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(2+1) DIMENSIONAL DOMAIN WALL COSMOLOGICAL MODEL IN THE $f(R,T)$ GRAVITATION THEORY

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ABSTRACT

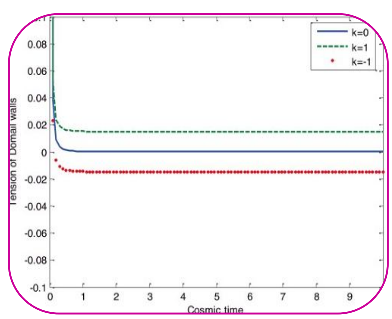
In this paper we have investigated cosmological model in the frame work of (2+1) dimensional space-time in $f(R,T)$ theory of gravitation in presence of domain wall, where R is Ricci scalar curvature and T is the trace energy momentum tensor. The functional form $f(R,T) = R + 2\mu T$, with μ being a constant is chosen for derivation and analysis of the field equation of $f(R,T)$ gravity. The solution of the field equations are obtained by using the special law of variation for Hubble's parameter proposed by M.S. Berman (1983) that yields a constant deceleration parameter. The physical and kinematical parameters of the model are discussed. The model is tested for physically acceptable cosmologies by using stability. It is observed that acceleration is due to gravitation and also observed that the model is expanding and remains isotropic throughout the evolution.

KEYWORDS: *(2+1) dimension, Domain wall, $f(R,T)$ gravity, Constant deceleration parameter.*

1. INTRODUCTION

Lower dimensional models have been of enormous use in practically every other branch of physics. Such models are important because they help to generate new ideas, and to stimulate new insights into their higher dimensional counterparts. Moreover, they provide a simple setting in which certain basic physical phenomena can be easily demonstrated, while avoiding the mathematical complexities often encounter in four dimensions. Therein lies the motivation for studying gravity in three space-time dimensions. Einstein gravity in three space-time dimensions exhibits some unusual features, which can be deduced from the properties of the Einstein field Equations and the curvature tensor. (2+1) dimensional gravity does contain interesting features in common with four dimensional gravity theory.

Deser, Jackiw and 't Hooft [1] have obtained the solutions to three dimensional Einstein gravity with mass-less, spinning point source, and Clement [2] has generalized their results to include many massive spinning sources. The generalization to coupled Einstein-Maxwell theory has been considered by Deser and Mazur [3], Melvin [4] and Gott, Simon and Alpert [5]. The Regge calculus version of three dimensional gravity with point masses has been developed by Rocek and Williams [6]. Many of the basic aspects of classical Einstein gravity in three dimensions are covered in the article by Giddings, Abbott and Kuchar [7] Gott and Alpert [8] and Barrow, Burd and Lancaster [9]. They discussed the lack of correspondence between Einstein and Newtonian gravity in three dimensions; the conic geometry associated with a point mass and also includes cosmological solutions for perfect fluids. In addition, Barrow, Burd and Lancaster present two cosmological solutions containing scalar field that produce inflation, and discussed cosmological singularities for three dimensional space-time. Deser and Laurent [10] have studied the interior and exterior solutions to various matter distributions assuming the space-time is axially symmetric and stationary. Deser [11] has shown that there are no nontrivial statics solutions to the coupled Einstein gravity-Yang Mills system in three dimensions. Edward Witten [12] has shown that (2 + 1) dimensional gravity (with or without a cosmological constant) is exactly soluble at the classical and quantum levels and it is closely related to Yang-Mills



theory with purely the Chern-Simons action. N.J. Cornish and N.E. Frankel [13] have investigated gravitational field theories in (2+1) space-time dimensions and reviewed the consequences of the lack of a Newtonian limit to general relativity. The cosmic holographic principle suggested by Fischler and Susskind has been examined in (2+1) dimensional cosmological models by Bin Wang and Elcio Abdalla [14]. Ranjan Sharma et.al [15] have investigated Gravitational collapse of a circularly symmetric star in an (2+1) anti-deSitter space-time and analyzed the impacts of various factors on the evolution of the star, which begins its collapse from an initial static configuration. Yun He and Meng-Sen Ma [16] have constructed (2 +1)-dimensional regular black holes with nonlinear electrodynamics sources and studied the thermodynamic properties of the regular black holes.

The late time accelerated expansion of the universe is one of the most challenging problems in modern cosmology. This has encouraged eminent researchers to explore modified theories of gravitation. Harko et al. [17] have proposed an *f (R, T)* modified theory, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and the trace T of the energy–momentum tensor. They have discussed some particular models corresponding to a specific choice of the function *f (R, T)*. S.Ram and P. Yadav [18] and Singh and Singh [19] have reconstructed the cosmological models in the *f (R, T)* theory of gravitation in the presence of a perfect fluid. Naidu et al. [20] have investigated a spatially homogeneous and anisotropic Bianchi type V cosmological model in *f (R, T)* theory. A dark energy model with an equation of state parameter for a non-static plane-symmetric space-time filled with a perfect fluid source in the *f (R, T)* theory of gravity has been studied by Chirde and Shekh [21]. Various forms of topological defects are formed due to the phase transitions in the early universe. The topological defects are cosmic string, domain walls, textures and monopoles [22], Mermin [23] and Linde [24]. Among these three topological defects, domain walls and cosmic strings are more important as they do not contradict observation. Vilenkin [25] has pointed out that domain walls and cosmic strings are important in the formation of galaxies and large scale structure of the universe.

