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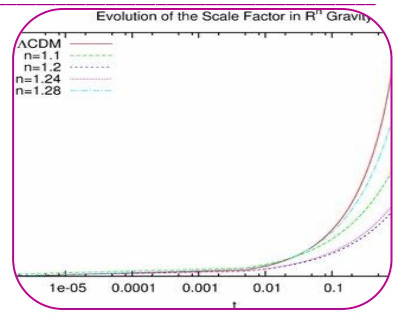
# COMPARATIVE STUDY OF SYMMETRIC UNIVERSE FILLED WITH SCALAR FIELD COUPLED WITH ELECTROMAGNETIC FIELDS IN $f(R,T)$ THEORY OF GRAVITY.

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**ABSTRACT :**

*In $f(R,T)$ theory of gravity, we have studied the interacting scalar and electromagnetic fields in Bianchi types space-time, by considering the particular cases $(R,T) = R + \lambda T$ . It is observed that, even though the line element of the space-time is distinct, the convergent and identical solution of metric functions can be evolved in each universe along with the components of vector potential.*

**KEYWORDS :** *Bianchi types, symmetric line elements, scalar field, Electromagnetic field, isotropy,* $f(R,T) = R + \lambda T$

## LIST OF SYMBOLS

1) $R$ Ricci scalar
2) $T$ Trace of energy momentum tensor
3) $g_{ij}$ Metric tensor
4) $R_{ij}$ Ricci tensor
5) $T_{ij}$ Energy momentum tensor
6) $G_{ij}$ Einstein tensor
7) $g$ determinant of metric tensor
8) $L_m$ Matter Lagrangian
9) $V_i$ electromagnetic four potential
10) $F_{ij}$ Electromagnetic field tensor
11) $\theta_{ij}$ tensor
12) $\varphi$ Scalar field
13) $\psi$ psi is function of I
14) $f(R,T)$ Function of R and T

## 1. INTRODUCTION

Cosmological data from wide range of source have indicated that our universe is undergoing an accelerating expansion [2-8]. To explain this fact, two alternative theories are proposed: one concept of dark energy and other the amendment of general relativity leading to $f(R)$ and $f(R,T)$ theories [7, 9, 11] where $R$ stands for Ricci scalar $R = g^{ij} R_{ij}$, $R_{ij}$ being Ricci tensor, $T$ stands for trace of energy momentum tensor and $T = g^{ij} T_{ij}$ ,$T_{ij}$ being energy momentum tensor. The field equations of $f(R,T)$ theories due to Harko [10] are deduced by varying the action

$$s = \int f(R,T)\sqrt{-g}d^4 x + \int L_m\sqrt{-g}d^4 x , \qquad (1.1)$$

where $L_m$ is lagrangian and the other symbols have their usual meaning. Energy momentum tensor is given by

$$T_{ij} = L_m g_{ij} - 2\frac{\delta L_m}{\delta g^{ij}} \ . \tag{1.2}$$

Varying the action (1.1) with respect to $g^{ij}$, which yields as

$$\delta s = \frac{1}{2\chi}\int \left\{f_R(R,T)\frac{\delta R}{\delta g^{ij}} + f_T(R,T)\frac{\delta T}{\delta g^{ij}} + \frac{f(R,T)}{\sqrt{-g}}\frac{\delta(\sqrt{-g})}{\delta g^{ij}} + \frac{2\chi}{\sqrt{-g}}\left(\frac{\delta(L_m\sqrt{-g})}{\delta g^{ij}}\right)\right\}\sqrt{-g}\, d^4x \ . \tag{1.3}$$

Here we define $\theta_{ij} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$ . (1.4)

Defining the generalized kronecker symbol $\frac{\delta g^{\alpha\beta}}{\delta g^{ij}} = \delta_i^\alpha \delta_j^\beta$ , we can reduce

$$\frac{\delta g^{\alpha\beta}}{\delta g^{ij}} T_{\alpha\beta} = \delta_i^\alpha \delta_j^\beta T_{\alpha\beta} \ = g^{p\alpha} g_{pi} g^{q\beta} g_{qj} T_{\alpha\beta} = T_{ij} \ .$$

Using above equations we can write

$$\frac{\delta T}{\delta g^{ij}} = \frac{\delta(g^{\alpha\beta}T_{\alpha\beta})}{\delta g^{ij}} = \frac{\delta g^{\alpha\beta}}{\delta g^{ij}} T_{\alpha\beta} + g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}} = T_{ij} + \theta_{ij} \ .$$

Integrating (1.3), yield

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + \left(g_{ij} - \nabla_i\nabla_j\right)f_R(R,T) = \chi T_{ij} - f_T(R,T)\left[T_{ij} + \theta_{ij}\right] . \tag{1.5}$$

Taking trace of (1.5), we obtain

$$f_R(R,T) = \frac{2}{3}f(R,T) - \frac{1}{3}f_R(R,T)R + \frac{\chi}{3}T - \frac{1}{3}f_T(R,T)[T+\theta]. \tag{1.6}$$

Since the expression of the Ricci tensor in (1.5) is complicated, the solutions of the field equations in general cannot be obtained. With this reality we take recourse to the particular case of the function $f(R,T)$ and there upon try to obtain the solution.

