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APPLICATION OF DIFFERENTIAL EQUATIONS IN EXPONENTIAL AND LOGISTIC GROWTH MODEL FOR INDIA'S POPULATION GROWTH

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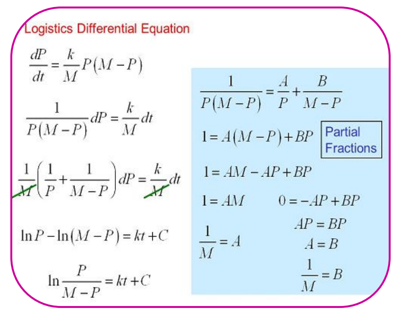
ABSTRACT

The intention of this paper is to use mathematical models to predict the population growth of India. India is a South Asian country and a most popular democracy in the world. It is banded by Indian ocean in the south, the Arabian sea on the south west, and the bay of Bengal on the south east. It borders with Pakistan to the west, China, Nepal and Bhutan to the north east and Myanmar (Burma) and Bangladesh to the east. In the Indian ocean, India is the vicinity or proximity of Shrilanka and the Maldives. India's Andaman and Nicobar Island share a maritime border with Thailand and Indonesia. India seventh-largest country by area, India is second – most populous country in the world, with over 1.2 billion people. The exponential and the logistic growth models were applied to model the population growth of India using data from 1970 to 2017. The exponential model predicated a growth rate of 2% per annum and also predicted the population to be 3.3491 billion in 2050. We determined the carrying capacity and the vital coefficients 0.0225 and 0.97 respectively. Thus the population growth of India, according to the logistic model is 5% and predicted India's population to be 6.1009 billion in 2050.



KEY WORDS: *Exponential growth model logistic growth models, differential equation, population mode MAPE, carrying capacity, vital coefficients.*

1 .INTRODUCTION

In The Economic Development of any country is population places a important role as well as in the decision making for the socio-economic and demographic development. Today the major issue of the world is the tremendous growth of the population especially in the developing countries like India. A population model is a type of mathematical model that is applied to the to the study of population dynamics. This paper reviews two population growth models and tries to find out a proper way to explain and predict population growth. It shows that two growth models that is exponential growth model and logistic growth models used to predict population growth. In this paper we also fours that the factor affecting population growth based on data available. As we know that India has a huge number of people, which has provided plenty of labor force which yield the development of the Indian economy. After our nations adopt the open economic policy in the year 1990's after that nation developing vigorously for more than 27 years. In some years we hope that India became one of the largest economy in the world. Such huge population base began to crub the farther advancement of India. The population advantage gradually turned to be a population disadvantage. [10] [12] [13] [14] [15] [16]

A present mathematical model is set of formulas or equations based on quantitative description or real world phenomenon and created in the hope that the conducts itself predicts that corresponds the real behavior on which the model is based (Glenn Ledder, 2005). It involves the following steps.

(1) The formulation of a real-word problem in mathematical expression: that is the formulation of mathematical model.

(2) The calculations, or solutions of the resulting mathematical model.

(3) The analysis of the mathematical results in the context of the original situation.

A model can be in many shapes, sizes and styles. It is important to emphasize that a model is not the real world, but exclusively a human contract to help us better understanding of the real-word system, particularly one uses models in all attitude of our life, in order to extract the important habitude from complex processes to permit similitude among systems to facilitate analysis of causes of process acting on the system and to make a attitude about the future. In this research article we compute the population growth of India using the exponential and the logistic growth models. The speed and the element of population growth to the present all-around situation come across the India's vast demand for continuous economic splendor.[1] [14] [15][17] [18]

2 A literature Review of population growth in India:

A research is best understood as a process of arriving aided actions to the problem through the systematic collection, scrutiny and analysis of data. In this research article secondary population data were taken from world development Indicator and Global Development finance- Google public data explore (http://www.worldometers.info/)

The exponential and logistic growth, mathematical models were used to compute the projected population values, calculating by using a calculator and Mape. The goodness of fit of the models is assessed closing the mean absolute percentage Error (MAPE.)