In recent years, there has been a lot of interest in the study of cosmological models in the presence of domain walls. Reddy and Naidu [26] have studied cosmic string and domain walls in a scale-covariant theory of gravitation. Pradhan et al. [27] have considered plane-symmetric inhomogeneous bulk viscous domain walls in Lyra geometry. Khadekar et al. [28] have analyzed the Kaluza-Klein type Robertson Walker (RW) cosmological model with variable cosmological constant in the presence of strange quark matter with domain wall. Adhav et al. [29] have obtained n-dimensional Kaluza-Klein cosmological model in presence of string cloud and domain walls with quark matter in general relativity. Sahoo and Mishra [30, 31] have studied string cloud and domain walls with quark matter in a plane symmetric metric and Kink space time respectively.

Recently, Katore et al. [32] have discussed Bianchi type-II, VIII and IX cosmological model with domain wall as matter source in $f(R,T)$ gravity. Biswal et al. [33] have investigated the five dimensional Kaluza-Klein cosmological model in the frame work of $f\ (R,T)$ theory of gravity in presence of domain walls. Katore and Hatkar [34] have investigate Bianchi type III and Kantowski–Sachs cosmological models in the presence of domain walls in the framework of the modified *f (R, T)* theory of gravitation.

This motivates us to investigate the (2+1) dimensional cosmological models with domain wall in the frame work of *f (R, T)* theory of gravitation. The paper is organized as follows. In Section 2, the metric and energy momentum tensor are described. The field equations in (2+1) dimensional Robertson-Walker space-time with domain wall in the frame work of *f (R, T)* theory of gravity are derived. Section 3 is devoted to the solutions of the field equations under some physical conditions. In Section 4, we discuss some physical and kinematical properties of the cosmological model and concluding remarks are given in Section 5.

2. FIELD EQUATIONS

The Einstein's field equations in $f(R,T)$ theory of gravitation are given by [17]

$$R_{ij} - \frac{1}{2}g_{ij}R = 8\pi T_{ij} + 2f^{'}(T)T_{ij} + \{2Pf^{'}(T) + f(T)\}g_{ij} \quad (i,j = 0,1,2), \qquad (1)$$

where T_{ij} is the energy momentum tensor, T the stress of energy momentum tensor, $f(T)$ the arbitrary function of T and all other symbols have their usual meaning as in the Riemannian geometry. In $f(R,T)$

gravity theory, the function $f(R,T)$ depends on the nature of the matter source. Harko et al.[17] have given three types of models as

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases} \qquad (2)$$

They have stated that the cosmic acceleration may result not only from a geometrical contribution to the total cosmic energy density, but it also depends on the matter content of the universe. We intend to study simple form such as

$$f(R,T) = R + 2\mu(T) \qquad (3)$$

where μ is a constant. The energy momentum tensor for the domain walls is given by

$$T_{ij} = \rho\left(g_{ij} + u_i u_j\right) + P g_{ij}\,, \qquad (4)$$

where, ρ is the energy density, P stands for pressure, u^i is velocity vector satisfying $u_i u^i = -1$. Then, we have

$$T_0^0 = -P,\ T_1^1 = T_2^2 = \rho,\ T = 3\rho - P \qquad (5)$$

The energy momentum tensor of domain walls contains the normal matterρ_m, p_m and the tension σ_d . They are related with the relations $p = p_m - \sigma_d$, $\rho = \rho_m + \sigma_d$ and

$$p_m = (\gamma - 1)\rho_m, 1 \leq \gamma \leq 2. \qquad (6)$$

Here we consider (2+1) dimensional Robertson-Walker line element [35]

$$ds^2 = dt^2 - A^2(t)(dr^2 + r^2 d\theta^2), \qquad (7)$$

The field equation (1) with energy momentum tensor (2) for the metric (7) gives

$$\frac{\dot{A}^2}{A^2} = -(8\pi + \mu)P + 2\mu\rho \qquad (8)$$

$$\frac{\ddot{A}}{A} = (8\pi + 4\mu)\rho + \mu P \qquad (9)$$

Here, dot $(\cdot)$ denotes differentiation with respect to time t only.

