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# SECOND - ORDERNEIGHBOUR EFFECTS USING EUCLIDEAN GEOMETRY 

RANI, S. AND MEENA KUMARI

Assistant Professor Department of Statistics Hindu Girls College, Sonipat Haryana, India


#### Abstract

: The treatment applied to one experimental plot may affect the response on neighbouring plots as well as response on the plot to which it is applied. Neighbour Design can be constructed using Euclidean Geometry for OS1 series with parameters v $=s 2, b=s(s+1), r=s+1, k=s$ and $\lambda=1$. After obtaining the treatmentsfurther it is observed that neighbour treatments follow the property of circularity of second-order.


## KEYWORDS:

Galois field, BIBD, Neighbour Design, left neighbours, Right neighbours, Circularity.

## INTRODUCTION

A Balanced Incomplete Block Designs (BIBD)is an ordinary 2-(n, m, $\lambda$ ) design i.e. a set of msubsets of an $n$-set such that each 2 -subset is contained in exactly $\lambda$ blocks. There are two main kinds of finite plane geometry: affine and projective. A finite plane of order s is one such that each line has s points (for an affine plane). The usual notation for the parameters of these designs will be used: BIBD - ( $\mathrm{v}, \mathrm{b}, \mathrm{r}, \mathrm{k}$, $\lambda)$; SBIBD $-(\mathrm{v}, \mathrm{k}, \lambda)$ for affine plane of order $\mathrm{s}-\left\{\mathrm{s}^{2}, \mathrm{~s}(\mathrm{~s}+1), \mathrm{s}+1, \mathrm{~s}, 1\right\}$; projective plane of order $\mathrm{s}-\left\{\mathrm{s}^{2}+\mathrm{s}\right.$ $+1, \mathrm{~s}+1,1\}$ respectively. Affine planes of order s exist whenever v is a prime power by using affine plane over the finite field with $v=s k$ elements. Generally, a finite affine plane of order $s$ has $s^{2}$ points, and $s^{2}+s$ lines; each line contains s points and each point is on $s+1$ points.Fisher and Yates (1938) have embodied a number of such solutions. Rao (1945) gives the cyclic solutions to combinatorial problem such as the BIB designs derivable from finite geometries. Thomas (1987) constructed the first nontrivial 2-design over a finite field which is a BIB design with parameters 2-(v, 3,$7 ; 2$ ) for $v \geq 7 \& v \equiv \pm 1 \bmod 6$. He used a geometric construction in a projective plane. Unfortunately, however BIB designs exist in a limited number of cases.

In forestry or agricultural experiments, competition effects on neighbouring plots can lead to badly biased comparisons between treatments if they are not taken into account. To avoid bias when comparing the effects of treatments in this situation, it is important to ensure that no treatment is unduly disadvantaged by its neighbour. The estimates of treatment differences may, therefore, deviate because of competition or interference from neighbouring units. Studies on interference between neighbouring units began with the work on neighbour designs by Rees (1967) on designing of plots to diffusion tests in virus research. Azais et al. (1993) defined the block design for competition effects to be balanced in the sense that every treatment has every other treatment appearing once as a left neighbour and once as a right neighbour. Pateria et al. (2007) considered a series of block designs by putting N-1 MOLS with N treatments one below another. Laxmi and Rani (2009) obtained the patterns of neighbour treatments of first - order neighbours for every treatment of neighbour designs of the OS1 series considering two sided (left \&right) border plots. Ahmed and Akhtar (2009) constructed all order neighbour balanced designs in circular blocks using the method of cyclic shifts only for $v$ is prime number or power of a prime number. Laxmi and Parmita (2010) suggested a method of construction neighbour design for OS2 series using method of MOLS. Laxmi and

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Parmita (2011) further suggested a method of finding right neighbours of a treatment for the OS2 series Hereneighbour design for OS1 series with parameters $v=s^{2}, b=s(s+1), r=s+1, k=s, \lambda=1$ is constructed for different values of s using Euclidean geometry. Neighbour designs of OS1 series can be obtained by first forming a finite 2-dimensional $E G(2, s)$, where s is either a prime number or power of a prime number, by using the elements of G.F.(s), treating the points as treatments, all possible lines as blocks, and then the points on a line as the contents of the block corresponding to the line. This method is considered for s being a prime number or power of a prime number by taking $\mathrm{s}=3$ and $\mathrm{s}=4$.

## 2 METHOD OF CONSTRUCTION OF BIBD FOR OS1 SERIES USING EUCLIDEAN GEOMETRY:

In (1936) Yates introduced the concept of orthogonal series for B.I.B.D. with parameters $v=s^{2}, b=$ $\mathrm{s}(\mathrm{s}+1), \mathrm{r}=\mathrm{s}+1, \mathrm{k}=\mathrm{s}, \lambda=1$ and $\mathrm{v}=\mathrm{b}=\mathrm{s}^{2}+\mathrm{s}+1, \mathrm{r}=\mathrm{k}=\mathrm{s}+1, \lambda=1$. The first series was named as OS1 series and the second series was named as Os2.