3 THE EXPONENTIAL GROWTH MODEL:

3.1 Exponential Growth Model and Differential equation.

Let us consider a special property of the function.

$$y = f(x) = e^x \quad ...(1)$$

$$\therefore \frac{dy}{dx} = e^x = y \quad ...(2)$$

This function satisfies the relationship

$$\frac{dy}{dx} = y \quad ....(3)$$

We call, this is a exponential differential equation because it associates one or more derivatives of a function with function itself.

Now we will discuss the significance of the above observation. Since most of the applications that we examine time dependent operations where t is the independent variables.[2] [5] [6] [7] [10]

Then we got following conclusion.

a) Let y be the function of time 't'.`

$$y = f(t) = e^t \qquad ...(4)$$

$$\frac{dy}{dt} = e^t = y \qquad ...(5)$$

The function $y = e^t$ satisfies the differential equation

b) Now let us consider

$$y = e^{kt} \qquad ...(6)$$

Then, using derivative of composite function

put $Kt = v$ and $y = e^v$

then we can find that

$$\frac{dy}{dt} = \frac{dy}{dv} \times \frac{dv}{dt} = e^v k = ke^{kt} = ky.$$

We observe that the function $y = e^{kt}$ satisfies the differential equation

$$\frac{dy}{dt} = ky \qquad ...(7)$$

c) If instead we take the function

$$y = e^{-kt} \qquad ...(8)$$

We can similarly show that the differential equation, it satisfies is

$$\frac{dy}{dt} = -ky \qquad ...(9)$$

d) If we take constant in front. We get the following function

$$y = 7e^{Kt}.$$

Then by simple differentiation and rearrangement we got

$$\frac{dy}{dt} = \frac{d}{dt}\left(7e^{Kt}\right)$$

$$\frac{dy}{dt} = 7\frac{d}{dt}e^{Kt} = 7Ke^{Kt} = Ke^{Kt}$$

$$\frac{dy}{dt} = ky \qquad ...(10)$$

e) We got the equation

$$y = ce^{Kt} \text{, c is any constant} \qquad ...(11)$$

Equation (11) is the algebraic equation and the differential equations have a solution for the above equation. From above, we conclude that *k* will be positive or negative exponent. This remark that two different types of behavior exponential growth and exponential delay above equation have. The graph for above expression represent the family of curves which represent a function that satisfies one of the differential equations.[5] [6] [7] [12]

We have the solution for the differential equation (11) at the initial value

The time t = 0 is $y(0) = y_0$

Where y_0 is some fixed value. This gives us a fixed value of the constant c in the desired solution

$$y(t) = ce^{Kt}$$

$$y(0) = ce^{K0} = ce^0 = c.1 = c$$

Famous Mathematical Economist Thomas R. Malthus prepared a mathematical model of population Growth in 1798. He proposed by the hypothesis that the population grows at a rate proportional to the size of the population. This is a rational hypothesis for a population of a bacteria or animal under ideal circumstances, i.e. infinite environment, satisfactory proportion of nutrients, absence of predators and privilege from disease.[2] [3] [5] [6] [7] [10] [11]

Assume that the population is P_0at some given time t = t_0 . And we are attending in calculated and getting the population p at some future time t = $t_n$in other words. We require to final a population function P(t) for to $t_0 \le t \le t_1$, satisfying

Then we assume the initial value problem.

$$\frac{dp}{dt} = Kp(t),\ t_0 \le t \le t_1,\ p(t_0) = p_0 \qquad ...(12)$$

Integrating equation 1. By using the variable separable method.

$$\therefore \int \frac{dp}{p} = K\int dt$$

$$\ln p = Kt + c, \quad p = e^{Kt+c}, \quad p = e^c e^{Kt}$$

$$\therefore p(t) = p_0 e^{Kt}$$

i.e. $k(t - t_0)$

$$p(t) = p_0 e^{Kt} \qquad ...(13)$$

Where K is a constant called the Malthus factor.
This is the multiple that determines the growth.
Equation (12) is the exponential growth model and equation (13) It is a solution of the equation (12).

Equation (13) is the differential equation and it contents an unknown function p and its derivative. $\frac{dp}{dt}$.

After formulating the models, we now look at its significance
If we eliminate significance a population of 0 Then $p(t) > 0$. For all t.

So if K>o. Then equation shows that $\frac{dp}{dt} > 0$, for all t.