3. SOLUTIONS OF FIELD EQUATIONS

There are two field equations (8) and (9) with three unknowns namely one scale factor A and two physical quantities ρ and P. Therefore to obtain an exact solution of the field equations, we need one more relations connecting these variables. Hence we use the following possible physical condition.

i) Variation of Hubble's parameter proposed by Berman [36] that yields a constant deceleration parameter models of the universe which is defined as

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = \text{constant}. \qquad (10)$$

Now, the Eqn. (10) admits the solution

$$R = (at+b)^{1/q+1}, \quad (11)$$

where a and b are constants of integration. Eq. (11) implies that the condition for expansion of the universe is $q+1>0$.

With suitable choice of coordinates and constants i.e. taking $a=1$ and $b=0$, the metric (7) can be written as

$$ds^2 = dt^2 - t^{2/q+1}(dr^2 + r^2 d\theta^2), \quad (12)$$

Eq. (12) represents (2+1) dimensional cosmological model in the $f(R,T)$ theory of gravitation.

4. PHYSICAL AND KINEMATICAL PROPERTIES AND DISCUSSION

The physical and kinematical quantities for the model (12) have the following expressions.

The scale factor A is given by

$$A = t^{1/q+1} \quad (13)$$

Spatial volume : $$V_2 = A^2 = t^{2/q+1} \quad (14)$$

Expansion scalar : $$\theta = u^i_{;i} = 2\frac{\dot{A}}{A} = \frac{2}{(q+1)t} \quad (15)$$

Hubble's parameter : $$H = \frac{1}{2}(H_1 + H_2) = \frac{1}{2}\left(\frac{\dot{A}}{A} + \frac{\dot{A}}{A}\right) = \frac{\dot{A}}{A} = \frac{1}{(q+1)t} \quad (16)$$

Energy density : $$\rho = -\left[\frac{8\pi q + (q-1)\mu}{2(8\pi+3\mu)(4\pi+\mu)}\right]\frac{1}{[(q+1)t]^2} \quad (17)$$

Matter Pressure : $$P = -\left[\frac{4\pi + \mu(q+2)}{2(8\pi+3\mu)(4\pi+\mu)}\right]\frac{1}{[(q+1)t]^2} \quad (18)$$

Tension of the domain wall is expressed as

$$\sigma_d = \frac{8\pi[q(1-\gamma)+1] + \mu[\gamma(1-q)+3(1+q)]}{2\gamma(8\pi+3\mu)(4\pi+\mu)[(q+1)t]^2} \quad (19)$$

Shear scalar and anisotropic parameter are given as

$$\sigma^2 = \frac{1}{2}\left[\sum_i H_i^{\,2} - \frac{1}{2}\theta^2\right] = 0 \quad (20)$$

$$A_m = \frac{1}{2}\sum_i \left(\frac{H_i - H}{H}\right)^2 = 0 \quad (21)$$

Stability of the model can be discussed by observing the ratio $\frac{dP}{d\rho}$. When the ratio $\frac{dP}{d\rho}$ is positive, we have a stable model, whereas when the ratio is negative, we have an unstable model. In this model, the ratio is obtained as

$$\frac{dP}{d\rho} = \frac{8\pi + 2(q+2)\mu}{8\pi q + (q-1)\mu} \quad (22)$$

Using the above results, we now discuss the behavior of the cosmological model (12).

It can be observed that the model given by (12) has initial singularity at $t = 0$. The result (14) shows that the model is expanding with time since $q + 1 > 0$. The energy condition $\rho > 0$ is satisfied for $4\pi q < \mu$. The pressure p is negative. The model is stable for $4\pi > \mu$ and unstable for $4\pi < \mu$. The tension of the domain wall is positive. The tension of the domain walls vanishes at infinite time where as it tends infinity as $t \to 0$. It can also be observed that, the Hubble parameter H, expansion scalar θ, matter density ρ and decreases with time and approach zero as $t \to \infty$ and all diverges at $t = 0$. Also shear scalar σ and anisotropic parameter A_m vanishes, which indicates that shape of the universe remains unchanged during the evolution and universe becomes isotropic and shear free.

5. CONCLUSIONS

In this paper we have investigated domain wall cosmological model in the frame work of (2+1) dimensional space-time in $f(R,T)$ theory of gravitation. The model is obtained using the special law of variation for Hubble's parameter proposed by Berman [36] that yields a constant declaration parameter. The model is expanding and possesses an initial singularity. It is observed from Eq. (22) that the physical viable model is unstable. It is also observed that all the physical and kinematical parameters of the model diverges when $t = 0$ and vanish when t is infinitely large. As $\frac{\sigma^2}{\theta^2} = 0$ and anisotropic parameter $A_m = 0$, which indicate that the model is not anisotropic in nature and remains isotropic throughout the evolution.

Also the tension of the domain wall is positive; i.e. the domain wall could exist at all evolutions of the times of the universe, which is not in accordance with Zel'dovich et al. [37]. In the presence of the matter content of $f(R,T)$ gravity, we have established the acceleration of the universe and observed that acceleration is due to gravity.

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