### 2.1 When $\mathrm{s}=3$

The parameters of OS 1 series corresponding to $\mathrm{s}=3$ i.e. a prime number are, $\mathrm{v}=9, \mathrm{~b}=12, \mathrm{k}=3, \mathrm{r}=4$, $\lambda=1$. The elements of the Galois fields for $\mathrm{s}=3$ are 0,1 and 2 . There are $\mathrm{s}^{2}=9$ points in the 2 -dimensional EG $(2,3)$ which can be written using these three elements. In EG $(2,3)$ each point is represented by two coordinates ( $\mathrm{x}, \mathrm{y}$ ), where each coordinates is $0,1 \& 2$. The following 9 possible pairs ( $\mathrm{x}, \mathrm{y}$ ) give the 9 points:

| Point number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Points | 00 | 01 | 02 | 10 | 11 | 12 | 20 | 21 | 22 |

The equations of the possible lines and writing the points on the lines the resulted solution is as follows:

| 1) $x=i(i=0,1,2)$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{x}=0$ |  |  | 00 | 01 | 02 |
|  | $\mathrm{x}=1$ |  |  | 10 | 11 | 12 |
|  | $\mathrm{x}=2$ |  |  | 20 | 21 | 22 |
| 2) $y=i(i=0,1,2)$ |  |  |  |  |  |  |
|  | $y=0$ |  |  | 00 | 10 | 20 |
|  | $\mathrm{y}=1$ |  |  | 01 | 11 | 21 |
|  | $y=2$ |  |  | 02 | 12 | 22 |
| 3) $x+y=i(i=0,1,2)$ |  |  |  |  |  |  |
| $x+y=0$ | 000 | 12 | 21 |  |  |  |
| $x+y=1$ | $1 \quad 01$ | 10 | 22 |  |  |  |
| $x+y=2$ | $2 \quad 02$ | 11 | 20 |  |  |  |
| 4) $x+2 y=i(i=0,1,2)$ |  |  |  |  |  |  |
|  | $x+2 y=0$ |  |  | 00 | 11 | 22 |
|  | $x+2 y=1$ |  |  | 02 | 10 | 21 |
|  | $x+2 y=2$ |  |  | 01 | 12 | 20 |

Thus there are 4 sets of lines with 3 parallel lines in each set. There is no common point in any two lines belonging to a set. In general, there are ( $s+1$ ) such sets of lines each set containing s parallel lines. So the total number of lines is $\mathrm{s}(\mathrm{s}+1)$ and the number of points is in a line, say
$x+p y=i$
denote the total number of lines where $p$ and $i$ are two elements in the G.F. For each value of $x$ of the Galois field, one value of $y$ is obtained from (A). These two values of $x$ and $y$ give a point on the line (A). As $x$ can

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be any of the s elements in the field, these are, in all s such points on the line. Given a point there is one line in each set which passes through the point. As there are $(s+1)$ sets, $(s+1)$ lines passes through a point. Hence the replication of each point is $\mathrm{s}+1$. As only one line can pass through two points, a pair of treatments occur together in one block only i.e. $\lambda=1$. After giving the numbering to the points lies to above 12 lines the resulted BIBD is as follows (only the point numbers have been shown):

| 1 | 2 | 3 |
| :--- | :--- | :--- |
| 4 | 5 | 6 |
| 7 | 8 | 9 |
| 1 | 4 | 7 |
| 2 | 5 | 8 |
| 3 | 6 | 9 |
| 1 | 6 | 8 |
| 2 | 4 | 9 |
| 3 | 5 | 7 |
| 1 | 5 | 9 |
| 3 | 4 | 8 |
| 2 | 6 | 7 |
| 2.2 | When $\mathrm{s}=4:$ |  |

The parameters of OS1 series corresponding to $s=4$ i.e. power of a prime number, are $v=16, b=20$, $\mathrm{k}=4, \mathrm{r}=5, \lambda=1$. The elements of the Galois fields for $\mathrm{s}=4$ are $0,1, \alpha, \alpha^{2}$ with $\alpha^{2}+\alpha+1$ as the minimal function. There are $s^{2}=16$ points in the 2 -dimensional $E G(2,4)$ which can be written using these four elements. In EG $(2,4)$ each point is represented by two coordinates $(x, y)$, where each coordinate is either $0,1, \alpha \& \alpha^{2}$. The following 16 possible pairs $(\mathrm{x}, \mathrm{y})$ give the 16 points:

| Point number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Points | 00 | 01 | $0 \alpha$ | $0 \alpha^{2}$ | 10 | 11 | $1 \alpha$ | $1 \alpha^{2}$ |
| Point number | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| Points | $\alpha 0$ | $\alpha 1$ | $\alpha \alpha$ | $\alpha \alpha^{2}$ | $\alpha^{2} 0$ | $\alpha^{2} 1$ | $\alpha^{2} \alpha$ | $\alpha^{2} \alpha^{2}$ |

The equations of the possible lines can be obtained from equation number $(A)$ i.e. $x+p y=i$ where $p$ and $i$ are again the elements of G.F. (s). By writing the points on the lines the resulted solution is shown below:

1) $\mathrm{x}=\mathrm{i}\left(\mathrm{i}=0,1, \alpha, \alpha^{2}\right)$

| $\mathrm{x}=0$ | 00 | 01 | $0 \alpha$ | $0 \alpha^{2}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{x}=1$ | 10 | 11 | $1 \alpha$ | $1 \alpha^{2}$ |
| $\mathrm{x}=\alpha$ | $\alpha 0$ | $\alpha 1$ | $\alpha \alpha$ | $\alpha \alpha^{2}$ |
| $\mathrm{x}=\alpha^{2}$ | $\alpha^{2} 0$ | $\alpha^{2} 1$ | $\alpha^{2} \alpha$ | $\alpha^{2} \alpha^{2}$ |

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1) $\mathrm{y}=\mathrm{i}\left(\mathrm{i}=0,1, \alpha, \alpha^{2}\right)$

2) $x+\alpha^{2} y=i\left(i=0,1, \alpha, \alpha^{2}\right)$

| $x+\alpha^{2} y=0$ | 00 | $1 \alpha$ | $\alpha \alpha^{2}$ | $\alpha^{2} 1$ |
| :--- | :---: | :--- | :--- | :--- |
| $x+\alpha^{2} y=1$ | $0 \alpha$ | 10 | $\alpha 1$ | $\alpha^{2} \alpha^{2}$ |
| $x+\alpha^{2} y=\alpha$ | $0 \alpha^{2}$ | 11 | $\alpha 0$ | $\alpha^{2} \alpha$ |
| $x+\alpha^{2} y=\alpha^{2}$ | 01 | $1 \alpha^{2}$ | $\alpha \alpha$ | $\alpha^{2} 0$ |

Thus there are 5 sets of lines with 4 parallel lines in each set. There is no common point in any two ines belonging to a set. Given a point there is one line in each set which passes through the point. As there are 5 sets, 5 lines pass through a point. Hence the replication of each treatment is 5 . As only one line can pass through two points, a pair of treatments occurs in one block only i.e. $\lambda=1$. After giving the numbering to the points to above said 20 lines BIBD can be given as (only the point numbers have been shown)

| 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: |
| 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | 16 |
| 1 | 5 | 9 | 13 |
| 2 | 6 | 10 | 14 |
| 3 | 7 | 11 | 15 |
| 4 | 8 | 12 | 16 |
| 1 | 6 | 11 | 16 |
| 2 | 5 | 12 | 15 |
| 3 | 8 | 9 | 14 |
| 4 | 7 | 10 | 13 |
| 1 | 8 | 10 | 15 |
| 4 | 5 | 11 | 14 |
| 2 | 7 | 9 | 16 |
| 3 | 6 | 12 | 13 |
| 1 | 7 | 12 | 14 |
| 3 | 5 | 10 | 16 |
| 4 | 6 | 9 | 15 |
| 2 | 8 | 11 | 13 |

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## 3 CONSTRUCTION OF NEIGHBOUR DESIGN WITH BIBD:

Neighbour design for OS1 series can be constructed by using the border plots. The arrangement of border treatments at either end of the block is the same as the treatment on the interior plot at the other end of block. Arrangement of treatments at border plots is not used for measuring the response variable. Plots other than border plots are described as internal plots. Neighbour design corresponding to $s=3$ i.e. a prime number are, $v=9, b=12, k=3, r=4, \lambda=1$ is as
3.1 When $\mathrm{s}=3$ :

| 3 | 1 | 2 | 3 | 1 |
| :--- | :---: | :---: | :---: | :---: |
| 6 | 4 | 5 | 6 | 4 |
| 9 | 7 | 8 | 9 | 7 |
| 7 | 1 | 4 | 7 | 1 |
| 8 | 2 | 5 | 8 | 2 |
| 9 | 3 | 6 | 9 | 3 |
| 8 | 1 | 6 | 8 | 1 |
| 9 | 2 | 4 | 9 | 2 |
| 7 | 3 | 5 | 7 | 3 |
| 9 | 1 | 5 | 9 | 1 |
| 8 | 3 | 4 | 8 | 3 |
| 7 | 2 | 6 | 7 | 2 |

### 3.1.1 Second-order neighbours of a treatment :

Now, from the neighbour design for OS1 series when $s=3$, the second-order left and right neighbours can be obtained as one plot away in left direction and one plot away in right direction. In the neighbour design in block 1 treatment number 2 is the second- order left neighbour of treatment number 1 . In block 4 treatment number 4 is the second- order left neighbour of treatment number 1. Similarly, all the second- order left neighbours can be obtained for treatment number 1 in which block the treatment number 1 appears. Thus a list of second-order left neighbours for treatment number 1 is 2, 4, 6 and 5

In the neighbour design, for $\mathrm{s}=3$, in block 1 treatment number 3 is the second- order right neighbour of treatment number 1. In block 4 treatment number 7 is the second- order right neighbour of treatment number 1. Similarly, all the second- order right neighbours can be obtained from the blocks in which block the treatment number 1 appears. Thus a list of second-order right neighbours for treatment number 1 is $3,7,8$ and 9 . Similarly, all the second-order right neighbours for every other treatment can be obtained. All the second-order left and right neighbours for every treatment are shown in Table 3.1.1.1

