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ORIGINAL ARTICLE

S-ENTROPY AND FAMILY OF DISTRIBUTIONS

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Abstract:

Entropy is a measure of uncertainity in a randome variable. In this context, the term usually refers to the Shannon's measure of entropy, which quantifies the expected value of the imformation contained in a massage. It has vast application in the social sciences and all branches of science. Researchers have defined different form of entropy as Mathai and Rathie, Aczel and Z Darcozy. But A Renyi and Shannon's measure of entropy has sought wide attraction as extensive and non-extensive measure of imformation. In this study our interest in defined S-entropy using lambert functions.

KEYWORDS:

Entropy, S-entropy, extensive and nonextensive measure, Lambert function.

INTRODUCTION

There is hardly any area of sciences which is not associated with term 'uncertainty' or 'randomness' Shannon provide a quantitative measure of this uncertainty known as 'Entropy'. Entropy is considered as a measure of disorder or randomness associated with the system which is some total of the large number of constituents. Entropy assumes optimum value when the system can be in any number of states randomly with equal probability and it takes value zero when the system is in a specific state means no uncertainty is there. Landsberg studies various possible variations in the function form of different states. Some of these states turn out to be extensive where the entropy of a combination of a system is simply the sum of the entropies of the system. In this type of situation Shannon's and Renyi's entropy are being used and known as extensive from of entropy. Tsallis has defined the non extensive measure of entropy. In probabilistic optimization, we may therefore consider a fractional size of the register, or equivalently, an integral number of cells in the register with fractional sized cells to accommodate a given amount of information. Probabilistic optimization in place of the deterministic parameterization of classical Shannon information theory (Nielsen and Chuang) becomes invetable in quantum computing contexts, and hence, our use of the fractional cell sizes may be a classical precursor of the inevitable departure from stringent Shmmon type concepts. Shafee has defined another form of entropy defined as S-entropy using the lambert function. It turned out to be an appropriate candidate in a situation where the probability distribution does not confer to any of the previously defined forms, expecially when the probability density functions sought is expected to be stiffer than that resulting from maximizing the other measures of information i.e. entropies. We have defined S-entropy of various statistical probability density function.

Measure of Entropy
The infromaion scheme is represented by

$$s \quad \begin{matrix} x_1, x_2 \ldots\ldots\ldots x_n & x \\ p_1, p_2 \ldots\ldots\ldots p_n & p \end{matrix} \qquad 1$$

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where P is the Probability dist^{n} $p_1, p_2 \quad p_n \;; p_i \quad \sum_{2}^{n} p_1 \quad 1$

Shnnon's measure is given by

$$H_{sh}(p) \quad H(p_1, p_2 ....p_n) \quad \sum_{i2i}^{n} p_i \log p_i \qquad 2$$

The entropy (2) may be interpreted as the expected valve of the random variable (-log pi) which is the uncertainty with xi whose probability is pi.

Renyi (1961) defined the parametric measure of entropy as

$$H_R(P) \quad \frac{1}{1} In \frac{\sum_{i\ 1}^{n} p_i}{\sum_{i\ 1}^{n} p_i}, \quad 1 \qquad 0$$

Which is of order
It may be noted that it

$$1 H_R(P) \qquad Pi \log Pi$$

So that Shannon's measures of entropy is a limiting case of Renyi's measure of entropy. Renyi entropy for a continuous variable is defined as

$$H_R(\ ) \quad H_R(\ , fo) \quad \frac{1}{1} \quad fo \;\; du \qquad 0 \qquad 1 \qquad 3$$

And Shannon's entropy for the continuous variable is

$$H_{Sh}(f) \quad \int_x f(x) \log f(x) du \qquad 4$$

Is obtained from (3) for $\quad 1$. $H_R(\ )$ is monotonically decreasing and pseudoconcave function of and share similar properties as that of Shannon entropy. In this chapter our measure of interest is Fariel Shafee entropy also known as S-entropy. The cases are as

$$H_S(\ ) \qquad p(x) \;\; \log p(x) (\text{for discrete dist}^{n})$$

And

$$H_S(\ ) \quad \int_x f(x) \;\; \log f(x) dx \quad (\text{for continuous dist}^{n})$$