We consider the case $(R,T) = R + \lambda T$

We follow the notations $f_R(R,T) = \frac{\partial f(R,T)}{\partial R} = 1$ , $f_T(R,T) = \frac{\partial f(R,T)}{\partial T} = \lambda$ . (1.7)

Using (1.7), the field equation (1.5) reduces to

$$R_{ij} - \frac{1}{2}[R + \lambda T] = \chi T_{ij} - \lambda\left[T_{ij} + \theta_{ij}\right] . \tag{1.8a}$$

Using (1.7), the equation (1.6) reduces to

$$R + \lambda T = \lambda\theta - \chi T \ . \tag{1.8b}$$

Inserting (1.8b) in (1.8a) we obtain

$$R^i_j = \chi\left[T^i_j - \frac{1}{2}T g^i_j\right] - \lambda[T^i_j + \theta^i_j] + \frac{1}{2}\lambda\theta g^i_j \tag{1.9}$$

Let us now calculate the tensor $\theta_{ij}$. Varying (1.2) with respect to metric tensor $g^{ij}$ and using the definition (1.4), we obtain

$$\theta_{ij} = -T_{ij} + 2\left[\frac{\delta L_m}{\delta g^{ij}} - g^{\alpha\beta}\frac{\delta^2 L_m}{\delta g^{ij}\delta g^{\alpha\beta}}\right]. \tag{1.10}$$

## 2. Matter field Lagrangian $L_m$

The electromagnetic field tensor is given by

$$F_{ij} = \frac{\partial V_i}{\partial x^j} - \frac{\partial V_j}{\partial x^i},$$

where $V_i$ is electromagnetic four potential.
The aforesaid the matter Lagrangian $L_m$ can be expressed as

$$L_m = \left[\frac{1}{4}F_{\eta\tau}F^{\eta\tau} - \frac{1}{2}\varphi_{,\eta}\varphi^{,\eta}\psi\right], \tag{2.1}$$

where $\psi = \psi(I), \quad I = V_i V^i$.
The function $\psi$ characterizes the interaction between the scalar $\varphi$ and electromagnetic field [1].
Then the matter tensor in (1.2) can conveniently be expressed in the mixed form

$$T^i_j = \left(F^i_\alpha F^\alpha_j + \frac{1}{4}g^i_j F_{\alpha\beta}F^{\alpha\beta}\right) - \left[\frac{1}{2}\psi g^i_j - \dot{\psi}V^i V_j\right]\varphi_{,\eta}\varphi^{,\eta} + \psi\varphi^{,i}\varphi_{,j}\,. \tag{2.2}$$

Similarly (1.10) can be written as

$$\theta^i_j = -T^i_j - (\psi - I\dot{\psi})\varphi^{,i}\varphi_{,j} + I\ddot{\psi}\varphi_{,\eta}\varphi^{,\eta}V^i V_j\,. \tag{2.3}$$

The equations (2.2) and (2.3), after contraction yield

$$T = -(\psi - I\dot{\psi})\varphi_{,\eta}\varphi^{,\eta} \tag{2.4}$$

$$\theta = I^2\ddot{\psi}\varphi_{,\eta}\varphi^{,\eta}\,. \tag{2.5}$$

## 3. Bianchi type I cosmological model in $f(R,T) = R + \lambda T$

We consider the Bianchi type I metric

$$ds^2 = dt^2 - a^2dx^2 - b^2dy^2 - c^2dz^2\,, \tag{3.1}$$

where $a, b, c$ are functions of t only. In this case non-zero Ricci tensors are

$$R^1_1 = \frac{\ddot{a}}{a} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac}\,, \quad R^2_2 = \frac{\ddot{b}}{b} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}\dot{c}}{bc}\,, \quad R^3_3 = \frac{\ddot{c}}{c} + \frac{\dot{c}\dot{a}}{ca} + \frac{\dot{c}\dot{b}}{cb}\,, \quad R^4_4 = \frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c}$$