Table 3.1.1.1

| treatment no. ' $i$ ' | Second-order left nbhrs obtained | Second- order left nbhrs | Second- order right nbhrs obtained $\quad$ | Second- order right nbhrs | Series in which ' $i$ ' lies |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2,4,6,5 | 4,5,6, 2 | 3,7,8,9 | 7,8,9, 3 | $1=\mathrm{i}=\mathrm{s}$ |
| 2 | 3,5,4,6 | $4,5,6,3$ | 1,8,7,9 | $7,8,9,1$ | i.e. |
| 3 | 1,6,5,4 | $4,5,6,1$ | 2,9,7,8 | $7,8,9,2$ | $1=\mathrm{i}=3$ |
| 4 | 5,7,9,8 | 7,8,9, 5 | 6,1,2,3 | 1,2,3, 6 | $\mathrm{s}+1=\mathrm{i}=2 \mathrm{~s}$ |
| 5 | 6,8,7,9 | $7,8,9, \quad 6$ | 4,2,3,1 | 1,2,3, 4 |  |
| 6 | 4,9,8,7 | 7,8,9, 7 | 5,3,1,2 | 1,2,3, 5 | $4=\mathrm{i}=6$ |
| 7 | 8,1,3,2 | $1,2,3,8$ | 9,4,5,6 | $4,5,6,9$ | $2 \mathrm{~s}+1=\mathrm{i}=\mathrm{s}^{2}$ |
| 8 | 9,2,1,3 | 1,2,3, 9 | 7,5,6,4 | 4,5,6, 7 |  |
| 9 | 7,3,2,1 | $1,2,3,7$ | 8,6,4,5 | $4,5,6,8$ | $7=\mathrm{i}=9$ |

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The parameters of the neighbour design for OS1 series are: $\mathrm{v}=\mathrm{s}^{2}, \mathrm{~b}=\mathrm{s}(\mathrm{s}+1), \mathrm{r}=\mathrm{s}+1, \mathrm{k}=\mathrm{s}, \lambda=1$ for which both-sided neighbours are to be found whether s is a prime number or power of a prime number. (i) Consider the treatment number ' i ' where $\mathrm{i}=1,2, \ldots, \mathrm{~s}^{2}$.
(ii) Then find the series in which the treatment number 'I' lies

The series is defined in such a way that the sequence of first 's' treatments i.e. (1 to s ) of the design form the first series, the sequence of next 's' treatments i.e. ( $s+1$ to $2 s$ ) form the second series and the sequence of next ' $s$ ' treatments i.e. $(2 s+1$ to 3 s$)$ form the third series and so on. Hence the last series consists of the extreme last 's' treatments from $(\mathrm{s}(\mathrm{s}-1)+1)$-th to $s 2$-th treatment. Thus, there are 's' series upto the treatment number $s^{2}$. The ( $s+1$ )-th series of treatment numbers ( $s^{2}+1$ to $s 2+s$ ) reduces to ( 1 to $s$ ) with mod $\mathrm{v}=\mathrm{s}^{2}$. So the ( $\mathrm{s}+1$ )-th series is again the first series of the design. It again holds true for the next ( $\mathrm{s}+2$ )-th and so on series, which proves that the design is circular.

From the Table 3.1.1.1, it is observed that the treatment number $i=1$ lies in the series $(1 \leq i \leq s)$ so the second-order common left neighbour series of it shall be ( $s+1 \leq i \leq 2 s$ ) which are there as $s+1, s+2, s+3(2 s$ i.e. 4,5 and 6 namely). As the immediate second-order left neighbour of treatment number i shall be i-2. One more neighbour of treatment number 1 is treatment number 2 which simply can't be defined as i-2 as the immediate second-order left neighbour. As the property of circularity hold not only for the series in design but it also holds within each set of s treatments of the series in which i-th treatment lies so the other member of second-order left neighbour treatment of treatment number 1 is treatment number 2 .

The treatment number $\mathrm{i}=2$ has 4,5 and 6 as second-order common left neighbours. As the treatment number 2 lies in series ( $1 \leq \leq \leq s$ ) so the second-order common left neighbour series of it shall be the series ( $\mathrm{s}+1 \leq \mathrm{i} \leq 2 \mathrm{~s}$ ) as discussed above. Immediate second-order left neighbour of treatment number 2 is 3 which simply can't be defined as i-2. Because of the circularity within the series in which i-th treatment lies, for the treatment number 2 immediate second-order left neighbour is treatment number 3. For treatment number $\mathrm{i}=3$ immediate second-order left neighbour is simply $\mathrm{i}-2$ i.e. treatment number 1 and second-order common left neighbours are again 4,5 and 6 which lies in the series ( $s+1 \leq i \leq 2 s$ ) as discussed earlier..

