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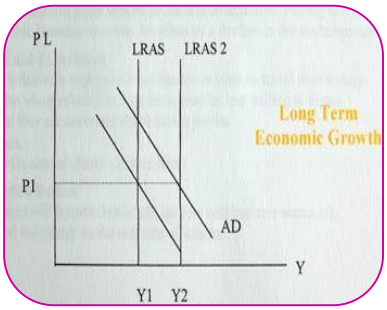
A DIFFERENTIAL EQUATION OF THE NEOCLASSICAL ECONOMICAL GROWTH MODEL



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ABSTRACT:

In the present research article we study the fundamental differential equation of the neoclassical economical growth model subject to foreign financing. We achieved a threshold parameter $\eta - 1$, which can be consumed to command a national financial network for exportation and its foreign liability.It is observed that the fundamental equation of neoclassical economic growth is achieved from the association of the income identity, gross investment identity and production function of time (t). Gross Domestic Product (GDP) can be consumed, invested or used to assist foreign liability. The durability characters of the differential model are also studied. This model relies on simplifying assumption. These assumptions extract the development of the model. An assumption is best when it helps us to build a model for the observations and forecast well.

KEYWORDS: *Differential equation, export-import, interest rate, input-output factors, balance of payment, production function.*

1.1 INTRODUCTION:

The prime intention of this study is to present the fundamental differential equation of economic growth of the neoclassical model if afford to allow for foreign financing. International capital flows have a close association to domestic saving and investment. Capital arrivals expand the domestic saving pool for growing the funds.Available for investment in physical capitalwhile capital out flowing, reduce the amount of savings available for investment. Thus capital inflows help to promote economic growth within a country and capital outflows to restrain it.

The effect of the existence of externally held debt off on the domestic economy arises solely. The taxes needed to finance a part of interest cost not covered by increased debt. We would expect the utility of an individual living at the time of long-run equilibrium to decrease because of increased taxes. Change in the equilibrium wage interest rate caused by the impact of these taxes on the supply side of the capital market. The relation between external debt, investment and economic growth has been established in many studies. Over the years that studies have established mixed conclusions either positive, negative, mixed and uncertain results linking external debt through investment to economic growth.[9]

It is observed that excessive debt effects a country's economic development in a number of ways. Firstly, the large debt service requirements shrink foreign exchange and capital, because they are transferred to lenders to payback principal and interest. A country benefits only partially from an increase in output or experts because a growing fraction of the increase gets used to service the occurred debt. Secondly, when the debtor countries are unable to fulfill their debt service obligation promptly, they are precise, high risk countries and they find it difficult to borrow. As a result, they have to pay high interest rates to obtain new credit. Thirdly, of the accumulation of debt causes a reduction in an economy's efficiency. It is difficult to adjust efficaciously to some shocks and international financial fluctuations. Finally, to save more foreign

exchange to meet debt obligations, many debtor countries out down on imports and restrict the quantity goods available for citizens. [4]

In an open economy the term of savings applicable as domestic investment consists of funds from savers of a abroad and saving of the domestic private and public sectors. Inspire of capital outflows is generally beneficial to the countries that receive them otherwise they are cost less. Countries that finance domestic capital formation primarily by capital arrival impact of paying interest and dividends to the foreign financial investors from whom they have financing. Many developing countries have experienced debt crisis, arising because the domestic investment they made with foreign funds turned out poorly leaving them insufficient income to pay what they owed their foreign creditors. The convenience of financing domestic capital formation chiefly with domestic saving is that the returns from the country's capital investment accumulate to domestic savers rather than flowing abroad. [12]

Financial links and trades with other countries has becomes in an open economy. Work done by fore*x* market is the exchange of one national currency to the other. The price at which the two currency exchange is the exchange rate. [2]. One country trades with other country in goods and service and may be financial assets is becoming in an open economy. A country's total export (E-I) is one factor of its combined requirement C + V + (E – I), for a country to be capable to service its foreign debt, reduce there is an independent rise in exports or reduce in imports which will have a multiplier effect on the economy. Interest rule of differential and capital flow are naturally the essential determinants of exchange rules change short in period.