### Electromagnetic field tensor $F_{ij}$

To achieve the compatibility with the non-static space -time (3.1), we consider the electromagnetic vector potential in the form

$$V_i = [V_1(t), V_2(t), V_3(t), V_4(t)]\,. \tag{3.2}$$

Then it is easy to deduce

$$I = -\left[\frac{{V_1}^2}{a^2} + \frac{{V_2}^2}{b^2} + \frac{{V_3}^2}{c^2} - {V_4}^2\right], \tag{3.3}$$

$$F_{14} = \dot{V}_1\,, \quad F_{24} = \dot{V}_2\,, \quad F_{34} = \dot{V}_3\,, \tag{3.4}$$

$$F_{ij}F^{ij} = -2\left[\frac{{\dot{V}_1}^2}{a^2} + \frac{{\dot{V}_2}^2}{b^2} + \frac{{\dot{V}_3}^2}{c^2}\right], \tag{3.5}$$

$$\varphi_{,i}\varphi^{,i} = \dot{\varphi}^2\,. \tag{3.6}$$

With these quantities at our disposal the components of energy momentum tensors in (2.2) becomes

$$T_1^1 = \frac{1}{2}\frac{{\dot{V}_1}^2}{a^2} - \frac{1}{2}\frac{{\dot{V}_2}^2}{b^2} - \frac{1}{2}\frac{{\dot{V}_3}^2}{c^2} - \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2\frac{{V_1}^2}{a^2}, \tag{3.7a}$$

$$T_2^1 = \frac{\dot{V}_1\dot{V}_2}{a^2} - \dot{\psi}\dot{\varphi}^2\frac{V_1V_2}{a^2}, \tag{3.7b}$$

$$T_3^1 = \frac{\dot{V}_1\dot{V}_3}{a^2} - \dot{\psi}\dot{\varphi}^2\frac{V_1V_3}{a^2}, \tag{3.7c}$$

$$T_2^2 = -\frac{1}{2}\frac{{\dot{V}_1}^2}{a^2} + \frac{1}{2}\frac{{\dot{V}_2}^2}{b^2} - \frac{1}{2}\frac{{\dot{V}_3}^2}{c^2} - \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2\frac{{V_2}^2}{b^2}, \tag{3.7d}$$

$$T_3^2 = \frac{\dot{V}_2\dot{V}_3}{b^2} - \dot{\psi}\dot{\varphi}^2\frac{V_2V_3}{b^2}, \tag{3.7e}$$

$$T_3^3 = -\frac{1}{2}\frac{{\dot{V}_1}^2}{a^2} - \frac{1}{2}\frac{{\dot{V}_2}^2}{b^2} + \frac{1}{2}\frac{{\dot{V}_3}^2}{c^2} - \frac{1}{2}\psi\dot{\varphi}^2 - \dot{\psi}\dot{\varphi}^2\frac{{V_3}^2}{c^2}, \tag{3.7f}$$

$$T_3^3 = \frac{1}{2}\frac{{\dot{V}_1}^2}{a^2} + \frac{1}{2}\frac{{\dot{V}_2}^2}{b^2} + \frac{1}{2}\frac{{\dot{V}_3}^2}{c^2} + \frac{1}{2}\psi\dot{\varphi}^2 + \dot{\psi}\dot{\varphi}^2{V_4}^2, \tag{3.7g}$$

$$T = -(\psi - I\dot{\psi})\dot{\varphi}^2\,. \tag{3.7h}$$

Similarly the components of tensor $\theta_j^i$ in (2.3) assume the values

$$\theta_1^1 = -T_1^1 - I\ddot{\psi}\dot{\varphi}^2\frac{{V_1}^2}{a^2}, \tag{3.8a}$$

$$\theta_2^1 = -T_2^1 - I\ddot{\psi}\dot{\varphi}^2\frac{V_1V_2}{a^2}, \tag{3.8b}$$

$$\theta_3^1 = -T_3^1 - I\ddot{\psi}\dot{\varphi}^2\frac{V_1V_3}{a^2}, \tag{3.8c}$$

$$\theta_2^2 = -T_2^2 - I\ddot{\psi}\dot{\varphi}^2\frac{{V_2}^2}{b^2}, \tag{3.8d}$$

$$\theta_3^2 = -T_3^2 - I\ddot{\psi}\dot{\varphi}^2\frac{V_2V_3}{b^2}, \tag{3.8e}$$

$$\theta_3^3 = -T_3^3 - I\ddot{\psi}\dot{\varphi}^2\frac{{V_3}^2}{c^2}, \tag{3.8f}$$

$$\theta_4^4 = -T_4^4 - (\psi - I\dot{\psi})\dot{\varphi}^2 + I\ddot{\psi}\dot{\varphi}^2 {V_4}^2, \tag{3.8g}$$

$$\theta = I^2\ddot{\psi}\dot{\varphi}^2 . \tag{3.8h}$$

Variation of lagrangian in (2.1) with respect to the electromagnetic field gives

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^j}\left(\sqrt{-g}F^{ij}\right) - (\varphi_{,j}\varphi^{,j})\dot{\psi}A^i = 0 , \qquad \text{where} \quad \dot{\psi} = \frac{\partial\psi}{\partial I}$$

for $i = 1, j = 4 \Rightarrow \left(\frac{\dot{V}_1}{V_1}\right)^{\cdot} + \frac{{\dot{V}_1}^2}{{V_1}^2} + \frac{\dot{V}_1}{V_1}\left[\frac{\dot{b}}{b} + \frac{\dot{c}}{c} - \frac{\dot{a}}{a}\right] = \dot{\psi}\dot{\varphi}^2$ , (3.9a)