It is observed that treatment number $i=4$ has second-order common left neighbours as 7,8 and 9 . As the treatment number 4 lies in the series ( $\mathrm{s}+1 \leq i \leq 2 \mathrm{~s}$ ) so the second-order common left neighbour series of it shall be $\left(2 s+1 \leq i \leq s^{2}\right)$ which are there as $2 s+1,2 s+2,2 s+3\left(s^{2}\right)$. One more second-order left neighbour of treatment number 4 is treatment number 5 which again can't be defined as i-2 immediate second-order left neighbour. As circularity not only hold for the series in design but it also holds within each set of s treatments in the series, hence for the treatment number 4 immediate second-order left neighbour is treatment number 5. Similarly, for treatment number 5 immediate second-order left neighbour is treatment number 6 because of the property of circularity within the series and common second-order left neighbour series is again $\left(2 s+1 \leq i \leq s^{2}\right)$ i.e. 7,8 and 9 . Further for treatment number 6 the second-order common left neighbour series is $\left(2 \mathrm{~s}+1 \leq \mathrm{i} \leq \mathrm{s}^{2}\right)$ and the other member of neighbour treatments is simply written as i-2 immediate second-order left neighbour, i.e. treatment number 4.

Lastly, for treatment number 7, 8 and 9, the second- order common left neighbours are 1, 2 and 3 . As the treatment number $7,8 \& 9$ lies in the series $\left(2 s+1 \leq i \leq s^{2}\right)$ so the second-order common left neighbour series of these treatments shall be ( $1 \leq i \leq s$ ) (as the circularity holds between the series in the design as well as for each set of s treatments in which series the i-th treatments lies) which are there as 1, 2 and 3 namely. The other member of treatment number 7 is 8 which is immediate second-order left neighbour because of the property of circularity. One more neighbour of treatment number 8 is 9 because of the same reason. Similarly for treatment number 9 immediate second-order left neighbour is treatment number 7 as discussed earlier

From the Table 3.1.1.1 it is observed that the treatment number 1 lies in the series ( $1 \leq i \leq s)$ so the second-order common right neighbour series of it shall be the series $(2 s+1 \leq i \leq 3 s)$ which are there as $2 \mathrm{~s}+1$, $2 \mathrm{~s}+2,2 \mathrm{~s}+3$ (s2) i.e. 7,8 and 9 . As the immediate second-order right neighbour of treatment number $\mathrm{i}=1$ shall be i +2 so the other member of right neighbours is treatment number 3 .

The treatment number 2 which lies in the series ( $1 \leq i \leq s)$ and again ( $2 \mathrm{~s}+1 \leq \mathrm{i} \leq 3 \mathrm{~s}$ ) is the series of second-order common right neighbours, immediate second-order right neighbour of treatment number 2 is 1 which simply can't be defined as $i+2$. As the property of circularity holds not only for the series in design but it also holds within each set of s treatments of the series in which i-th treatment lies. For the same reason, treatment number 2 is the immediate second-order right neighbour of treatment number $\mathrm{i}=3$ and second-order common right neighbours are again 7,8 and 9 i.e. the series ( $2 \mathrm{~s}+1 \leq i \leq 3 \mathrm{~s}$ ) because the

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treatment number 3 also lies in the series ( $1 \leq i \leq s$ ).
It is observed that treatment number 4 has second-order common right neighbours as 1,2 and 3. As the treatment number $i=4$ lies in the series $(s+1 \leq i \leq 2 s)$ so the second-order common right neighbour series of it shall be $(1 \leq i \leq s)$ due to the right circularity of second-order with in the series of the design. The other immediate second-order right neighbour of treatment number i shall be $i+2$ so the other member of secondorder right neighbours is treatment number 6. Further, the treatment number 5 has second-order right neighbours are 1, 2 and 3. As the treatment number 5 lies in the series ( $\mathrm{s}+1 \leq i \leq 2 \mathrm{~s}$ ), it has ( $1 \leq \mathrm{i} \leq \mathrm{s}$ ) as the series of second-order common right neighbour series for the same reason as discussed earlier. One more secondorder right neighbour of treatment number 5 is treatment number 4 which again can't be simply defined as $i+2$. As circularity not only holds for series within the design but it also holds within each set of s treatments in which i-th treatment lies, hence for the treatment number 5 immediate second-order right neighbour is treatment number 4. Similarly, for treatment number 6 immediate second-order right neighbour is treatment number 5 and common second-order right neighbour series is $(1 \leq i \leq s)$ i.e. 1, 2 and 3 as discussed earlier.

Lastly, for treatment number $i=7,8$ and 9 the second- order common right neighbours are 4,5 and 6. As the treatment number 7,8 and 9 lies in the series $\left(2 s+1 \leq i \leq s^{2}\right)$ so the second-order common right neighbour series of this shall be the series $\left(4 s+1 \leq i \leq s^{2}+2 s\right)$. As the circularity holds within the series in the design, $\left(4 \mathrm{~s}+1 \leq i \leq \mathrm{s}^{2}+2 \mathrm{~s}\right)$ reduces to $(\mathrm{s}+1 \leq \mathrm{i} \leq 2 \mathrm{~s})$ with $\bmod \mathrm{v}=\mathrm{s}^{2}$ which are there as $\mathrm{s}+1, \mathrm{~s}+2, \mathrm{~s}+3(2 \mathrm{~s}$ in this case) i.e, 4,5 and 6 namely. As the immediate second-order right neighbour of treatment number i shall be $\mathrm{i}+2$ so the other member of second-order right neighbours of treatment number 7 is 9 . Further one more neighbour of treatment number 8 is 9 which simply can't be defined as $i+2$. As the property of circularity holds within each set of $s$ treatments in which the i-th treatment lies so treatment number 8 has immediate second-order right neighbour treatment as 7. Similarly, for the same reason for treatment number 9 immediate second-order right neighbour is treatment number 8.
3.2 when $s=4$ :