It means that the population is always increasing in case as $P(t)$ increases, equation (12), is assigned for modeling population growth under ideal conditions, Thus we have to organize that have more realistic must reflect the fact a given environment has a limited resource.[7] [8] [11]

4 THE LOGISTIC GROWTH MODEL:-

The logistic growth model was prepared by the Belgian mathematical biologist Verhulst in the year 1840 as the model for world population growth. The logistic growth model integrated the concept of carrying capacity. Thus the population growth not only on how to depend on the population size but also on how for this size is from the its upper limit i.e. maximum supportable population. A Mathematical biologist modified the Malthus model to make a population size proportional to both the previous population and new term[2] [5] [9] [10] [11]

$$p = \frac{a - bp(t)}{a} \qquad ...(14)$$

Where a and b are the vital coefficients of the population. This term describes how for the population is from its maximum limit. Now, as the population value gets closed to $\frac{a}{b}$, this new term will become very small and tend to zeros providing the right feedback to limit the population growth. Thus the second term models the competition for available resources. This tends to limit the population growth. So the modified equation using this term is new .

$$\frac{dp}{dt} = \frac{ap(t)(a-bp(t))}{a} \qquad ...(15)$$

Where $t_0 \le t \le t_1$, p $(t_0) = p_0$

This equation is known as the logistic low of population growth. Solving equation 4.
Applying the initial condition, equation (15)becomes

$$\frac{dp}{dt} = ap - bp^2 \qquad ...(16)$$

in which is a linear differential equation (15).

To solve differential Equation (15) we can integrate above equation by variable separable from.

$$\int \frac{dp}{dt} = \int ap - bp^2$$

$$\therefore \int \frac{dp}{dt} = \int p\,(a - bp)$$

$$\therefore \int \frac{dp}{p(a-bp)} = \int dt. \qquad ...(17)$$

To integrate above equation we can use partials fraction method.

$$\therefore \frac{1}{p\,(a-bp)} = \frac{x}{p} + \frac{y}{(a-bp)}$$

$$\frac{1}{P(a-bp)} = \frac{x(a-bp)+yp}{p\,(a-bp)}$$

By equality, we can write

X (a-bp) +yp = 1 ...(18)

To find the value of x we can write

P = 0 in equation (18).

$\therefore x$ (a-0) + 0 = 1

$\therefore x = \frac{1}{a}$

Now, put p = $\frac{a}{b}$ in equation (18) ...(19)

$\therefore x$ (0) + $y\left(\frac{a}{b}\right)$ = 1.

$\therefore y = \frac{b}{a}$

$\therefore$ equation (17) becomes

$$\therefore \int \left(\frac{1}{a}\left(\frac{1}{p}\right) + \frac{b}{a}\left(\frac{1}{a-bp}\right)\right) dp = \int dt$$

$$\therefore \int \frac{1}{a}\left(\frac{1}{p} + \frac{b}{a-bp}\right) \text{dp} = \int dt$$

$$\therefore \frac{1}{a}\left(\frac{1}{p} + \frac{b}{a-bp}\right) \text{dp = dt}$$

$$\therefore \frac{1}{a}(\ln p - \ln(a - bp)) = \text{t + c} \qquad ...(20)$$

At t= 0 and p = p_0

C = $\frac{1}{a}(\ln p_0 - \ln(a - bp_0))$

Substituting c in equation 9. And solving for p yields.

$$P = \frac{\frac{a}{b}}{1+\left(\frac{\frac{a}{b}}{p_0}-1\right)e^{-at}} \quad ...(21)$$

Now taking the limit as t→ ∞ of equation 10.

$$P_{max} = \lim_{t\to\infty} P = \frac{a}{b} \quad (k>0) \quad ...(22)$$

Putting t= 1 and t=2 the values of p are p, and p_2 respectively. Them we obtain from 10. The following. [10]

$$\frac{b}{k}(1-e^{-a}) = \frac{1}{n_1} - \frac{e^{-a}}{n_0} \quad ...(23)$$

$$\frac{b}{k}(1-e^{-2a}) = \frac{1}{n_2} - \frac{e^{-2a}}{n_0} \quad ...(24)$$

Dividing equation 13. By 12. We get.