for $i = 2, j = 4 \Rightarrow \left(\frac{\dot{V}_2}{V_2}\right)^{\cdot} + \frac{{\dot{V}_2}^2}{{V_2}^2} + \frac{\dot{V}_2}{V_2}\left[\frac{\dot{c}}{c} + \frac{\dot{a}}{a} - \frac{\dot{b}}{b}\right] = \dot{\psi}\dot{\varphi}^2$ , (3.9b)

for $i = 3, j = 4 \Rightarrow \left(\frac{\dot{V}_3}{V_3}\right)^{\cdot} + \frac{{\dot{V}_3}^2}{{V_3}^2} + \frac{\dot{V}_3}{V_3}\left[\frac{\dot{a}}{a} + \frac{\dot{b}}{b} - \frac{\dot{c}}{c}\right] = \dot{\psi}\dot{\varphi}^2$ , (3.9c)

for $i = 4, j = 4 \Rightarrow V_4 = 0$ . (3.9d)

Since for the space-time (3.1) $R_2^1 = 0, R_3^1 = 0, R_3^2 = 0$ the field equation (1.9), yield

$$\frac{\dot{V}_1\dot{V}_2}{V_1V_2} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}I\ddot{\psi}\dot{\varphi}^2, \tag{3.10a}$$

$$\frac{\dot{V}_1\dot{V}_3}{V_1V_3} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}I\ddot{\psi}\dot{\varphi}^2 , \tag{3.10b}$$

$$\frac{\dot{V}_2\dot{V}_3}{V_2V_3} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}I\ddot{\psi}\dot{\varphi}^2 , \tag{3.10c}$$

From (3.10) we can write

$$\frac{\dot{V}_1\dot{V}_2}{V_1V_2} = \frac{\dot{V}_2\dot{V}_3}{V_2V_3} = \frac{\dot{V}_1\dot{V}_3}{V_1V_3} = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}I\ddot{\psi}\dot{\varphi}^2 , \tag{3.11}$$

or $\frac{\dot{V}_1}{V_1} = \frac{\dot{V}_2}{V_2} = \frac{\dot{V}_3}{V_3} \equiv \frac{\dot{h}_1}{h_1}$, say (3.12)

where $h_1$ is some unknown function of t .
Inserting (3.12) in (3.11) we get

$$\left(\frac{\dot{h}_1}{h_1}\right)^2 = \left(\frac{\dot{h}_1}{h_1}\right)^2 = \left(\frac{\dot{h}_1}{h_1}\right)^2 = \dot{\psi}\dot{\varphi}^2 - \frac{\lambda}{\chi}I\ddot{\psi}\dot{\varphi}^2 . \tag{3.13}$$

Integrating equations (3.12) with respect to t, yield

$$V_1 = k_1h_1 , \qquad V_2 = k_2h_1 , \qquad 3 = k_3h_1 , \tag{3.14}$$

where $k_1, k_2, k_3$ are constants of integration
Now our plan is to express the components of $T_j^i$ in (3.7) in terms of $T_4^4$. For this we consider the expression

$$\frac{\dot{V_1}^2}{a^2}+\frac{\dot{V_2}^2}{b^2}+\frac{\dot{V_3}^2}{c^2}=\left(\frac{{V_1}^2}{a^2}+\frac{{V_2}^2}{b^2}+\frac{{V_3}^2}{c^2}\right)\left(\frac{\dot{h}_1}{h_1}\right)^2 \quad \text{by (3.12)}$$

$$=-I\left(\frac{\dot{h}_1}{h_1}\right)^2 \quad \text{by (3.3) and (3.9d)}$$

$$=\frac{\lambda}{\chi}I^2\ddot{\psi}\dot{\varphi}^2-I\dot{\psi}\dot{\varphi}^2 \quad \text{by (3.13)}. \tag{3.15}$$

We attempt to express the components of $T_j^i$ in (3.7) in terms of $T_4^4$ by using (3.12) , (3.13) and (3.15)

$$T_4^4=\frac{1}{2}\psi\dot{\varphi}^2-\frac{1}{2}I\dot{\psi}\dot{\varphi}^2+\frac{1}{2}\frac{\lambda}{\chi}I^2\ddot{\psi}\dot{\varphi}^2\ , \tag{3.16a}$$

$$T_1^1=-T_4^4-\frac{\lambda}{\chi}I\ddot{\psi}\dot{\varphi}^2\frac{{V_1}^2}{a^2}\ , \tag{3.16b}$$

$$T_2^2=-T_4^4-\frac{\lambda}{\chi}I\ddot{\psi}\dot{\varphi}^2\frac{{V_2}^2}{b^2}\ , \tag{3.16c}$$