S-Entropy for Different Probability Distributions

1.1 Uniform distribution (Discrete)

$$p(x/n) \quad \frac{1}{n}, x \quad 1,2,\ldots\ldots\ldots n.$$

$$H_s(\ ) \quad \sum_{x\ 1}^{n} p(x) \quad \log p(x) \qquad \sum_{x\ 0} \frac{1}{n} \quad \log \frac{1}{n} \ ,$$

$$= \frac{\log n}{n^{\ \ 1}},$$

$$H_{sh}(X) \quad \log n.$$

1.2 Uniform Distribution (continuous)

$$f(x) \quad \frac{1}{\ }, \qquad x \qquad \frac{\ }{2},$$

$$H_s(\ ) \quad \int_{a\ \frac{\ }{2}}^{a\ \frac{\ }{2}} \frac{1}{\ } \quad \log \frac{1}{\ } \ dx,$$

$$= \frac{1}{\ } \qquad \frac{\ }{2} \qquad \frac{\ }{2} \ \log \qquad \frac{\log}{\ \ \ 1},$$

and

$$H_{sh}(X) \quad \log \ .$$

1.3 Geometric Distribution.

$$p(x/p) \quad p(1 \quad p)^x, x \quad 0,1,\ldots\ldots\ldots$$

$$H_s(\ ) \quad \sum_{x\ 0} p(1 \quad p)^x \quad \log \ p(1 \quad p)^x \ ,$$

$$= \sum_{x\ 0} p(1 \quad p)x \quad \log p \qquad \sum_{x\ 0} p(1 \quad p)^x \quad \log(1 \quad p),$$

$$H_s(\ ) \quad p \ \log p \ \frac{1}{1 \quad (1 \quad p)} \qquad p \quad \frac{(1 \quad p)}{[1 \quad (1 \quad p)\ ]^2} \ \log(1 \quad p),$$

and

$$H_{sh}(X) \quad \log p \quad \frac{(1 \quad p)\log(1 \quad p)}{p} \ .$$

1.4 Pareto Distribution

$$f(x) \quad - \frac{x}{} ^{a\ 1} \quad \frac{a}{x^{a\ 1}}, x \quad ,$$

$$H_s(\ ) \quad \frac{a}{x^{a\ 1}} \quad \log \frac{}{xa\ \ 1} \quad dx,$$

$$= -(\quad) \quad x^{(a\ 1)} dx \ \log(\quad) \ (\quad) \ (\quad 1) \quad x^{(\ 1)} \log x dx,$$

$$= -(\quad) \quad \frac{x^{(\ 1)\ 1}}{(\quad 1)\ \ 1} \quad \log(\quad) \ (\quad) \ (\quad 1) \quad \log x \frac{x^{(\ 1)\ 1}}{(\quad 1)\ \ 1} \qquad \frac{x^{(\ 1)\ 1}}{(\quad 1)\ \ 1} \frac{1}{x} dx$$

$$= (\quad) \quad \frac{^{(\ 1)\ 1}}{(\quad 1)\ \ 1} \quad \log(\quad) \ (\quad) \ (\quad 1) \frac{^{(\ 1)\ 1}}{(\quad 1)\ \ 1} \log \quad \frac{(\quad)\ (\quad 1)}{[\quad (\quad 1)\ \ 1]^2} \quad ^{(\ 1)\ 1}$$

$$H_{sh}(X) \quad \log \quad \log \quad \frac{1}{} \quad 1.$$

1.5 Triangular Distribution.

$$f(x) \quad \begin{matrix} \frac{2}{} \ \frac{x}{} . & 0 \quad x \\ \frac{2}{1} \ 1 \ \frac{x}{} . & x \end{matrix}$$