$$1 + e^{-a} = \frac{\frac{1}{p_2}-\frac{e^{-2a}}{p_0}}{\frac{1}{p_1}-\frac{e^{-a}}{p_0}} \quad ...(25)$$

Hence solve for e^{-a} we have

$$e^{-a} = \frac{p_0(p_2-p_1}{p_2(p_1-p_2} \quad ...(26)$$

Substituting e^{-a} into the equation 12. We get

$$\frac{b}{a} = \frac{p_1^2 - p_0p_2}{p_1(p_0p_1 - 2p_0p_1 + p_1p_2)} \quad ...(27)$$

Therefore the limiting value of p is given by

$$P_{max} = \lim_{t\to\infty} P = \frac{a}{b} = \frac{p_1(p_0p_1 - 2p_0p_1 + p_1p_2)}{p_1^2 - p_0p_2} \quad ...(28)$$

5 MEAN ABSOLUTE PERCENTAGE ERROR:

Mean absolute a percentage error is the method of evaluation of data which is used to access the goodness of fit of different models in national and substantial population projections. This data is expressed in percentage. The concept of mean absolute percentage error (MAPE) seems to be the very elementary but of numerous significance in the selecting a parsimonious model than the other data. A model with smaller is MAPE is preferred to the other models.

The mathematical model form of MAPE is given under

$$\text{MAPE} = \frac{1}{n}\sum_{t=0}^{n}\left|\frac{y_t - \bar{y}}{y_t}\right| \times 100 \qquad ...(29)$$

Where y_t - Actual observation of population

$\bar{y}$ - Fitted observation of population

n - Number of observations of population

Lower MAPE values are better because they indicate that smaller percentage errors are produced by the prediction model. The following interpretation of MAPE values was suggested by Lewis (1982) as follows: Lewis mentioned that if MAPE values less than 10% is a highly accurate prediction 10% to 20%is good predication 21% to 50% is reasonable prediction and 51% and above is inaccurate prediction.[1] [2] [3] [5]

6 RESULT AND DISCUSSION :

To estimate the future population of India, we need to determine the growth rate of India using the Exponential Growth model in (2). Using the actual population of India in Billion on table 1 below with $t = 0$ corresponding to the year 1970, we have $p_0 = 0.5536$. We can solve for the growth rate

k , the fact that $p_1 = 0.5662$, when t = 1

$$0.5662 = 0.5536e^{k}$$

$$k = ln\left(\frac{0.5662}{0.5536}\right)$$

$$k = 0.0225$$

Hence the general solution

$$p(t) = 0.5536e^{0.0225t} \qquad ...(30)$$

This suggests that the predicted rate of India population growth is 2% with the Exponential growth model. With this we projected the population of India to 2050.

Again, based on table 1, let t = 0,1 and 2 correspond to the year 1970, 1971 and 1972 respectively. Then p_0, p_1 and p_2 also correspond 0.5536, 0.5662 and 0.5790

Substituting the values of p_0, p_1 and p_2 into (17) we get $p\max = \frac{a}{b}$165.7282951707. This is the predicted carrying capacity of the population of India.

From equation (15) , we obtain e^{-a} = 0.97 hence

$$a = -\ln(0.97)$$

Therefore the values of a = 0.030. This also implies that the predicted rate of India population growth is approximately 3% , with the logistic growth model from $\frac{a}{b} = 165.7282951707$ and euation (29) we obtained b = 18.101×105

Substituting the values of p_0, e^{-a} and $\frac{a}{b}$ into equation (10) we obtain

$$P(t) = \frac{165.7282951707}{1 + (299.3647)0.97^{t}}$$

As the general solution and we use this to predict population of India to 2050. The predicted populations of India with both models are presented on the table 1 below.