$$T_3^3=-T_4^4-\frac{\lambda}{\chi}I\ddot{\psi}\dot{\varphi}^2\frac{{V_3}^2}{c^2}\ , \tag{3.16d}$$

$$T=-(\psi-I\dot{\psi})\dot{\varphi}^2\ , \tag{3.16e}$$

We consider the non-vanishing components of Ricci tensor $G_1^1, G_2^2, G_3^3$ from (1.9) by using (3.16) and (3.8)

$$\frac{\ddot{a}}{a}+\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{a}\dot{c}}{ac}+=0\ , \tag{3.17a}$$

$$\frac{\ddot{b}}{b}+\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{b}\dot{c}}{bc}=0, \tag{3.17b}$$

$$\frac{\ddot{c}}{c}+\frac{\dot{a}\dot{c}}{ac}+\frac{\dot{b}\dot{c}}{bc}=0\ , \tag{3.17c}$$

With the help of (3.12) we can write equation (3.9) as

$$\left(\frac{\dot{h}_1}{h_1}\right)^{\cdot}+\frac{\dot{h_1}^2}{{h_1}^2}+\frac{\dot{h}_1}{h_1}\left[\frac{\dot{b}}{b}+\frac{\dot{c}}{c}-\frac{\dot{a}}{a}\right]=\dot{\psi}\dot{\varphi}^2\ , \tag{3.18a}$$

$$\left(\frac{\dot{h}_1}{h_1}\right)^{\cdot}+\frac{\dot{h_1}^2}{{h_1}^2}+\frac{\dot{h}_1}{h_1}\left[\frac{\dot{c}}{c}+\frac{\dot{a}}{a}-\frac{\dot{b}}{b}\right]=\dot{\psi}\dot{\varphi}^2\ , \tag{3.18b}$$

$$\left(\frac{\dot{h}_1}{h_1}\right)^{\cdot}+\frac{\dot{h_1}^2}{{h_1}^2}+\frac{\dot{h}_1}{h_1}\left[\frac{\dot{a}}{a}+\frac{\dot{b}}{b}-\frac{\dot{c}}{c}\right]=\dot{\psi}\dot{\varphi}^2\ , \tag{3.18c}$$

Further this equation imply

$$\frac{\dot{b}}{b}+\frac{\dot{c}}{c}-\frac{\dot{a}}{a}=\frac{\dot{c}}{c}+\frac{\dot{a}}{a}-\frac{\dot{b}}{b}=\frac{\dot{a}}{a}+\frac{\dot{b}}{b}-\frac{\dot{c}}{c}$$

or
$$\frac{\dot{a}}{a}=\frac{\dot{b}}{b}=\frac{\dot{c}}{c}\ , \tag{3.19}$$

Inserting (3.19) in (3.17) we obtain

$$\frac{\ddot{a}}{a}+2\left(\frac{\dot{a}}{a}\right)^2=0\,, \qquad \frac{\ddot{b}}{b}+2\left(\frac{\dot{b}}{b}\right)^2=0\,, \qquad \frac{\ddot{c}}{c}+2\left(\frac{\dot{c}}{c}\right)^2=0\,. \tag{3.20}$$

Upon integration of the equations in (3.20), yield

$$a=(3k_4t+3k_5)^{\frac{1}{3}}\,, \qquad b=(3k_6t+3k_7)^{\frac{1}{3}}\,, \qquad c=(3k_8t+3k_9)^{\frac{1}{3}}\,, \tag{3.21}$$

where $k's$ are constant of integration.

Since $\frac{\dot{a}}{a}=\frac{\dot{b}}{b}=\frac{\dot{c}}{c}$ we get $k_4=k_6=k_8$ and $k_5=k_7=k_9$

Let $k_4=k_6=k_8=d_1$, say and $k_5=k_7=k_9=d_2$ , say

$$a=b=c=(3d_1t+3d_2)^{\frac{1}{3}} \tag{3.22}$$

Inserting (3.22) in (3.18), we get

$$\left(\frac{\dot{h}_1}{h_1}\right)^{\cdot}+\frac{\dot{h}_1{}^2}{h_1{}^2}+\frac{\dot{h}_1}{h_1}\left[\frac{d_1}{3d_1t+3d_2}\right]=\dot{\psi}\dot{\varphi}^2\,. \tag{3.23}$$

But from (3.13) and (3.23) we have

$$\left(\frac{\dot{h}_1}{h_1}\right)^{\cdot}+\frac{\dot{h}_1{}^2}{h_1{}^2}+\frac{\dot{h}_1}{h_1}\left[\frac{d_1}{3d_1t+3d_2}\right]=\frac{\lambda}{\chi}I\ddot{\psi}\dot{\varphi}^2. \tag{3.24}$$