Interest rate in a particular country rises when foreign interest rate endures unchanged. It results that attraction of capital flow to the country whose interest rate is rises and in consequence make the country's currency to appreciate. From this we remarkably suggest that an increase in interest rate affects to compact the economy by recognizing the country's currency and decreasing net export (E-I). Superior interest rate prevents investment cost and causes reduction in the investment (V) constituent of Y = C + V + (E – I). Also a decrease in interest rate affects to develop the economy by decreasing the county's currency and climbing net export (E-I). [15]

According to [1], there are three hypotheses by which circumstance causes a nation's currency to estimate or reduce in the market for foreign exchange they are:

(I) Ifthe requirement for nation's currency inflation, then the currency will admire; but if the requirement decreases that currency will devalue.

(II) If the availability of a nation's currency appreciates, that currency will depreciate, but if the supply decreases that affects currency will appreciate.

(III) If a country's currency admire there some foreign currency depreciates relative to it.

The interest rate is decided at the point where the descending direction demand for money curve intersects the perfectly inelastic supply of money curve. Changes in interest rate would appear if there is a change in market strength. It suggests that with a constant supply of money, in increase in demand for money increases the equilibrium interest rate while the interest rate decreases if the requirement for currency fall. Correspondingly, a decrease is a supply of currency increases the interest rate while an increase reduces it when the demand for money is constant. [6]

1.2 Export and Economic Growth:

Particularly concentrate on the effect of economic growth, the profit of export to economic growth about analysis of the supply policy on the economy is primly important in every nation.Inconsistent with neo-classical economic growth theory, the mainorigination of economic growth lies in two fields. He component is input increases and skill improvements, further more evaluation of the neoclassical economic growth theory frequently regards exports as an influence that can impress technological progress or

additional influences that are related to economic inadequacy. Regarding the economic potential of a country, the subscription of exports is concerned to be a part in the balances of growth accounting. [7]

The "new growth theory" indogenizes the appliance through which exports influences economic growth. For illustration, [5] expected a two-country growth model with internal technological development. In their model exports help to promote technology transfer and knowledge diffusion, increase bilateral relationship between two countries and as well increases economic growth. To introduce exports of a country into a production function is one of the major problems that are involved in econometric analysis which follows the neoclassical approach. Some analysts directly include exports in the production function as the third variable beside labor and capital. Others use more sophisticated methods, for illustration, [3] Divided the economy into two sectors.The export sector and the domestic sector. The export sector has to cope with foreign producers and consumers more often than the domestic sector does; it is more efficient than domestic sector. In order to capture the diffusion process of technology and knowledge, [3] introduced the output of the export sector (total exports) into the production function of the domestic sector as an element that could affect its economic efficiency.

In the study of neoclassical economic theory, it shows the contributions of exports through analysis on the demand side of the economy. Since this is the approach adopted in the study of [7] one export and economic growth in China, much review was made in their following discussions. This demand side approach is also called demand oriented analysis or Post – Keynesian analysis. According to the traditional Keynesian theory, an increase in exports is one of the factors that can stimulate increases in demand and thus will surely lead to increases in outputs. However, this approach has not been used widely. According to [8], this is because of the remnant of Say's Law in people's mind. Most people believe that the major constraints of modern economic growth lie on the supply side instead of on the demand side. In other words, only increases in factor inputs and improvements in economic efficiency can stimulate economic growth.

1.3ASSUMPTIONS OFTHE MODEL:

1. The model is open to allow foreign financing
2. D which is the foreign debt and, η the interest rate on the foreign debt is assumed given.
3. Capital accumulation is measured by the rate of change of the capital stock.
4. Existing capital stock depreciates at a constant proportionate rate.
5. The production function is invariant overtime and twice differentiable.
6. Production function exhibits constant returns to scale.
7. The Labor Force grows at a given experiential rate.

2. DIFFERENTIAL EQUATION MODEL:

The income equation of the neoclassical growth model if opened to allow for foreign borrowing is given as

$$Y = C + V + (X – M)$$

Where:

X represents an export component
M represents import component
C represents consumption component and
Y represents the GDP

According to [6], exports are domestic goods and services purchased by foreigners while imports are foreign goods and services purchased domestically. Net export (E – I) can be negative, positive or zero. It is

negative if the domestic expenditure on foreign goods exceeds foreign expenditure on domestic goods and services, positive if the reverse is the case and zero if both values are equal.

According to the balance of payments equation, however **E + D = I +** η **D**[9] is the foreign debt, and η is the interest rate on the foreign debt.

From the balance of the payment equation above, we have: **E – I =** η **D – D**

Upon substitution in the income equation model above, we have:

Y = C + V + (η **– D)**

Y = C + I + (η **- 1) D**

If output at the time (t) is Y (t), consumption at time t is the C(t) and investment at time (t) is V (t) and the foreign debt at a time (t) is D (t), then according to the income identity,

Y (t) = C (t) + v (t) + (η **-1)D (t)** (2.1)

Which states that output (Gross National Product) can be consumed, invested or used to service foreign debt?