$$H_s \quad _0 \ \frac{2}{} \frac{x}{} \ \log \ \frac{2}{} \frac{x}{} \ dx \qquad \frac{2}{1} \ 1 \ \frac{x}{} \quad \log \ \frac{2}{1} \ 1 \ \frac{x}{} \quad dx,$$

$$\frac{2}{} \quad \frac{x}{} \quad _0 \ x \ \log \ \frac{2}{_2} \ dx \quad \frac{2}{} \quad \frac{1}{} \quad _0 \ x \ \log x dx$$

$$\frac{2}{1} \quad 1 \ \frac{x}{} \quad \log \ \frac{2}{1} \quad dx$$

$$\frac{2}{1} \quad 1 \ \frac{x}{} \quad \log \ 1 \ \frac{x}{} \ dx$$

Put $1 \ \frac{x}{} \quad y \quad x \quad 1 \ \ y \ ,$

$$\frac{2}{} \quad \frac{1}{} \quad \frac{x^{\ 1}}{1} \ _0 \quad \log \ \frac{2}{_2} \quad \frac{2}{} \quad \frac{1}{} \quad \log x \ \frac{x^{\ 1}}{1} \ _0 \quad _0 \ \frac{x^{\ 1} 1}{1 x} dx$$

$$\int_{1}^{0} \frac{2}{1} \quad y \ \log \ \frac{2}{1} \quad dy \quad \int_{1}^{0} \frac{2}{1} \quad y \ \log y \ dy,$$

$$\frac{2}{2} \quad \frac{1}{1}\log \ \frac{2}{2} \quad \frac{2}{2} \quad \frac{1}{1}\log \quad \frac{2}{2} \quad \frac{1}{1^{\ 2}}$$

$$\frac{2}{1} \quad \log \ \frac{2}{1} \quad \frac{x^{\ 1}}{1}\Big|_{0}^{1} \quad \frac{2}{1} \quad \log y \ \frac{y^{\ 1}}{1}\Big|_{0}^{1} \quad \int_{0}^{1} \frac{y^{\ 1}}{1} \frac{1}{y} dy \ ,$$

$$H_s \quad \frac{2}{2} \quad \frac{1}{1}\log \ \frac{2}{2} \quad \frac{2}{2} \quad \frac{1}{1}\log \quad \frac{2}{2} \quad \frac{1}{1^{\ 2}}$$

$$\frac{2}{1} \quad \frac{1}{1}^{1}\log \ \frac{2}{1} \quad \frac{2}{1} \quad \frac{1}{1}^{1}\log \ 1$$

$$\frac{2}{1} \quad \frac{1}{1^{\ 2}}^{1} \ ,$$

and

$$H_{sh} \ X \quad \log \quad \log 2 \ \ \frac{1}{2}.$$

1.6 Lomax Distribution.

$$f(x) \quad ak^{a}(x \quad k)^{\ (a \ 1)}, x \quad 0, a \quad 0 \text{ and } k \quad 0$$

$$H_s \quad \int_{0} ak^{a}(x \quad k)^{\ (a \ 1)} \quad \log \ ak^{a}(x \quad k)^{\ (a \ 1)} \ dx,$$

$$a \ k^{a} \ \log(ak^{a}) \int_{0} (x \quad k)^{\ (a \ 1)} dx \quad (a \quad 1) a \ k \int_{0} (x \quad k)^{\ (a \ 1)} \log(x \quad k) dx,$$

put $x \quad k \quad y, \ dx \quad dy,$

$$a \ k^{a} \ \log(ak^{a}) \int_{k} y^{\ (a \ 1)} dx \quad (a \quad 1) a \ k \int_{k} y^{\ (a \ 1)} \log y dy,$$

$$H_S(\ ) \quad \frac{a \ k^{1}}{(a \quad 1) \quad 1}\log(ak^{a}) \quad \frac{(a \quad 1) a \ k^{1}}{(a \quad 1) \quad 1}\log k \quad \frac{(a \quad 1) a \ k^{1}}{(a \quad 1) \quad 1^{\ 2}},$$