If we confine $\psi$ as linear function $\psi$ ($\psi=k_{10}I+k_{11}$ or $\ddot{\psi}=0$) then (3.24) have perfect solution

$$h_1=k_{13}exp\left\{k_{14}(3d_1t+3d_2)^{\frac{2}{3}}\right\}. \tag{3.25}$$

With the help of (3.25) the equations in (3.14) convert into

$$V_1=k_{15}exp\left\{k_{14}(3d_1t+3d_2)^{\frac{2}{3}}\right\}, \tag{3.26a}$$

$$V_2=k_{16}exp\left\{k_{14}(3d_1t+3d_2)^{\frac{2}{3}}\right\}, \tag{3.26b}$$

$$V_3=k_{17}exp\left\{k_{14}(3d_1t+3d_2)^{\frac{2}{3}}\right\}, \tag{3.26c}$$

From (3.13) and (3.25), we get

$$\varphi=k_{18}(3d_1t+3d_2)^{\frac{2}{3}}+k_{19}\,,$$

where $d's$ and $k's$ are constants of integration.

## 4. Bianchi type III cosmological model in $f(R,T)=R+\lambda T$

We consider the Bianchi type III metric

$$ds^2=-dt^2+A^2dx^2+B^2e^{-2mx}dy^2+C^2dz^2\,, \tag{4.1}$$

where $A, B, C$ functions of t and m is constant.
In this case non-zero Ricci tensors are

$$R_1^1 = \frac{m^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{C}}{AC}, \quad R_4^1 = \frac{m\dot{A}}{A^3} - \frac{m\dot{B}}{A^2 B}, \quad R_2^2 = \frac{m^2}{A^2} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC}$$

$$R_3^3 = -\frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{B}\dot{C}}{BC}, \quad R_1^4 = \frac{m\dot{B}}{B} - \frac{m\dot{A}}{A}, \quad R_4^4 = -\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C}$$

We assume the vector potential as

$$A_i = [u(x)V_1(t),\ V_2(t),\ V_3(t), V_4(t)] . \tag{4.2}$$

By applying the same procedure as in section 3 we obtain metric functions as

$$A = B = C = (3d_3 t + 3d_4)^{\frac{1}{3}} . \tag{4.3}$$

And components of vector potential as

$$u(x) = n_1 e^{mx} , \tag{4.4a}$$

$$V_1 = n_{19} exp\left\{n_{18}(3d_3 t + 3d_4)^{\frac{2}{3}}\right\}, \tag{4.4b}$$

$$V_2 = n_{20} exp\left\{n_{18}(3d_3 t + 3d_4)^{\frac{2}{3}}\right\}, \tag{4.4c}$$

$$V_3 = n_{21} exp\left\{n_{18}(3d_3 t + 3d_4)^{\frac{2}{3}}\right\}, \tag{4.4d}$$

$$V_4 = 0, \tag{4.4e}$$

and $$\varphi = n_{22}(3d_3 t + 3d_4)^{\frac{2}{3}} + n_{23} , \tag{4.5}$$

where $d's$ and $n's$ are constants of integration.

## 5. Bianchi type VI$_0$ cosmological model in $f(R,T) = R + \lambda T$

We consider the Bianchi type VI$_0$ metric

$$ds^2 = dt^2 - {a_1}^2 dx^2 - {a_2}^2 e^{-2m^2 x} dy^2 - {a_3}^2 e^{2m^2 x} dz^2 . \tag{5.1}$$

In this case non-zero Ricci tensors are

$$R_1^1 = -\frac{2m^4}{{a_1}^2} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1\dot{a}_2}{a_1 a_2} + \frac{\dot{a}_1\dot{a}_3}{a_1 a_3}, \quad R_4^1 = \frac{m^2}{{a_1}^2}\left[\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3}\right], \quad R_2^2 = \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3}$$

$$R_3^3 = \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1 a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3}, \quad R_1^4 = m^2\left[\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_2}{a_2}\right], \quad R_4^4 = \frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3}$$

We assume the electromagnetic vector potential in the from

$$A_i = [u(x)V_1(t), V_2(t), V_3(t), V_4(t)] . \tag{5.2}$$

By applying the same procedure as in section 3 we obtain metric function as

$$a_1 = a_2 = a_3 = (3d_5t + 3d_6)^{\frac{1}{3}} . \tag{5.3}$$

And components of vector potential as

$$u(x) = constant , \tag{5.4a}$$

$$V_1 = m_{17} exp\left\{m_{15}(3d_5t + d_6)^{\frac{2}{3}}\right\} , \tag{5.4b}$$

$$V_2 = m_{18} exp\left\{m_{15}(3d_5t + d_6)^{\frac{2}{3}}\right\} , \tag{5.4c}$$

$$V_3 = m_{19} exp\left\{m_{15}(3d_5t + d_6)^{\frac{2}{3}}\right\} , \tag{5.4d}$$

$$V_4 = 0, \tag{5.4e}$$

and $$\varphi = m_{21}(3d_5t + 3d_6)^{\frac{2}{3}} + m_{20} , \tag{5.5}$$

where $d's$ and $m's$ are constants of integration.