The investment component is used both to augment the stock capital and to replace depreciated capital. By capital stock of an economy, we mean the sum total of machines, buildings and other manufactured, non-labor resources that are in existence at some point in time. The study of the theory of capital as a factor of production is extremely important in economics. Two areas for which we need an understanding of the nature of capital are the study of economic growth and the explanation of business cycles. Economics has traditionally assigned an important role to capital as a factor of growth in the production process. One of the reasons for increases in per capita output over time is the increasing amount of production equipment that workers have at their disposal [14].

Using two homogenous factor inputs, laborL(t)and capital K(t)where t is assumed to vary continuously and letting K(t) be the stock of capital at a time (t), capital accumulation is measured by the rate of change of the capital stock,

$$K = \frac{d}{dt}\mathrm{K(t)}$$

Assuming that the existing capital stock depreciates at the constant proportionate rate $\xi\,\mathrm{K(t)}$the depreciated capital to be replaced at time t is $\xi\,\mathrm{K(t)}$and the gross investment identity states that,

$$V(t) = k(t) + \xi k(t) \qquad (2.2)$$

The net capital accumulation is that part of the investment not used to replace depreciated capital. The output in equation (2.1) is determined by an aggregate production function, which summaries the technically efficient possibilities for production of output from capital and labor.

$$Y = F(K, L) \qquad (2.3)$$

The production function in 2, 3 is assumed invariant overtime and twice differentiable, where, for all positive factor inputs;

$$\frac{\partial F}{\partial K} > 0 \qquad \frac{\partial^2 F}{\partial K^2} < 0 \qquad (2.4)$$

$$\frac{\partial F}{\partial L} > 0 \qquad \frac{\partial^2 F}{\partial L^2} < 0$$

and taking limits:

$$\lim_{k\to 0} \frac{\partial F(K,L)}{\partial K} = \infty, \qquad \lim_{k\to \infty} \frac{\partial F(K,L)}{\partial K} = 0$$

$$\lim_{l\to 0} \frac{\partial F(K,L)}{\partial L} = \infty, \qquad \lim_{l\to \infty} \frac{\partial F(K,L)}{\partial K} = 0 \qquad (2.5)$$

The above imply that both marginal products start at infinity and diminishes to zero. Assuming the production function exhibits constant returns to scale. So, for any positive scale factor β, we have:

$$F(\beta K, \beta L) = \beta F(K,L) = \beta Y \qquad (2.6)$$

Particularly, choosing $\beta = \frac{1}{L}$, we have:

$$\frac{Y}{L} = F\left(\frac{K}{L}, 1\right) = f\left(\frac{K}{L}\right) \qquad (2.7)$$

The function $f(.)$ gives output per worker as a function of capital per worker. If we denote per-worker quantities of lower-case letters, we can write

$$y = f(k) \qquad (2.8)$$

where $y(t)$ is output per worker and k(t) is the capital per worker.

$$y(t) = \frac{Y(t)}{L(t)}, \quad k(t) = \frac{K(t)}{L(t)} \qquad (2.9)$$

by assumptions (2.4) and (2.5). It follows that:

$$f'(k) = \frac{df(k)}{dk} > 0, \quad f''(k) = \frac{d^2 f(k)}{dk^2} < 0$$

for all positive k.
taking limits

$$\lim_{k\to 0} f'(k) = \infty \qquad \lim_{k\to \infty} f'(k) = 0 \tag{2.10}$$

The per-capita production function $F(.)$ is a strictly concave monotonic increasing function, with its slope decreasing from infinity at $k = 0$ to zero $k = \infty$.

The variables and equations stated earlier can also be rewritten in per-worker terms.

Let c (t) is consumption per worker and v (t) is investment per worker at time **t**, and d (t) is foreign debt per worker.

Where:

$$c(t) = \frac{C(t)}{L(t)}, \quad i(t) = \frac{I(t)}{L(t)}, \quad d(t) = \frac{D(t)}{L(t)} \tag{2.11}$$

A characteristic of most industrialized economies is that variables like output per capita, consumption per capita etc, and exhibit sustained growth over long periods of time. This long-run growth occurs at rates that are roughly constant over time within economies but differ across economies [2].

