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KALUZA- KLEIN SPACE TIME WITH COSMOLOGICAL CONSTANT IN SCALAR TENSOR THEORY

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ABSTRACT

Kaluza-Klein type cosmological model with time dependent cosmological term-Λ *in the framework of Saez and Ballester (1986) theory of gravitation has been studied. In order to find the exact solution of the field equations, we have used the equation of state and the fact that scalar expansion is proportional to the shear scalar. The cosmological constant term is found to decreasing function of cosmic time. Some physical and kinematical properties of the model are also discussed.*

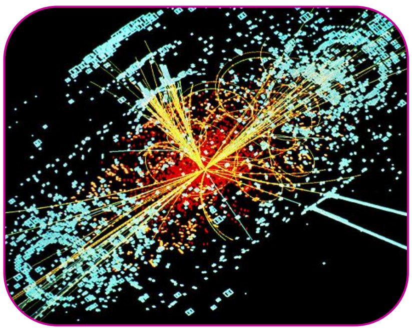
KEYWORDS: *Cosmological constant term;scalar- tensor theory;Kaluza-Klein cosmological space-time.*

1. INTRODUCTION

Einstein's general theory of relativity has successfully described gravitational phenomena. It has also served as a basis for models of the universe. However since Einstein first publishedhis theory of gravitation, there have been many criticismsof general relativity because of the lack of certain desirable features in the theory. For example Einstein himself pointed out that the general relativity does not account satisfactorily for inertial properties of matter,i.e.Mach's principle is not substantiated by general relativity.Since last few decades, there is a growing interest in alternative theories of gravitation, especially scalar-tensor theories of gravity, which are very useful tools in understanding the early stages of evolution of the universe.The most important among them are scalar-tensor theories of gravitation formulated by Brans and Dicke [1], Nordtvedt[2] and Saez and Ballester [3]. All version of the scalar tensor theories are based on the introduction of a scalar field ϕ into the formulation of general relativity, this scalar field along with the metric tensor field forms a scalar- tensor field representing the gravitational field.

The Saez-Ballester theory [3]have developed a new scalar - tensor theory of gravitation in which metric is coupled with a dimensionless scalar field in a simple manner .This coupling gives a satisfactory description of weak fields. In spite of the dimensionless character of the scalar field, an antigravity regime appears in the theory. Also, this theory suggests a possible way to solve missing matter problem in non-flat FRW cosmologies.

Some of the authors, Sing and Agrawal [4], Shri Ram and Tiwari [5], Reddy and VenkateswaraRao [6], Reddy et.al.[7] have studied several aspects of the Saez-Ballester scalar-tensor theory. Adhav et al.[8] investigatedaxially symmetric non-static domain walls in scalar-tensor theories formulated by Brans and Dick (1961) and Saez-Ballester.RecentlyEinstein-Rosen, Axially symmetry and Plane symmetry cosmological models in Saez-Ballester theory of gravitation have been investigated by Mete et.al [9, 10,11].



The Kaluza-Klein theory was introduced to unify Maxwell's theory of electromagnetism and Einstein's gravity theory by adding the fifth dimension. Kaluza-Klein theory has been regarded as a candidate of the fundamental theory due to its potential theory function to unite the fundamental interactions. Kaluza-Klein cosmological model has been studied with different matters [12-16]. In Kaluza-Klein theory, the inflation was considered [17] and the Schwarzschild solution for three space and n dimensions were formed [18]. String cloud and domain walls with quark matter in n-dimensional Kaluza-Klein cosmological model have also been studied by Adhav et.al [19].

Higher dimensional cosmology is important because it has physical relevance to the early stages of evolution of the universe before it has undergone compactification transitions. Hence several authors(Witten[20],Chodos and Detweller[21] Appelquist et al.[22],Marchiano[23])were attracted to the study of higher dimensional cosmology. Also in the context of Kaluza-Klein and super string theories higher dimensions have recently acquired much significance. Several investigations have been made in higher dimensional cosmology in the frame work of different scalar- tensor theories. In particular, Reddy et al. [24] have investigated a five dimensional Kaluza-Klein cosmological model in the presence of perfect fluid in $f(R,T)$ gravity.

The effect of cosmological constant has been extensively studied in the literature within the framework of general relativity and its alternative theories. Singh and Singh[25] investigated a cosmological model in Brans-Dicke theory by considering cosmological constant as a function of scalar field ϕ. Pimentel [26]obtained exact cosmological solutions in Brans-Dicke theory with uniform cosmological constant.A class of flat FRW cosmological models with cosmological constant in Brans- Dicke theory have also been obtained by Azar and Riazi [27]. The age of the universe from a view point of the nucleosynthesis with Λ term in Brans-Dicke theory was investigated byEtoch et al.[28]. Azad and Islam [29]extended the idea of Singh and Singh [25]to study cosmological constant in Bianchi type-I modified Brans-Dicke cosmology.Recently Qiang et al. [30] discussed cosmic acceleration in five dimensional Bran-Dicke theory using interacting Higgs and Brans-Dicke fields.