## 6. Spatially homogeneous and anisotropic Kantowaski-sachs cosmological model in $\boldsymbol{f(R,T) = R + \lambda T}$

We consider the spatially homogeneous and anisotropic Kantowaski-sachs metric

$$ds^2 = dt^2 - A^2 dr^2 - B^2(d\theta^2 + sin^2\theta \, d\phi^2) , \tag{6.1}$$

where $A, B$ are functions of t.

In this case non-zero Ricci tensors are

$$R_1^1 = \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} , \qquad R_2^2 = \frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 + \frac{1}{B^2} + \frac{\dot{A}\dot{B}}{AB} ,$$

$$R_3^3 = \frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 + \frac{1}{B^2} + \frac{\dot{A}\dot{B}}{AB} , \qquad R_4^4 = \frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B}$$

### Electromagnetic field tensor $F_{ij}$

We assume the electromagnetic vector potential in the from

$$A_i = [V_1(t), u(\theta)V_2(t), V_3(t), V_4(t)] . \tag{6.2}$$

Applying the procedure similar in section 3 we obtain metric functions as

$$A = B = (3d_7t + d_8)^{\frac{1}{3}} \tag{6.3}$$

and components of vector potential as

$$u(\theta) = l_1 cosec\theta , \tag{6.4a}$$

$$V_1 = l_{15} exp\left\{l_{13}(3d_7t + d_8)^{\frac{2}{3}}\right\}, \tag{6.4b}$$

$$V_2 = l_{16} exp\left\{l_{13}(3d_7t + d_8)^{\frac{2}{3}}\right\}, \tag{6.4c}$$

$$V_3 = l_{17} exp\left\{l_{13}(3d_7t + d_8)^{\frac{2}{3}}\right\}, \tag{6.4d}$$

$$V_4 = 0 , \tag{6.4e}$$

$$\varphi = l_{18}(3d_7t + 3d_8)^{\frac{2}{3}} + l_{19} \tag{6.5}$$

where $d's$ and $l's$ are constants of integration.

## 7. Robertson-walker universe cosmological model in $f(R,T) = R + \lambda T$

We consider the metric of universe given by Robertson-walker metric

$$ds^2 = dt^2 - a^2(dx^2 + dy^2 + dz^2) , \tag{7.1}$$

where $a$ is function of t.

In this case non-zero Ricci tensors are

$$R_1^1 = \frac{\ddot{a}}{a} + 3\left(\frac{\dot{a}}{a}\right)^2, \quad R_2^2 = \frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2, \quad R_3^3 = \frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2, \quad R_4^4 = 3\frac{\ddot{a}}{a}.$$

We consider the electromagnetic vector potential in the from

$$A_i = [V_1(t), V_2(t), V_3(t), V_4(t)] . \tag{7.2}$$

Applying the procedure similar in section 3 we obtain metric functions as

$$a = (3q_4t + 3q_5)^{\frac{1}{3}} \tag{7.3}$$

and vector potential as

$$V_1 = q_{10} exp\left\{q_8(3q_4t + q_5)^{\frac{2}{3}}\right\}, \tag{7.4a}$$

$$V_2 = q_{11} exp\left\{q_8(3q_4t + q_5)^{\frac{2}{3}}\right\}, \tag{7.4b}$$

$$V_3 = q_{12} exp\left\{q_8(3q_4t + q_5)^{\frac{2}{3}}\right\}, \tag{7.4c}$$

$$V_4 = 0 , \tag{7.4d}$$

and $$\varphi = q_{13}(3q_4t + q_5)^{\frac{2}{3}} + q_{14} , \tag{7.5}$$

where $q's$ are constants of integration.

## 8. Bianchi type II cosmological model $f(R,T) = R + \lambda T$

We consider the Bianchi type II metric

$$ds^2 = -dt^2 + A^2(dx^2 + dz^2) + B^2(dy - xdz)^2, \qquad (8.1)$$

where $A, B$ are functions of t.

In this case non-zero Ricci tensors are

$$R_1^1 = -\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} - \frac{\dot{A}\dot{B}}{AB} + \frac{B^2}{2A^4}, \quad R_2^2 = -\frac{\ddot{B}}{B} - \frac{2\dot{A}\dot{B}}{AB} - \frac{B^2}{2A^4}, \quad R_3^2 = \frac{\ddot{A}x}{A} - \frac{\ddot{B}x}{B} - \frac{\dot{A}^2x}{A^2} + \frac{\dot{A}\dot{B}x}{AB} + \frac{B^2x}{A^4}$$

$$R_3^3 = -\frac{\ddot{A}}{A} - \frac{\dot{A}^2}{A^2} - \frac{\dot{A}\dot{B}}{AB} + \frac{B^2}{2A^4}, \quad R_4^4 = -2\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B}$$