We can rewrite the income identity as:

$$y(t) = c(t) + v(t) + \left(\eta - 1\right)d(t) \tag{2.12}$$

And gross investment identity is rewritten as:

$$v(t) = \frac{K(t)}{L(t)} = \xi\,(K(t) \tag{2.13}$$

Keynesian economic theory assigns an important role to investment as one component of aggregate demand. Net investment comes about because firms desire to change the stock of capital they have available [11].

But the rate of change of capital per worker is:

$$\dot{k} = \frac{d}{dt}\left(\frac{K}{L}\right)$$

$$\therefore \dot{k} = \frac{\dot{K}}{L} - \frac{K\,\dot{L}}{L}, \text{ Where, } \frac{K}{L} = k$$

$$\Rightarrow \frac{\dot{K}}{L} = \dot{k} + \frac{k\,\dot{L}}{L} \tag{2.14}$$

So the gross investment identity is:

$$V(t) = \frac{\dot{K}(t)}{L(t)} + \xi K(t)$$

$$= \frac{\dot{K}(t) + \xi K(t)L(t)}{L(t)}$$

$$= \frac{\dot{K}(t)}{L(t)} + \xi\left(\frac{K(t)L(t)}{L(t)K(t)}\right)$$

$$\therefore v(t) = \frac{\dot{K}(t)}{L(t)} + \xi k$$

Upon substitution, we have

$$v(t) = \dot{K} + k\frac{\dot{L}}{L} + \xi k = \dot{k} + \left(\xi + \frac{\dot{L}}{L}\right)k \qquad (2.15)$$

Assuming the labor force grows at the given exponential rate E:

$$\frac{\dot{L}}{L} = E \qquad (2.16)$$

So upon substitution, we have:

$$v(t) = \dot{k} + (\xi + E)k = \dot{k} + \theta k \qquad (2.17)$$

Where $\xi + E = \theta$ (2.18)

i.e. the sum of the depreciation rate and the rate of growth of the labor forceand is assumed to be a positive constant.

The three basic equations derived so far; the income identity:

$$y(t) = c(t) + v(t) + (\eta + 1)d(t)$$

the gross investment identity:

$v(t) = \dot{k} + \left(\xi + \frac{\dot{L}}{L} \right) k$, and the production functions: $\theta = \xi + E$, can be combined to form the fundamental differential equation of neoclassical economic growth.

Recall that:

$$\frac{Y}{L} = F\left(\frac{K}{L}, 1\right) = f\left(\frac{K}{L}\right)$$

i.e. $y = f(k)$

$$y(t) = c(t) + v(t) + (\eta - 1)d(t)$$

but $$v(t) = k + \left(\xi + \frac{\dot{L}}{L} \right) k = k + (\xi + E)k = \theta k$$

upon substitution, we have:

$$f(k(t)) = c(t) + \theta k(t) + k(t) + (\eta - 1)d(t) \quad (2.19)$$

$$\therefore f(k(t)) = c(t) + \theta k(t) + k(t) + \omega d(t) \text{ Where } (\eta - 1) = \omega \quad (2.20)$$

equation (2.20) yields the fundamental differential equation of economic growth of the neoclassical model. It states that output per worker $f(k)$ is allocated among four uses: consumption per worker, ***c;*** maintenance of level of foreign debt per worker, ωd. Maintenance of level of capital per worker, θk; and net increase in the capital per worker is k.

This fundamental equation is illustrated in fig. The upper diagram shows per worker production function, $f(k)$ and the ray $(\theta k + \omega d)$. Subtracting the ray from the curve gives*c + k*as indicated in the lower diagram. Two critical points $\hat{k}$ and $\tilde{k}$, designate levels of capital per worker at which *c + k*is a maximum and zero, respectively [2]

$$f(\hat{k}) - \theta\tilde{k} \geq f(k) - \theta k \text{ All } k > 0$$
$$f(\tilde{k}) - \theta\hat{k} = 0$$

