in D$.
Let v be any vertex in C.
Then every (v, y)Path in G contains x.



But then Q = (P_1 (v,x),P_2(v,y)) is a path of length greater than the length of P.

It is a contradiction to the maximality of P.
Thus x is not a cut node.
Hence x is not weak fuzzy cut node.
Similarly y is not a weak fuzzy cut node.

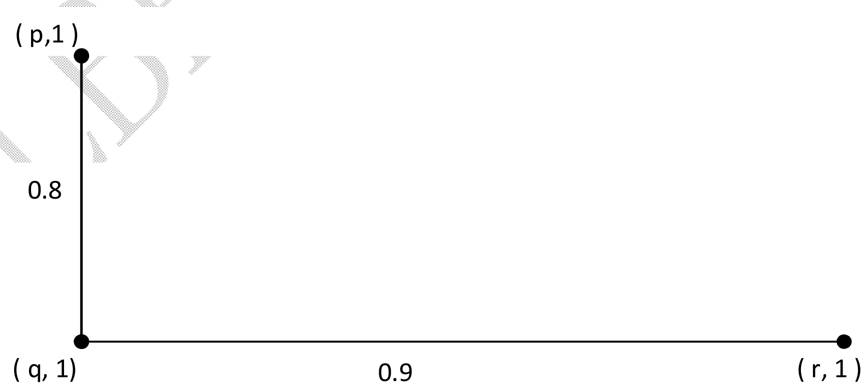
Note:

Weak Fuzzy Point is also known as Weak Fuzzy vertex (or) Weak Fuzzy Node

Example 3.4

Let V = {p, q, r, s}. Define the fuzzy subsets σ of V and μ of E= {pq, qr,} as follows:
σ(p) = σ(q) = σ(r) = σ(s) =1 and μ(pq) = 0.8, μ(qr) = 0.9,Then d (μ) = 0.8 and h (μ) = 0.9.

For $0 < t \leq 0.8$, G^t= (V, {pq, qr})
Here b and c not a weak fuzzy cut vertex
For $0.6 < t \leq 0.7$, G^t= (V, {qr}).
Here G^t has 2 vertices.



Theorem 3.6

A vertex v of a fuzzy graph is a weak fuzzy cut vertex if and only if there exist vertices x and y ($\neq$ v) such that every (x , y) – Path contains v

Proof:

Suppose v is a weak fuzzy cut node.

Then there exist a t $\in$(0, h(μ)] such that v is aweak fuzzy cut node for G^t.

Then G^t-v is disconnected, so it contains atleast two components say C and D.

Let x $\in$ V(C) and y $\in$V(D).

Since there is no (x, y) path in G-v.It follows that every x-y path in G contains v.

Conversely,

Suppose there exist x and y ($\neq$v) such that every (x,y) path contains v.

It follows that there is no (x,y) – path in G-v.

Hence G-v is disconnected, that is v is a cut node.

Then u is a weak fuzzy cut node

Theorem 3.7

Let G be a fuzzy graph. Then the following statements are equivalent

(i) xy is a weak fuzzy bridge.

(ii) xy is not a weakest edge of any cycle.

Proof:

(i)$\Rightarrow$ (ii).

Assume xy is a weak fuzzy bridge.

Then there exist a t$\in$(0, h(μ)] such that v is a weak fuzzy cut node for G^t.

Suppose xy is a weakest edge of anycycle.

Since xy is a weakest edge we first remove xy from G to find G^t.

But the edge is not a bridge because G^t is a cycle.

Hence xy is not a weak fuzzy bridge.

It is a contradiction.

Thus xy is not a weakest edge of any cycle.

(ii) $\Rightarrow$ (i)

Suppose xy is not a weakest edge of any cycle. But G has at least one edge as weakest edge we first remove the weakest edge. Then remove the second weakest edge if xy is the second weakest edge then it should be a bridge because G^t is a tree here. Thus xy is a weak fuzzy bridge.

Theorem 3.8

Let G be a fuzzy graph such that G* is a cycle. Then a node of G is a weak fuzzy cut node iff it is a common node of two weak fuzzy bridges.

Proof:

Let w be a weak fuzzy cut node of G.

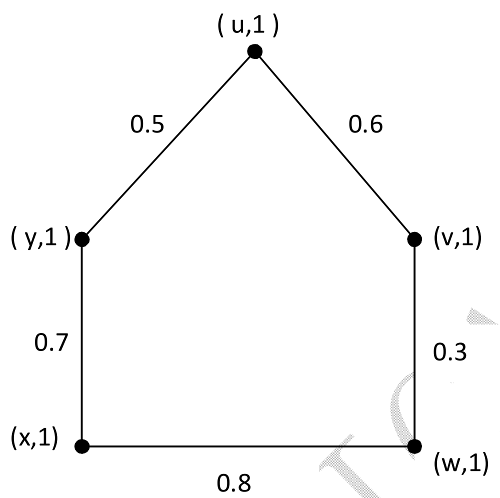
Then there exist u and v, other than w such that w is on every strongest u-v path. Because G* is a cycle then three exist only one strongest u-v path containing w and all its edges are weak fuzzy bridges.

Thus w is a common vertex of two weak fuzzy bridge .

Conversely,

Suppose w is a common node of two fuzzy bridges uw and wv.
Then both uw and wv are not weakest edge of G.
If we remove w then u and v should lie in different components.
Hence w is a cut node.
Thus w is a weak fuzzy cut node.

Example 3.9



Here all the edges except vw are weak fuzzy bridge and also except v and w are weak fuzzy cut nodes.

4. CONCLUSION

We gave necessary and sufficient condition here. We use Bridge as friendship and node as person. If we remove then they becomes separated.

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