We consider the electromagnetic vector potential in the from

$$A_i = [u(x)V_1(t), V_2(t), V_3(t), V_4(t)], \qquad (8.2)$$

Applying the procedure similar in section 3 we obtain metric functions as

$$A = B = (3d_9t + 3d_{10})^{\frac{1}{3}} \qquad (8.3)$$

and vector potential as

$$u(x) = Constant, \qquad (8.4a)$$

$$V_1 = p_{13}exp\left\{p_{12}(3d_9t + 3d_{10})^{\frac{2}{3}}\right\}, \qquad (8.4b)$$

$$V_2 = p_{14}exp\left\{p_{12}(3d_9t + 3d_{10})^{\frac{2}{3}}\right\}, \qquad (8.4c)$$

$$V_3 = p_{15}exp\left\{p_{12}(3d_9t + 3d_{10})^{\frac{2}{3}}\right\}, \qquad (8.4d)$$

$$V_4 = 0, \qquad (8.4e)$$

and $$\varphi = p_{17}exp\left\{p_{12}(3d_9t + 3d_{10})^{\frac{2}{3}}\right\} + p_{18},$$

where $d's$ and $p's$ are constant of integration.

## 9. Bianchi type VIII cosmological model

We consider the Bianchi type VIII metric

$$ds^2 = dt^2 - A^2dx^2 - [A^2cosh^2x + B^2sinh^2x]dy^2 - B^2dz^2 - 2B^2sinhxdydz, \qquad (9.1)$$

where $A, B$ are functions of t.

In this case non-zero Ricci tensors are

$$R_1^1 = \frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} - \frac{B^2}{2A^4} - \frac{1}{A^2} + \frac{B^4}{4A^6}tanh^2x, \quad R_4^1 = -\frac{B\dot{B}}{2A^4}tanhx,$$

$$R_2^2 = -\frac{1}{A^2 cosh^2 x} + \frac{2tanh^2 x}{A^2} + \frac{2B^2 tanh^2 x}{A^4} + \frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{1}{A^2}\frac{sinhx}{cosh^3 x} - \frac{B^2}{A^4}\frac{sinhx}{cosh^3 x} + \frac{\dot{A}\dot{B}}{AB} +$$

$$2B\dot{B}\frac{\dot{A}}{A^3} tanh^2 x - 2B\dot{B}\frac{\dot{A}}{A^3} tanhxsechx - \frac{3}{2}\frac{B^2}{A^4} tanhxsechx - \frac{B^4}{A^6} tanhxsechx$$

$$R_3^2 = \frac{3B^2}{2A^4}\frac{sinhx}{cosh^2 x}\ , \qquad R_3^3 = \frac{\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{B^2}{A^4} - \frac{3B^2}{2A^4} tanh^2 x\ , \qquad R_4^4 = 2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B}$$

**Electromagnetic field tensor $F_{ij}$**

We consider the electromagnetic vector potential in the form

$$A_i = [u(x)V_1(t), V_2(t), V_3(t), V_4(t)]\ . \tag{9.2}$$

By applying the procedure similar in section 3 we obtain metric functions as

$$A = B = (3r_5 t + r_6)^{\frac{1}{3}} \tag{9.3}$$

And vector potential as

$$u(x) = r_1 sechx\ , \tag{9.4a}$$

$$V_1 = r_{12} exp\left\{r_{11}(3r_5 t + r_6)^{\frac{2}{3}}\right\}\ , \tag{9.4b}$$

$$V_2 = r_{13} exp\left\{r_{11}(3r_5 t + r_6)^{\frac{2}{3}}\right\}, \tag{9.4c}$$

$$V_3 = r_{14} exp\left\{r_{11}(3r_5 t + r_6)^{\frac{2}{3}}\right\}, \tag{9.4d}$$

$$V_4 = 0\ , \tag{9.4e}$$

and $$\varphi = r_{16}(3r_5 t + r_6)^{\frac{2}{3}} + r_{15}\ , \tag{9.5}$$

where $r's$ are constants of integration.

## 10. CONCLUSION

1) In the present paper we have considered particular cases of $f(R,T)$ theory of gravity $f(R,T) = R + \lambda T$ model in different Bianchi types and some symmetric metric.
2) It is observed that, even though the line elements are distinct, the convergent, non-singular, isotropic solution can be evolved in each metric along with the components vector potential.
3) we believe firmly that, due to the interacting scalar and electromagnetic field in $f(R,T)$ theory, the metric functions and vector potentials convert into isotropic.
4) Each models show that universe expand algebraically in $f(R,T) = R + \lambda T$ theory of gravity.
5) The metric functions (scale factor) in each non-static space-time admits constant value at early time of the universe ($t \to o$) and after that metric functions start increasing with increase in cosmic time, and finally diverge to $\infty$ as $t \to \infty$. This shows that universe expands and approaches to infinite volume.
5) It is also interesting to note that the investigated models are free from singularity in each metric.

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