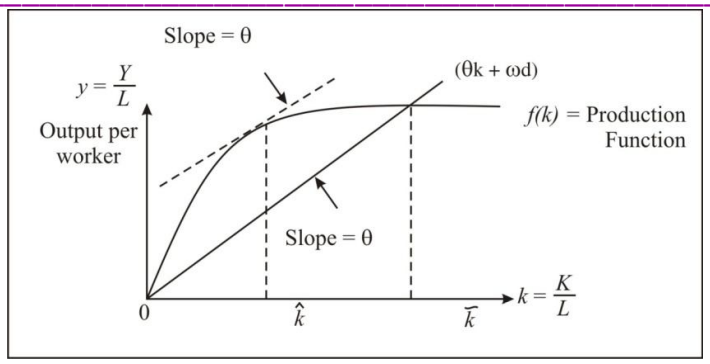

Figure 1.1

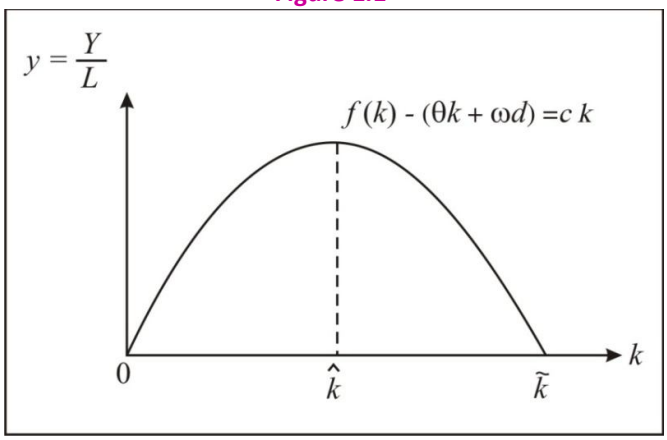

Figure 1.2

Figure 1.1 and 1.2 the fundamental Differential Equation of the Neoclassical Growth. **Under the assumptions above, the points $\hat{k}$ and $\tilde{k}$ exist and are unique.**

3. STABILITY ANALYSIS OF THE DIFFERENTIAL EQUATION MODEL:

The stability properties of the fundamental equation of economic growth depend on the level of consumptionper worker as illustrated in figure (2). in case (a). Consumption per worker is zero, so the vertical axis is *k*, andthe diagram is a phase diagram, at the point $\tilde{k}$ the derivative*k* is zero, so $\tilde{k}$ is an equilibrium point. To theleft of $\tilde{k}$ the derivative*k*is positive, so *k*moves to the right; to the right; to the right of $\tilde{k}$ the derivative*k*is negative, so *k*moves to the left. These directions are shown by arrows which make it clear that $\tilde{k}$ isan equilibrium that is locally stable. By the dynamics of the system, any small deviation of *k*from $\tilde{k}$ willeventually be eliminated and the equilibrium at $\tilde{k}$ will be restored.

In case (b) consumption per worker is at its maximum level at $\hat{c}$, the height of the curve at $\hat{k}$ k,whereas fig. 1makes it clear, $\hat{k}$ is defined by:

$$f'(k) = \theta = \xi + E \quad at \quad k = \hat{k} \tag{3.1}$$

The level of capital per worker, $\hat{k}$ called the golden rule level of capital per worker, is the equilibrium thatmaximizes the sustainable level of consumption per worker. The maximized level of consumption per worker $\hat{c}$ that can be maintained forever as an equilibrium level at $\hat{k}$ is:

$$\hat{c}f(\hat{k}) - \theta\hat{k}, \qquad (3.2)$$

(a) Zero consumption per worker; *c* = 0

(b)

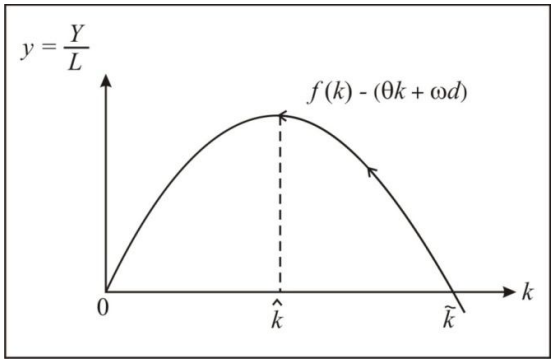

Figure 1.3

(c) Maximum (Gold – Rule) consumption per worker $c = \hat{c}$

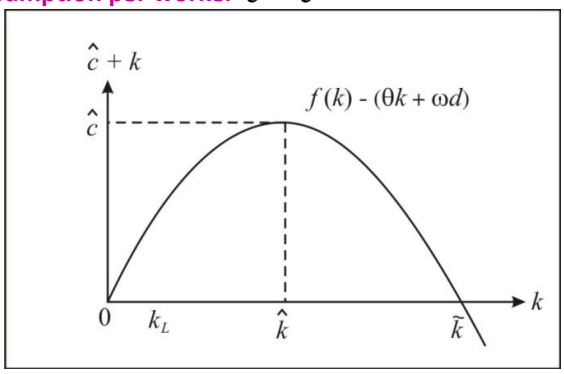

Figure 1.4

(d) Fixed Consumption per worker; $c = \overline{c}$ **, where** $0 < \hat{c} < \overline{c}$