7. Table 1. Projection of India's Population using Exponential and Logistic Growth Models

Year	***Actual Population (in Billions)***	***Projected Population (in Billions)***	
		Exponential Model	**Logistic Model**
1970	0.5536	0.5536	0.5536
1971	0.5662	0.5662	0.5688
1972	0.5794	0.5790	0.5741
1973	0.5931	0.5925	0.6043
1974	0.6071	0.6057	0.6229
1975	0.6213	0.6195	0.6421
1976	0.6358	0.6336	0.6579
1977	0.6505	0.6480	0.6823
1978	0.6655	0.6628	0.7034
1979	0.6809	0.6779	0.7250
1980	0.6968	0.6932	0.7473
1981	0.7131	0.7091	0.7703
1982	0.7298	0.7251	0.7941
1983	0.7469	0.7417	0.8185
1984	0.7642	0.7586	0.8437
1985	0.7817	0.7758	0.8695
1986	0.7992	0.7939	0.8963
1987	0.8169	0.8115	0.9239
1988	0.8345	0.8300	0.9524
1989	0.8523	0.8489	0.9816
1990	0.8701	0.8682	1.0180
1991	0.8880	0.8880	1.0428
1992	0.9060	0.9082	1.0749
1993	0.9240	0.9288	1.1079
1994	0.9422	0.9499	1.1420
1995	0.9605	0.9716	1.1770
1996	0.9789	0.9937	1.2131
1997	0.9974	1.0163	1.2504
1998	1.0160	1.0394	1.2888
1999	1.0345	1.0631	1.3284

2000	1.0531	1.0873	1.3691
2001	1.0715	1.1120	1.41105
2002	1.0898	1.1373	1.4543
2003	1.1080	1.1632	1.4988
2004	1.1261	1.1897	1.5448
2005	1.1441	1.2168	1.5919
2006	1.1620	1.2444	1.6410
2007	1.1797	1.2728	1.6912
2008	1.1971	1.3017	1.7428
2009	1.2143	1.3313	1.1760
2010	1.2310	1.3616	1.8512
2011	1.2472	1.3926	1.9080
2012	1.2631	1.4243	1.9663
2013	1.2786	1.4567	2.0260
2014	1.2939	1.4899	2.0879
2015	1.3091	1.5237	2.1520
2016	1.3242	1.5584	2.2175
2017	1.3392	1.5939	2.2853
2018		1.6302	2.3543
2019		1.6672	2.4265
2020		1.7052	2.4999
2021		1.7440	2.5767
2022		1.7837	2.6542
2023		1.8243	2.7359
2024		1.8658	2.8181
2025		1.9083	2.9038
2026		1.9517	2.9933
2027		1.9961	30834
2028		2.0451	3.1772
2029		2.0880	3.1772
2030		2.1355	3.2730
2031		2.1840	3.4743
2032		2.2338	3.5799
2033		2.2846	3.6872
2034		2.3366	3.7985
2035		2.3897	3.9140
2036		2.4441	4.0338
2037		2.4997	4.1548
2038		2.5566	4.2801
2039		2.6115	4.4062
2040		2.6743	4.5399
2041		2.7352	4.6780
2042		2.7974	4.8164
2043		2.8601	4.9632
2044		2.9261	5.1098
2045		2.9927	5.2653
2046		3.0771	5.4199

2047		3.1305	5.5783
2048		3.2017	5.7462
2049		3.2746	5.4182
2050		3.3491	6.1009

Fig. 1. Depicts that from 1970 the population of India has increased throughout. This may be attributed to the improvement in the education, agricultural productively, water and sanitation and health services. There was a belief in India that the more children one had, one would have a higher social and economic status, have higher work force in their farms and receive better care in old age. This coupled with other factors had an overall effect on the increase in population. The exponential model predicted India's population to be 3.3491 Billion in 2050 whereas the Logistic model projected it to be 6.1009. This in presented on figure 2

