INTERACTING DARK FLUIDS IN LRS BIANCHI TYPE-V UNIVERSE

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ABSTRACT:
This paper deals with the study of interacting holographic dark energy (DE) and cold dark matter (CDM) in Locally Rotationally Symmetric (LRS) Bianchi type-V. The solutions have been obtained for Einstein’s field equations. Also, the behaviors of the obtained solutions have been discussed.

KEYWORDS: LRS Bianchi types-V space-time; statefinder parameters; interacting dark fluids.

1. INTRODUCTION
The universe is expanding in an accelerating manner indicated by recent observations of SNeIa (Type Ia supernovae) [1, 2]. Many observations of CMB (cosmic microwave background) [3, 4] in coherence with LSS (Large Scale Structure) [5, 6] denote that the universe is spatially flat. An exotic component is known as ‘Dark energy’ (DE) having huge negative pressure dominated the universe. Analysis of cosmological observations under the study of Wilkinson Microwave Anisotropy Probe (WMAP)[7-10] state that $\frac{2}{3}$ of the total energy of the universe is occupied by dark energy whereas remaining $\frac{1}{3}$ is taken up by dark matter (DM) and baryonic matter (ordinary matter). The term ‘Dark Energy’ is specially used for the unknown form of energy which is not detected directly and do not cluster as other ordinary matter does. This observation can be explained by assuming that at large scale the Einstein gravity model of general relativity (GR) break down and a more general action describes the gravitational field. The simplest component of DE which is supported by current observational data is the cosmological constant ($\Lambda$) having equation of state $\omega = -1$. For the satisfaction of the current value of DE, it should be fine-tuned [9-11].

In recent years many dark energy models have been studied including quintessence scalar field models [12, 13], tachyon field [14, 15], K-essence [16-18], phantom field [19-21], Chaplygin gas [22, 23], Quinton[24] and so on [25-27]. Various DE cosmologies (isotropic) having early deceleration and delayed acceleration was previously reviewed by Bamba et al. [28]. The increasing expansion with the phantom/quintessence characteristic in detail together with cosmography test has been represented by cosmological model like Holographic dark energy, coupled dark energy, scalar field theory, $f(T)$ gravity, $f(R,T)$ gravity, $f(R)$ gravity and $\Lambda$CDM cosmological model. These models have been studied by Bamba et al. [28]. The scalar tensor theory of gravitation consisting of five dimensional DE was thoroughly investigated by Reddy et al. [29].

To investigate the problem of dark energy, holographic dark energy model provides a more simple and reasonable frame. The principle known as the holographic principle [25,27, 30-31] emerged as a new paradigm in quantum gravity in relation to black hole physics, it was...
initially put forward by Hooft [32] and latter extended by Fischer and Susskind [33] to string theory. According to this principle, the entropy of system scale on its surface area and not with its volume. Thus if we take whole universe in consideration the energy created by vacuum when related to holographic principle can be used as DE which is known as holographic dark energy. These new studies of string theory and black hole can provide a new solution of DE problem. Though, it is the most radical modification and proposed a relation \( \rho_\Lambda = H^2 \), which is the relation between \( \rho_\Lambda \) (holographic dark energy density) as well as \( H \) (Hubble parameter). It can’t support to play a part of present accelerated expansion of the universe [34,35]. A holographic density of the form \( \rho_\Lambda \approx \alpha H^2 + \beta H \), where \( \alpha \) and \( \beta \) are constant and \( H \) is Hubble parameter has been put forward by Granda and Oliverson [36] but it should satisfy the restriction imposed by the current observational data. Granda and Oliverson’s [37] new model of dark energy not only represent the accelerated expansion of the universe but also it is consistent with current observational data. A scalar phantom-nonphantom transition model and generalised holographic dark energy have been discussed by Nojiri and Odintsov [38].

An important role in the early stage of the universe was played by anisotropy thereby making it necessary to study the homogeneous as well as anisotropic cosmological models. So the models based on Bianchi type universe are considered to be spatially homogeneous model of cosmology that is in general anisotropic. The holographic dark energy is permitted to interact with dark matter are the models which are studied under special class [39-50]. In literature, many authors have studied holographic dark energy model in Bianchi type’s space-time in GR and alternative theories. Recently Sarkar [51-53] has been studied Bianchi type-I and V universe with the help of non-interacting holographic DE using linearly varying deceleration parameter. Adhav et. al. [54, 55] have investigated holographic dark energy model in Bianchi type-I and V space-time. The holographic dark energy model in Bianchi type-VI0 with the help of scalar tensor theory has been investigated by Reddy et. al. [56]. Mete et. al. [57] explored LRS Bianchi type-I universe filled with interacting cold dark matter and holographic dark energy. Interacting dark fluids in Bianchi type-I universe with variable deceleration parameter has been constructed by Mete et. al. [58].

This motivates us to investigate interacting holographic dark energy (DE) and (CDM) in LRS Bianchi type-V universe.