This motivatesus toinvestigate Kaluza-Klein type cosmological model with time dependent cosmological term-Λ in the framework of Saez and Ballester (1986) theory of gravitation.

2. THE METRIC AND FIELD EQUATION

The Einstein's field equations (in gravitational units, $8\pi c = 1$) in the scalar tensor theory proposed by (Saez and Ballester,1986) with time dependent Λ-term may be written as

$$R_{ij} - \frac{1}{2} R g_{ij} - \omega \phi^n \left(\phi_{,i}\phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k}\phi^{,k} \right) = -T_{ij} + \Lambda(t) g_{ij}, \quad (1)$$

where T_{ij} is the energy momentum tensor of matter and ϕ is the scalar field satisfying the equation

$$2\phi^n \phi^i_{;i} + n\phi^{n-1}\phi_{,k}\phi^{,k} = 0. \quad (2)$$

Here n is arbitrary constant, ω is the dimensionless coupling constant. Comma and semi-colon respectively denote partial and covariant derivativewith respective to t.

The energy momentum tensor T_{ij} of cosmic fluid can be define as

$$T_{ij} = (\rho + p) u_i u_j - p g_{ij}, \quad (3)$$

where ρ, p are the energy density and pressure respectively and $u_i = (0,0,0,0,1)$ is the flow vector satisfying the relation

$$g_{ij} u^i u^j = 1. \quad (4)$$

Here weconsider Kaluza-Klein type space time described by the line element

$$ds^2 = dt^2 - A^2(t)\left(dx^2 + dy^2 + dz^2\right) - B^2(t) dw^2, \quad (5)$$

where the metric potentials A and B are functions of the proper time t only.

The field equations (1) and (2) for the metric (5) with the help of (3) and (4) can be written as

$$2\frac{\ddot{A}}{A}+\left(\frac{\dot{A}}{A}\right)^2+2\frac{\dot{A}\dot{B}}{AB}+\frac{\ddot{B}}{B}-\frac{\omega}{2}\phi^n\dot{\phi}^2=-p-\Lambda \quad (6)$$

$$3\frac{\ddot{A}}{A}+3\left(\frac{\dot{A}}{A}\right)^2-\frac{\omega}{2}\phi^n\dot{\phi}^2=-p-\Lambda \quad (7)$$

$$3\left(\frac{\dot{A}}{A}\right)^2+3\frac{\dot{A}\dot{B}}{AB}+\frac{\omega}{2}\phi^n\dot{\phi}^2=\rho-\Lambda \quad (8)$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{2A_4}{A}+\frac{B_4}{B}\right)+\frac{n}{2}\left(\frac{\dot{\phi}^2}{\phi}\right)=0, \quad (9)$$

where suffix 4 at the symbols A, B, ϕ and ρ denotes ordinary differentiation with respective to t. The geometrical quantities;spatial volume V and average scale factor $a(t)$ for Kaluza-Klein space time are define by

$$V=a^4(t)=A^3B \quad (10)$$

The mean Hubble parameter H isgiven by

$$H=\frac{1}{4}\sum_{i=1}^{4}H_i$$

$$=\frac{1}{4}\left[3\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right] \quad (11)$$

Thescalar expansion θ and shear scalar σ^2 given by

$$\theta=4H=3\frac{\dot{A}}{A}+\frac{\dot{B}}{B} \quad (12)$$

$$\sigma^2=\frac{1}{2}\sigma^{ij}\sigma_{ij} \quad (13)$$

$$\sigma_{ij}=\frac{1}{2}\left[u_{i,j}-u_{j,i}\right]+\frac{1}{2}\left[u_{i,k}u^k u_j-u_i u_{j,k}u^k\right]-\frac{1}{3}\theta \quad (14)$$

The average anisotropic parameter A_m is define as

$$A_m = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2, \qquad (15)$$

where $H_i, i = 1,2,3,4$ represents the directional Hubble parameters in x, y, z and w directions respectively and $A_m = 0$ corresponds to isotropic expansion.

3. SOLUTION OF THE FIELD EQUATIONS

The set of field equation (6) – (9) are the system of fourindependent equations with six unknowns A, B, p, ρ, ϕ and Λ. To find determinate solution, extra condition should be needed. Here we use the scalar expansion θ is proportional to scalar expansion σ^2. So that we have (Collins et al. [31])

$$A = B^m, \qquad (16)$$

where m is a arbitrary constant.
From equations (6) and (7), we get

$$\frac{\ddot{B}}{B} + 3n\left(\frac{\dot{B}}{B}\right)^2 = 0 \qquad (17)$$

solving this differential equation, we obtain the expression for metric coefficients as

$$A = \left[(3m+1)(k_1 t + k_2)\right]^{\frac{m}{3m+1}} \qquad (18)$$

And

$$B = \left[(3m+1)(k_1 t + k_2)\right]^{\frac{1}{3m+1}}, \qquad (19)$$

where $k_1 \neq 0$ and k_2 are constants of integration.
From equation (9) ,we have