The resulting design is the neighbour design with parameters: $\mathrm{v}=16, \mathrm{~b}=20, \mathrm{r}=5, \mathrm{k}=4, \lambda=1$.

| 4 | 8 | 12 | 16 | 13 | 14 | 15 | 16 | 16 | 15 | 14 | 13 | 15 | 14 | 16 | 13 | 14 | 16 | 15 | 13 |
| :---: | :---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 5 | 9 | 13 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 4 | 2 | 3 | 1 | 3 | 4 | 2 |
| 2 | 6 | 10 | 14 | 5 | 6 | 7 | 8 | 6 | 5 | 8 | 7 | 8 | 5 | 7 | 6 | 7 | 5 | 6 | 8 |
| 3 | 7 | 11 | 15 | 9 | 10 | 11 | 12 | 11 | 12 | 9 | 10 | 10 | 11 | 9 | 12 | 12 | 10 | 9 | 11 |
| 4 | 8 | 12 | 16 | 13 | 14 | 15 | 16 | 16 | 15 | 14 | 13 | 15 | 14 | 16 | 13 | 14 | 16 | 15 | 13 |
| 1 | 5 | 9 | 13 | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 | 1 | 4 | 2 | 3 | 1 | 3 | 4 | 2 |

### 3.2.1 Second-Order neighbours of a treatment :

From the neighbour design of OS1 series for $s=4$ the second-order left neighbour treatments and right neighbour treatments can be obtained by observing the treatments as one plot away in left direction and one plot away in right direction for the desired treatment. It is observed that in block 1 treatment 3 is the second-order left neighbour of treatment number 1 . In block 5 treatment number 9 is the second-order left neighbour of treatment number 1. Similarly, one can find out all the second-order left neighbours from the blocks in which block the treatment number 1 appears. By doing so a list of second-order left neighbours for treatment number 1 is $3,9,11,10$ and 12 . Similarly, all the second-order left neighbours for every other treatment can be obtained.

From the neighbour design it is observed that in block 1 treatment number 3 is the second- order right neighbour of treatment number 1. In block 5 treatment number 9 is the second- order right neighbour of treatment number 1. Similarly, all the second- order right neighbours can be obtained for treatment number 1, in which block the treatment number 1 appears. Thus a list of second-order right neighbours for treatment number 1 is $3,9,11,10$ and 12 . Similarly, all the second-order right neighbours for every other treatment can be obtained. The second-order left neighbour treatments and second-order right neighbour treatments for every treatment are shown in Table 3.2.1.1

Table 3.2.1.1

| 'i' | Second- order <br> left <br> nbhrs | Second- order <br> left nbhrs | Second- order <br> right <br> obtained | Second- order <br> right nbhrs | Series <br> which <br> lies |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | $3,9,11,10,12$ | $9,10,11,12$, | 3 | $3,9,11,10,12$ | $9,10,11,12$, | 3 | $1=\mathrm{i}=\mathrm{s}$ |
| $\mathbf{2}$ | $4,10,12,9,11$ | $9,10,11,12$, | 4 | $4,10,12,9,11$ | $9,10,11,12$, | 4 | i.e. |
| $\mathbf{3}$ | $1,11,9,12,10$ | $9,10,11,12$, | 1 | $1,11,9,12,10$ | $9,10,11,12$, | 1 | $1=\mathrm{i}=4$ |
| $\mathbf{4}$ | $2,12,10,11,9$ | $9,10,11,12$, | 2 | $2,12,10,11,9$ | $9,10,11,12$, | 2 |  |
| $\mathbf{5}$ | $7,13,15,14,16$ | $13,14,15,16$, | 7 | $7,13,15,14,16$ | $13,14,15,16$, | 7 | $\mathrm{~s}+1=\mathrm{i}=2 \mathrm{~s}$ |
| $\mathbf{6}$ | $8,14,16,13,15$ | $13,14,15,16$, | 8 | $8,14,16,13,15$ | $13,14,15,16$, | 8 | i.e. |
| $\mathbf{7}$ | $5,15,13,16,14$ | $13,14,15,16$, | 5 | $5,15,13,16,14$ | $13,14,15,16$, | 5 | $5=\mathrm{i}=8$ |
| $\mathbf{8}$ | $6,16,14,15,13$ | $13,14,15,16$, | 6 | $6,16,14,15,13$ | $13,14,15,16$, | 6 |  |
| $\mathbf{9}$ | $11,1,3,2,4$ | $1,2,3,4$, | 11 | $11,1,3,2,4$ | $1,2,3,4$, | 11 | $2 \mathrm{~s}+1=\mathrm{i}=3 \mathrm{~s}$ |
| $\mathbf{1 0}$ | $12,2,4,1,3$ | $1,2,3,4$, | 12 | $12,2,4,1,3$ | $1,2,3,4$, | 12 | i.e. |
| $\mathbf{1 1}$ | $9,3,1,4,2$ | $1,2,3,4$, | 9 | $9,3,1,4,2$ | $1,2,3,4$, | 9 | $9=\mathrm{i}=12$ |
| $\mathbf{1 2}$ | $10,4,2,3,1$ | $1,2,3,4$, | 10 | $10,4,2,3,1$ | $1,2,3,4$, | 10 |  |
| $\mathbf{1 3}$ | $15,5,7,6,8$ | $5,6,7,8$, | 15 | $15,5,7,6,8$ | $5,6,7,8$, | 15 | $3 \mathrm{~s}+1=\mathrm{i}=\mathrm{s}^{2}$ |
| $\mathbf{1 4}$ | $16,6,8,5,7$ | $5,6,7,8$, | 16 | $16,6,8,5,7$ | $5,6,7,8$, | 16 | i.e. |
| $\mathbf{1 5}$ | $13,7,5,8,6$ | $5,6,7,8$, | 13 | $13,7,5,8,6$ | $5,6,7,8$, | 13 | $13=\mathrm{i}=16$ |
| $\mathbf{1 6}$ | $14,8,6,7,5$ | $5,6,7,8$, | 14 | $14,8,6,7,5$ | $5,6,7,8$, | 14 |  |