and

$$H_{sh} \ X \quad 1 \ \ \frac{1}{a} \quad \log \ \frac{k}{\ } \ .$$

1.7 Normal Distribution.

$$f(x) \quad \frac{1}{\sqrt{2}} e^{\frac{1}{2\ 2}(x \quad)^2}, \qquad x \quad , \qquad\qquad , \ ^2 \quad 0$$

$$H_S \qquad \frac{1}{\sqrt{2}} \quad e^{\frac{1}{2\ 2}(x \quad)^2} \quad \log \frac{1}{\sqrt{2}} e^{\frac{1}{2\ 2}(x \quad)^2} \quad dx,$$

$$\log \sqrt{2} \qquad \frac{1}{\sqrt{2}} \quad e^{\overline{2\ 2} y^2} \, dy \quad \frac{1}{2\ ^2} \frac{1}{\sqrt{2}} \qquad y^2 e^{\overline{2\ 2} y^2} \, dy$$

Put $x \qquad y, \quad dx \quad dy$

$$\log \sqrt{2} \quad \frac{1}{\sqrt{2}} \qquad e^{\overline{2\ 2} y^2} \, dy \quad \frac{1}{2\ ^2} \frac{1}{\sqrt{2}} \qquad y^2 e^{\overline{2\ 2} y^2} \, dy$$

$$\log \sqrt{2} \quad \frac{1}{\sqrt{2}} \quad 2 \ _0 \ e^{\overline{2\ 2} y^2} \, dy \quad \frac{1}{2\ ^2} \frac{1}{\sqrt{2}} \quad 2 \ _0 \ z e^{\overline{2\ 2} z} \ \frac{1}{2z^{1/2}}$$

Put $y^2 \quad z, \quad 2ydy \quad dz, \quad dy \quad \frac{1}{2z^{1/2}} dz$

$$\log \sqrt{2} \quad \frac{1}{\sqrt{2}} \quad 2 \ _0 \ e^{\overline{2\ 2} z} \frac{1}{2z^{1/2}} dz \frac{1}{2\ ^2} \quad \frac{1}{\sqrt{2}} \quad 2 \ _0 \ z e^{\overline{2\ 2} z} \ \frac{1}{2z}$$

$$\log \sqrt{2} \quad \frac{1}{\sqrt{2}} \quad \frac{1/2}{/ 2 \ ^2 \ ^{1/2}} \quad \frac{1}{2\ ^2} \frac{1}{\sqrt{2}} \quad \frac{3/2}{/ 2 \ ^2 \ ^{3/2}}$$

$$\log \sqrt{2} \quad \frac{1}{\sqrt{2}} \quad \frac{\sqrt{2}}{^{1/2}} \quad \frac{1}{2\ ^2} \frac{1}{\sqrt{2}} \quad \frac{1}{2} \frac{\sqrt{\ }}{/ 2 \ ^2 \ ^{3/2}}$$

$$\log \sqrt{2} \quad \frac{1}{\sqrt{2}} \quad \frac{\sqrt{2}}{^{1/2}} \quad \frac{1}{\sqrt{2}} \quad \frac{\sqrt{\ }}{^{3/2}} \ 2 \ ^{1/2},$$

$$H_s \quad \log \sqrt{2} \quad \frac{1}{\sqrt{2}} \quad \frac{\sqrt{2}}{^{1/2}} \quad \frac{1}{2} \frac{1}{\sqrt{2}} \quad \frac{\sqrt{2}}{^{3/2}}$$

and

$$H_{sh}\ X \quad \log \quad \frac{1}{2}\log 2\ \ e.$$

1.8 Generalized Pareto Distribution. For $x \quad 0(\text{if}\, c \quad 0 \text{ and } k \quad 0)$ **or**

for $0 \quad x \quad k/c$ (if $c \quad 0$ and $k \quad 0$),

$$f(x) \quad \frac{1}{k}\ 1 \quad \frac{cx}{k}^{\ 1/c\ \ 1},$$

Case 1. For $X \quad 0(\text{if}\, c$ and $k \quad 0)$,

$$H_S \qquad {}_0\ \frac{1}{k}\ 1 \quad \frac{cx}{k}^{\ 1/c\ \ 1} \quad \log\ \frac{1}{k}\ 1 \quad \frac{cx}{k}^{\ 1/c\ \ 1} \quad dx,$$