$$\dot{\phi}\phi^{\frac{n}{2}} A^3 B = \phi_0 \qquad (20)$$

using equations (18) and (19), equation (20) yields

$$\phi^{\frac{n+2}{2}} = \left(\frac{\phi_0}{2k_1}\right)\left(\frac{n+2}{3m+1}\right)\log(k_1 t + k_2) + \psi_0, \qquad (21)$$

where ϕ_0 and ψ_0 are constants of integration.
Therefore the investigated Kaluza-Klein space time (5) can be written as

$$ds^2 = dt^2 - \left[(3m+1)(k_1 t + k_2)\right]^{\frac{2m}{3m+1}}\left(dx^2 + dy^2 + dz^2\right) - \left[(3m+1)(k_1 t + k_2)\right]^{\frac{2}{3m+1}} dw^2 \qquad (22)$$

4. SOME PHYSICAL DISCUSSION

We assume the relation between pressure and density of matter i.e. the linear equation of state given by

$$p = \gamma\rho \quad (23)$$

using this relation one can obtain the following expressions for energy density , pressure and cosmological constant term - Λ as

$$\rho = \frac{6m(m+1)+\omega\phi_0^2}{(1+\gamma)(3m+1)(k_1t+k_2)^2} \quad (24)$$

$$p = \frac{6\gamma m(m+1)+\omega\phi_0^2}{(1+\gamma)(3m+1)(k_1t+k_2)^2} \quad (25)$$

And

$$\Lambda = \left[3m(m+1)+\omega\phi_0^2\right]\left(\frac{1-\gamma}{1+\gamma}\right)\left(\frac{1}{(3m+1)^2(k_1t+k_2)^2}\right) \quad (26)$$

From the relations (24) - (26), we can obtain three types of physical relevant models

- When $\gamma = 0$, we obtain empty model ,the energy density, pressure and cosmological term Λ are given by

$$\rho = \frac{6m(m+1)+\omega\phi_0^2}{(3m+1)(k_1t+k_2)^2} \quad (27)$$

$$p = 0 \quad (28)$$

and

$$\Lambda = \left[3m(m+1)+\omega\phi_0^2\right]\left(\frac{1}{(3m+1)^2(k_1t+k_2)^2}\right) \quad (29)$$

- When $\gamma = \frac{1}{3}$,we obtain radiating dominated model, the energy density, pressure and cosmological term Λ are given by

$$\rho = \frac{3\left[6m(m+1)+\omega\phi_0^2\right]}{4(3m+1)(k_1t+k_2)^2} \quad (30)$$

$$p = \frac{\left[6m(m+1)+\omega\phi_0^2\right]}{4(3m+1)(k_1t+k_2)^2} \quad (31)$$

And

$$\Lambda = \left[3m(m+1) + \omega\phi_0^2\right]\left(\frac{1}{2(3m+1)^2(k_1t+k_2)^2}\right) \qquad (32)$$

- When $\gamma = 1$,we obtain Zeldovich fluid or stiff fluid model ,the energy density, pressure and cosmological term Λ are given by

$$p = \rho = \frac{\left[6m(m+1) + \omega\phi_0^2\right]}{2(3m+1)(k_1t+k_2)^2} \qquad (33)$$

And

$$\Lambda = 0 \qquad (34)$$

The physical and kinematical quantities for the model (22) have the following expressions

The mean Hubble parameter $H = \dfrac{1}{4(k_1t+k_2)}$ (35)

Spatial volume $V = (3m+1)(k_1t+k_2)$ (36)

Scalar expansion $\theta = 4H = \dfrac{1}{k_1t+k_2}$ (37)

Shear scalar $\sigma^2 = \dfrac{2}{9}\left(\dfrac{1}{k_1t+k_2}\right)^2$ (38)

Deceleration parameter $q = \dfrac{d}{dt}\left(\dfrac{1}{H}\right) - 1 = 3$ (39)

and the anisotropic parameter is

$$A_m = \frac{1}{12} \qquad (40)$$

From equation (26) ,we observe that the cosmological term-Λ decreases as t increases i.e. it varies inversely as square of time therefore our solution is consistent with observation of the present day values of the cosmological constant term-Λ which are very small. The positive value of deceleration parameter indicates that the universe is decelerated. The spatial volume V of the model increases as cosmic time increases which shows the spatial expansion of the universe. The Hubble parameter H , scalar expansion θ and shear scalar σ are decreases at $t \to \infty$.

5. CONCLUSION

In this paper, we have studied Kaluza-Klein type cosmological model with time dependent cosmological term-Λ in the framework of Saez and Ballester (1986) theory of gravitation Here, we have discussed three cases corresponding the values of $\gamma = 0, \ \frac{1}{3}, \ 1$. When $\gamma = 0, \ \frac{1}{3}$, the cosmological term - Λ is decreasing function of time t and when $\gamma = 1$, the cosmological term - Λ becomes zero. Also in this investigated model, we observed that $\frac{\sigma^2}{\theta^2} =$ constant i.e. the model does not approach isotropy at any time. The energy density and pressure are also decreases as time $t \to \infty$.

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