From the Table 3.2.1.1 it is observed that treatment numbers 1, 2, 3 and 4 have $(s=4)$ second-order common left neighbours as $9,10,11$ and 12 . As the treatment number $1,2,3$ and 4 lies in the series $(1 \leq i \leq s)$ so the second-order common left neighbour series shall be the series ( $2 \mathrm{~s}+1 \leq i \leq 3 \mathrm{~s}$ ) which are there as $2 \mathrm{~s}+1$, $2 s+2,2 s+3,2 s+4$ ( 3 s , in this case). Other than the second-order common left neighbours, one more immediate second-order left neighbour of treatment number i should be i-2. One more neighbour treatment for treatment number 1 is 3 which simply cannot be written as $i-2$. It is due to the second-order left circularity within the series i.e. in each set of s series in which i-th treatment lies as discussed earlier, So, for the treatment number 1 , one more left neighbour other than the second-order common left neighbours is treatment number 3. Similarly, one more left neighbour treatment for treatment number 2 is treatment number 4. For the same reason as circularity holds within each set of s series in which i-th treatment lies. One more left neighbour for the treatment number 3 is treatment number 1 which is simply written as immediate second-order left neighbour i.e. i-2. Similarly, for treatment number 4 immediate second-order left neighbour is treatment number 2 .

Further, treatment numbers 5, 6, 7 and 8 have $(s=4)$ second-order common left neighbours as $13,14,15$ and 16. As the treatment number $5,6,7$ and 8 lies in the series ( $s+1 \leq i \leq 2 s$ ) so the second-order common left neighbour series shall be the series $(3 \mathrm{~s}+1 \leq \mathrm{i} \leq \mathrm{s} 2)$ which are there as $3 \mathrm{~s}+1,3 \mathrm{~s}+2,3 \mathrm{~s}+3,3 \mathrm{~s}+4\left(\mathrm{~s}^{2}\right)$, due to the second-order left circularity between the series in the design as discussed earlier. Immediate second-order left neighbour of treatment number i shall be i-2. One more neighbour treatment for treatment number 5 is 7 which simply cannot be written as i-2 but due to the second-order left circularity within the set of the series in which i-th treatment lies, it is treatment number 7. Similarly, one more left neighbour treatment for treatment number 6 is 8 which simply can't be written as i-2, but for the same reason as circularity holds within each set of the series in which i-th treatment lies, it is there the treatment number 8 as second-order left neighbour. One more left neighbour for the treatment number 7 is treatment number 5 which is simply written as immediate second-order left neighbour i.e. i-2. Similarly, for treatment number 8 immediate second-order left neighbour is treatment number 6.

Now, it is observed that the treatment numbers $9,10,11$ and 12 have ( $s=4$ ) second-order common left neighbours as $1,2,3$ and 4 . As the treatment number $9,10,11$ and 12 lies in the series ( $2 \mathrm{~s}+1 \leq i \leq 3 \mathrm{~s}$ ) so the

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second-order common left neighbour series shall be the series ( $1 \leq i \leq s$ ) which are there as $1,2,3$, $s$, due to the second-order left circularity between the series in the design as discussed earlier. Immediate second-order left neighbour of treatment number i shall be i-2. So one more neighbour treatment for treatment number 9 is 11 which is due to the second-order left circularity within the series as discussed earlier. Similarly, for the treatment number 9 one more left neighbour other than the second-order common left neighbours is treatment number 11. One more left neighbour treatment for treatment number 10 is 12 due to the secondorder circularity within series in which i-th treatment lies. One more left neighbour for the treatment number 11 is treatment number 9 which is simply written as immediate second-order left neighbour i.e. i-2. Similarly, for treatment number 12 immediate second-order left neighbour is treatment number 10

Lastly, it is observed that treatment number $i=13,14,15$ and 16 has ( $s=4$ ) second-order common left neighbours as 5, 6, 7 and 8 . As the treatment number $13,14,15$ and 16 lies in the series $\left(3 s+1 \leq i \leq s^{2}\right)$ so the second-order common left neighbour series shall be the series ( $\mathrm{s}+1 \leq i \leq 2 \mathrm{~s}$ ) due to the second-order left circularity between the series in the design as discussed earlier which are there as $s+1, s+2, s+3, s+4(2 s$ in this case). Immediate second-order left neighbour of treatment number i shall be i-2. One more neighbour treatment for treatment number 13 is 15 which is due to the second-order left circularity within the series in which i-th treatment lies. One more left neighbour treatment for treatment number 14 is 16 with the same reason. One more left neighbour for the treatment number 15 is treatment number 13 which is simply written as immediate second-order left neighbour i.e. i-2. Similarly, for treatment number 16 immediate second-order left neighbour is treatment number 14.