2. METRIC AND FIELD EQUATIONS

The LRS Bianchi type-V line element can be written as

\[
ds^2 = dt^2 - A^2 dx^2 - e^{2\nu} B^2 (dy^2 + dz^2),
\]

where \( A \) and \( B \) are the cosmic scale factors and functions of the cosmic time \( t \) only.

The Einstein’s field equations in GR are \((8\pi G = 1 \text{ and } c = 1)\)

\[
R_{ij} - \frac{1}{2} g_{ij} R = -\left(\rho_m + \rho_\Lambda\right) u_i u_j + g_{ij} \rho_\Lambda,
\]

The matter tensor for holographic dark energy and cold dark matter (pressureless i.e. \( w_m = 0 \)) are

\[
\rho_\Lambda = \left(\rho_\Lambda + \rho_\Lambda\right) u_i u_j + g_{ij} \rho_\Lambda,
\]

\[
\rho_m u_i u_j.
\]
Here $\rho_\Lambda$ and $p_\Lambda$ denote the energy density and pressure of holographic dark energy and $\rho_m$ is the energy density of dark matter.

For the metric (1), the Einstein’s field equations (2) using equations (3) and (4) convert to the following set of equations

$$\frac{2\dot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = -p_\Lambda, \tag{5}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} - \frac{1}{A^2} = -p_\Lambda, \tag{6}$$

$$\frac{\dot{B}^2}{B^2} + \frac{2\dot{A}\dot{B}}{AB} - \frac{3}{A^2} = \rho_\Lambda + \rho_m, \tag{7}$$

$$-2\frac{\dot{B}}{B} + \frac{\dot{A}}{A} = 0, \tag{8}$$

where an overdot (\cdot) indicates derivative with respect to time $t$.

We assumed that both components i.e. holographic dark energy and dark matter interact with each other and do not conserve separately in such a manner that balance equations in form as;

$$\dot{\rho}_m + \left(\frac{\dot{V}}{V}\right)\rho_m = Q, \tag{9}$$

$$\dot{\rho}_\Lambda + \left(\frac{\dot{V}}{V}\right)(1 + w_\Lambda)\rho_\Lambda = -Q. \tag{10}$$

Here $w_\Lambda$ denotes equation of state parameter (EoS) for holographic dark energy is given by $w_\Lambda = \frac{p_\Lambda}{\rho_\Lambda}$. The quantity $Q > 0$ denotes the interaction between the dark energy components. Wetterich [12] introduced models featuring an interacting matter-dark energy and first used by Horvat[60] alongside the holographic dark energy. Although this expression for interacting term may looks purely phenomenological. Continuity equations (9) and (10) suggest that $Q \propto \frac{1}{t}$ i.e. the interacting term ($Q$) proportional to a quantity with unit of inverse time. This occurred due to choosing $H$ and $Q$ term and is motivated purely by mathematical simplicity. To form energy density any combination of dark matter and dark energy can be expressed phenomenological in form such as [59, 61]

$$Q = 3b^2H\rho_m = b^2\frac{\dot{V}}{V}\rho_m, \tag{11}$$

where $b^2$ is known as coupling constant. The relation considered by Cai and Wang [62] for interacting phantom DE and DM is same as above relation used for neglecting the coincidence problem. The energy density of dark matter is given by equations (9) and (11) as
\[ \rho_m = \rho_0 V^{(b^2-1)} \]  
(2.14)

where \( \rho_0 > 0 \) is an integrating constant.

With the help of equations (11) and (12), the interacting term given as

\[ Q = 3 \rho_0 b^2 H V^{(b^2-1)}. \]  
(13)

3. Cosmological Solutions

Integrating equation (8), we obtain

\[ A = nB, \]  
(14)

where \( n > 1 \) is constant of integration.

Subtracting equation (5) from equation(6) we obtain,

\[ \frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{\dot{A}B}{AB} - \frac{\ddot{A}}{A} = 0. \]  
(15)

Using equation (14) in the equation (15) and then on integrating, we obtain the value of scale factors as

\[ A = n(c_1 t + c_2)^{\frac{1}{2}}, \]  
(16)

\[ B = (c_1 t + c_2)^{\frac{1}{2}}, \]  
(17)

where \( c_1 > 0 \) and \( c_2 \) are real constants of integration.