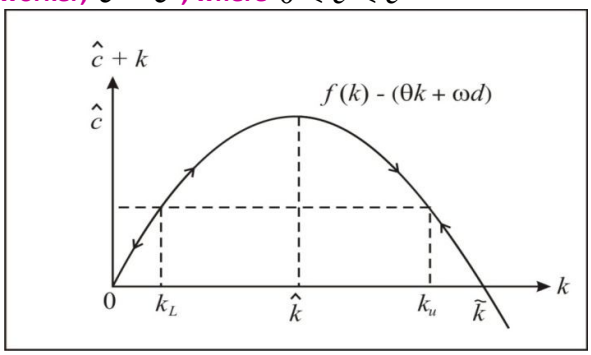

Figure 1.5

figure 1.3, 1.4 & 1.5 stability properties of the fundamental differential equation of the neoclassical economic growth, where $\hat{c}$ is called the golden-rule of level of consumption per worker condition (3.1) is called the goldenruleof accumulation.

While the golden-rule level of capital per worker, $\hat{k}$ is equilibrium. Fig (2)-(b) shows that this equilibriumis not stable. Deviations to the right of $\hat{k}$ are eliminated, but those to the left of k are not, as shown byarrows. If capital per worker falls below $\hat{k}$, it will continue to fall. Assuming *c* = 0 at *k*= 0, the only stableequilibrium in case (b) is at the origin. Finally, in case (c) consumption per worker is fixed at $\tilde{c}$, which is positive but less than the maximum consumption per worker $0 < \tilde{c} < \hat{c}$

In this case, the consumption per worker line, $c=\tilde{c}$, intersects the curve at the points, at a lower level, K_u, and an upper level of capital per worker. Both K_L and K_u are equilibrium points in that if either is attained, the system will not move away from it. The two equilibrium points differ in their stability properties,however. The upper point K_u is a stable equilibrium in that, as indicated by the arrows, slight deviations are eventually eliminated. The lower point K_L is an unstable equilibrium. As the arrows indicates, starting from level of *k* slightly below K_L, capital per worker falls toward zero, while starting from levels of *k* slightly above K_L, capital per worker rises towards K_u.

Thus, if consumption per worker is fixed at some intermediate level, such as a subsistence level, the level ofcapital per worker must be sufficiently large initially for the gravitate toward the upper stable equilibrium.This argument shows the need for a big "Push" in reaching a critical level of capital per worker beyond whichthe economy will, via its own dynamics, gravitate towards higher and higher levels of capital per worker and hence output per worker.

4. DISCUSSION

In this research article, we have been able to formulate a differential equation of the neoclassical growth model if opened to allow for foreign borrowing. From the balance of payment equation, we have $E - I = \eta D - D$ which, when introduced into the income equation yields $Y = C + V + (\eta D - D)$. Comparing $Y = C + V + (E - I)$ With $Y = C + V + (\eta - 1)D$ will mean that $(\eta - 1)$ is a thresh-hold factor to controlthe net export of a country as well as its foreign debt. When $0 \le \eta < 1$, i.e. $\eta = 0$ or $\eta < 1$, then the countrywill experience a boom in its net reduction in its net export because D will be negative. But when $\eta < 1$, then the country will experience a reduction in its net export because D will be positive. Furthermore, analysis revealsthat at time (t), Gross National Output can be consumed, invested or used to service foreign debt.

Finally, the fundamental equation of economic growth of the neoclassical model reveals that output perworker is allocated among four uses: Consumption per worker, maintenance of level of foreign debt perworker, maintenance of level of capital per worker and net increase in the level of capital per worker.

5. CONCLUSION:

The conclusion is therefore growth can be generated not only that growth can be generated not only by increase in aggregate level of labor and capital but also by the reallocation of existing resources from the less efficient non-export sector to the higher productivity export sector This study presented and analyzed a fundamental differential equation of the neoclassical economic growth. We say that $(\eta - 1)$ is a threshold parameter to checkmate the net export of a country. For a country not to be trapped by foreign

debt the η need to be negative and otherwise positive. Furthermore, the threshold parameter η need not be equal to one as that will make $(\eta - 1)$D, in equation 2.1 above to be unrealistic.

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