From the Table 3.2.1.1 it is further observed that treatment number $\mathrm{i}=1,2,3$ and 4 has $(\mathrm{s}=4)$ second-order common right neighbours as $9,10,11$ and 12 . As the treatment number 1,2,3 and 4 lies in the series $(1 \leq i \leq s)$ so, due to the second-order right circularity between the series in the design as discussed earlier the second-order common right neighbour series shall be the series ( $2 \mathrm{~s}+1 \leq i \leq 3 \mathrm{~s}$ ) which are there as $2 s+1,2 s+2,2 s+3,2 s+4$ ( 3 s in this case). Immediate second-order right neighbour of treatment number i shall be $i+2$. One more right neighbour treatment for treatment number 1 is 3 which simply can be written as i +2 immediate second-order right neighbour. Again one more right neighbour treatment for treatment number 2 is 4 which is simply as $i+2$ immediate second-order right neighbour. Again one more right neighbour for the treatment number 3 is treatment number 1 which simply cannot be written as $i+2$. Due to the second-order right circularity within the series in which i-th treatment lies, so, for the treatment number 3 one more right neighbour other than the second-order common right neighbours is treatment number 1. Similarly, for treatment number 4 one more right neighbour is treatment number 2, as discussed earlier.

It is observed that treatment number $i=5,6,7$ and 8 has ( $s=4$ ) second-order common right neighbours as $13,14,15$ and 16 . As the treatment number $5,6,7$ and 8 lies in the series ( $s+1 \leq i \leq 2 \mathrm{~s}$ ) so the second-order common right neighbour series shall be the series $\left(3 s+1 \leq i \leq s^{2}\right)$ which are there as $3 s+1,3 s+2$, $3 \mathrm{~s}+3,3 \mathrm{~s}+4\left(\mathrm{~s}^{2}\right)$ due to the second-order right circularity between the series in the design. Immediate secondorder right neighbour of treatment number i shall be $i+2$. One more right neighbour treatments for treatment numbers 5 and 6 are 7 and 8 respectively which simply can be written as i+2 immediate second-order right neighbour. One more right neighbour treatments for the treatment numbers 7 and 8 are treatment number 5 and treatment number 6 respectively which simply cannot be written as $i+2$. These are due to the secondorder right circularity within the series in which i-th treatment lies as discussed earlier.

In the same fashion, treatment number $i=9,10,11$ and 12 have $(s=4)$ second-order common right neighbours as $1,2,3$ and 4 . As the treatment number $9,10,11$ and 12 lies in the series ( $2 \mathrm{~s}+1 \leq i \leq 3 \mathrm{~s}$ ) so the second-order common right neighbour series shall be the series $(1 \leq i \leq s)$ due to the second-order right circularity between the series. Immediate second-order right neighbour of treatment number 9 is 11 and treatment number 10 is 12 which simply can be written as $i+2$ immediate second-order right neighbour. Second-order right neighbours for the treatment number 11 and 12 is treatment number 9 and 10 respectively, due to the second-order right circularity within the series in which i-th treatment lies.

Lastly, treatment number $\mathrm{i}=13,14,15$ and 16 have $(\mathrm{s}=4)$ second-order common right neighbours as 5, 6, 7 and 8 . As the treatment number 13, 14, 15 and 16 lies in the series $\left(3 s+1 \leq i \leq s^{2}\right)$ so the second-order common right neighbour series shall be the series ( $s+1 \leq i \leq 2 s$ ) which are there as $s+1, s+2, s+3,2 s(2 s$ in this case) due to the second-order right circularity between the series in the design. Immediate second-order right neighbours for treatment number 13 is 15 and for treatment number 14 is 16 which simply can be written as i+2 immediate second-order right neighbour. Second-order right neighbours for the treatment number 15 is treatment number 13 and for treatment number 16 is treatment number 14 due to the secondorder right circularity within the series in which i-th treatment lies as discussed earlier. Further, it is noted that the second-order common neighbour series of left neighbours is also the second-order common

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neighbour series of right neighbours for each treatment. This is due to the second-order left neighbours and second-order right neighbours occur at the same position for $s=4$.

## 4 CONCLUSION:

Here, the neighbour designs of OS1 series are constructed by method of Euclidean Geometry. Laxmi and Rani have verified that this method holds for different values of s, for neighbour design of OS1 series. Using this method, one can obtain the second order left- side neighbours and second-order right-side neighbours of a particular treatment of the design with OS1 series, whether s is a prime number or power of a prime number. It is concluded that the neighbour treatments of every treatment obtained from the neighbour design, constructed either using MOLS or Euclidean Geometry are exactly same.

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