Put $\frac{cx}{k} \quad y, dx \quad \frac{k}{c}dy,$

$$_0\ \frac{k}{c}\frac{1}{k}\ 1 \quad y \ \bar{c} \quad \log\ \frac{1}{k}(1 \quad y)^{\frac{1}{c}\ 1} \quad dy,$$

$$_0\ \frac{k^1}{c}\ 1 \quad y\ \bar{c} \quad \log k dy \qquad _0\ k^1\ \frac{(1 \quad c)}{c^2}\ 1 \quad y\ \bar{c} \quad \log(1 \quad y)\ dy,$$

$$\frac{k^1}{c}\ \frac{(1 \quad y)^{\bar{c}} \quad 1}{(\ 1)\ \frac{}{c} \qquad 1}_0 \quad \log k \quad k^1\ \frac{1 \quad c}{c^2}\ _0\ 1 \quad y\ \bar{c} \quad \log(1 \quad y)dy$$

Put $1 \quad y \quad z,\ dy \quad dz,$

$$\frac{k^1}{c}\ \frac{1}{c \quad c\ /c}\ \ 0\ \log k \quad k^1\ \frac{1 \quad c}{c^2}\ _0\ z^{\bar{c}}\ \ \log z dz,$$

$$\frac{k^1}{c \quad c}\log k \quad k^1\ \frac{1 \quad c}{c^2}\frac{c^2}{c \quad c^{\ 2}},$$

$$H_S \quad \frac{k^1}{c \quad c}\log k \quad k^1 \quad \frac{1 \quad c}{c \quad c^{\ 2}}.$$

Case 2. For $0 \quad x \quad k / c(\text{if } c \quad 0 \text{ and } k \quad 0)$,

$$S \quad {}_0^{k/c} \frac{1}{k} \ 1 \quad \frac{cx}{k}^{\frac{1}{c} \ 1} \log \frac{1}{k} \ 1 \quad \frac{cx}{k}^{\frac{1}{c} \ 1} \quad dx,$$

Put $1 \quad \frac{cx}{k} \quad y, dx \quad \frac{k}{c}dy,$

$${}_0^1 \frac{k}{c}\frac{1}{k} y^{\overline{c}} \quad \log \frac{1}{k} y^{\frac{1}{c} \ 1} \quad dy,$$

$${}_0^1 \frac{k^1}{c} y^{\overline{c}} \quad \log k dy \quad {}_0^1 k^1 \ \frac{(1 \quad c)}{c^2} y^{\overline{c}} \quad \log y dy,$$

$$\frac{k^1}{c}\log k \ \frac{1}{\frac{c \quad c}{c}} \quad 0 \quad k^1 \ \frac{1 \quad c}{c^2} \ \frac{y^{\overline{c}} \quad 1}{\overline{c} \quad 1} 2 \Big|_0^1 ,$$

$$\frac{k^1}{c \quad c}\log k \quad k^1 \ \frac{1 \quad c}{c^2} \frac{c^2}{c \quad c^{\ 2}},$$

$$H_S \quad \frac{k^1}{c \quad c}\log k \quad k^1 \quad \frac{1 \quad c}{c \quad c^{\ 2}},$$

and

$$H_{sh}(X) \quad \log k \quad 1 \quad c.$$

CONCLUSION.

We have computed the Shafee entropy for the various classes of density functions. From the expressions observed for the S entropy, we have shown that Shannon entropy becomes the particular case of S entropy. So, it can be used in the situation where extensive as well as non extensive measures of information are applicable.

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