The volume scale factor \( V \) is defined and obtained as,

\[ V = AB^2 = n(c_1 t + c_2)^{3}. \]  
(18)

Mean hubble parameter defined and obtained as,

\[ H = \frac{1}{3} \frac{V}{V} = \frac{c_1}{2(c_1 t + c_2)}. \]  
(19)

Using equation (18) in equations (12) and (13), we get energy density of dark matter and interacting term as,

\[ \rho_m = \rho_0 \left[n^{(b^2-1)}(c_1 t + c_2)^{3(b^2-1)}\right], \]  
(20)

\[ Q = \frac{3}{2} b^2 c_1 \rho_0 \left[n^{(b^2-1)}(c_1 t + c_2)^{3b^2-4}\right]. \]  
(21)
From equation (7) using equations (16), (17) and (20), the holographic dark energy density is given as,

$$
\rho_{\Lambda} = \frac{3c_1^2}{4(c_1t + c_2)^2} - \frac{3}{n^2(c_1t + c_2)} - \rho_0 n^{(b^2-1)}(c_1t + c_2)^3(b^2-1)
$$

(22)

From equations (5)-(6), the pressure of holographic dark energy is given by using equations (16) and (17) as,

$$
p_{\Lambda} = \frac{c_1^2}{4(c_1t + c_2)^2} + \frac{1}{n^2(c_1t + c_2)}
$$

(23)

With the help of equations (23) and (24), the EoS parameter of holographic dark energy is defined and obtained as,

$$
w_{\Lambda} = \frac{p_{\Lambda}}{\rho_{\Lambda}}
$$

$$
w_{\Lambda} = \frac{c_1^2}{4(c_1t + c_2)^2} + \frac{1}{n^2(c_1t + c_2)} - \frac{3}{4(c_1t + c_2)^2} - \frac{3}{n^2(c_1t + c_2)} - \rho_0 n^{(b^2-1)}(c_1t + c_2)^3(b^2-1).
$$

(24)

The coincidence parameter \(\bar{r} = \rho_m / \rho_{\Lambda}\), which is the ratio of two energy densities i.e. the ratio of dark matter energy density to the dark energy density is given by,

$$
\bar{r} = \frac{\rho_0 n^{(b^2-1)}(c_1t + c_2)^{3(b^2-1)}}{3c_1^2 - \frac{3}{4(c_1t + c_2)^2} - \frac{3}{n^2(c_1t + c_2)} - \rho_0 n^{(b^2-1)}(c_1t + c_2)^3(b^2-1)}
$$

(25)

5. STATEFINDER DIAGNOSTIC

Though, there have been several dark energy models constructed to describe and interpret the cosmic acceleration, to differentiate between those competing dark energy models is required. For this purpose, the parameter pair \(\{r, s\}\) known as ‘Statefinder diagnostic’ was proposed by Sahni et al. [63], gives the universe’s expansion dynamics through \(a\) (expansion factor) which is considered to be natural companion of ‘\(q\)’ (deceleration parameter).

The Statefinder parameter \(\{r, s\}\) is as follows,

$$
r = \frac{\ddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r - 1}{3(q - 1/2)}.
$$

The geometrical diagnostic pair known as ‘Statefinder’ depends upon metric describing space time and expansion factor. The \(s - r\) plane containing trajectories which corresponds to other cosmological models show qualitatively different behaviors. A fixed point \(\{s, r\}_{\Lambda CDM} = \{0, 1\}\) shown in a Fig.2 corresponds to flat \(\Lambda C\) CDM. A good way which is provided for demonstration of ‘distance’ of this model
from $\Lambda CDM$, depart a given dark energy model from the fixed point [64]. For differentiation amongst a large variety of DE models including quintessence, cosmological constant, the Chaplygin gas, interacting dark energy models and braneworld models, the statefinder may prove to be successful tool [59, 65, 66].

$$r = 3$$ and $$s = \frac{2(r - 1)}{r}$$.

6. DISCUSSION & CONCLUSION:

Here we discussed interacting holographic DE and DM in LRS Bianchi type-V space-time. Equation of state of DE $w_\Lambda < 0$ is go along with the decay of DE component into pressureless DM in the holographic DE model by engaging the apparent horizon as the IR cutoff [67]. In this model as $t \to 0, v \to $ constant and $t \to \infty, v \to \infty$ that there is no Big-Ban type of initial singularity and universe expand as time increases.

For Fig.1 As $t \to 0, \omega_\Lambda \to \infty$, As time increases $\omega_\Lambda$ starts from phantom region and for some finite time, it reaches to $\omega_\Lambda = -1$ (cosmological constant), which indicate the model reduces to $\Lambda CDM$. After some finite time then it goes in to quintessence region

$(-1 < \omega_\Lambda < -\frac{1}{3})$. The available observation in cosmology, especially the three year WMAP data [4], the SNeIa data [68, 69] and the SDSS data [70] pointed out that the $\Lambda CDM$ model or the model reducible to $\Lambda CDM$ are present as a standard model in cosmology which is similar to the present universe. The Fig. 2 shows the $s$-$r$ plane having evolving trajectory for the corresponding model is different from other dark energy models. We aspire that the future high accurate observations will be able for determination of these statefinder parameters as well as explore the DE’s nature. Also for any choice of constants, we cannot avoid the coincident problem in our model.

![Fig.1:Evolution of EoS parameter ($w_\Lambda$).](image-url)
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REFERENCES:

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