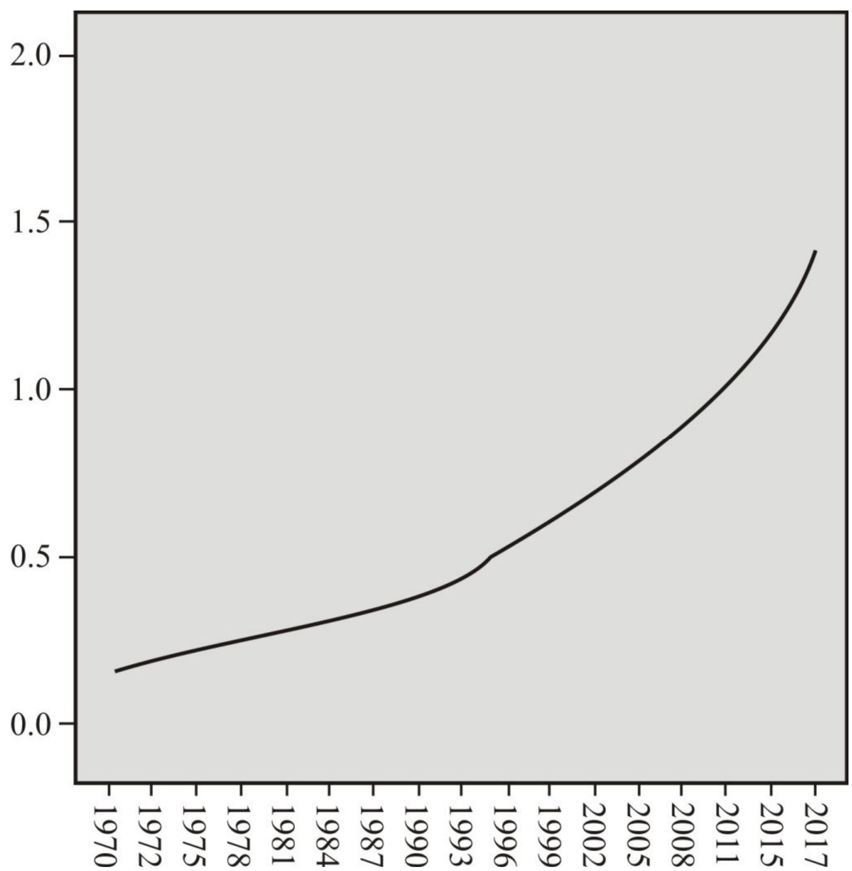


Fig. 1: Graph of actual population from 1970 to 2017

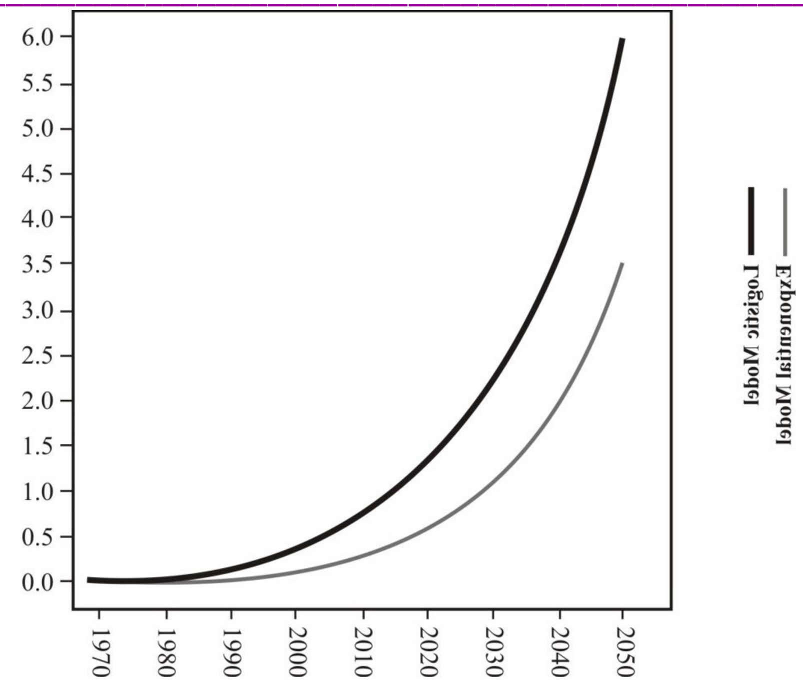


Fig 2: Graph of predicted population values of India

Figure 2 above show the graph of the predicted population of India with both models. The Logistic Model is in blue and it deviate far from the actual population. The lovercurve represent the forecast of the exponential model which is quiet similar to the actual population graph.

8. CONCLUSION:

Evidence from time series analysis indicates that, two factors the degree of carbonization and the Sex ratio, hence significant influences and population growth of India.

Present study not only setting up on efficient mathematical model for population growth Indian inspired theoretical research successfully but also be beneficial for the government to make corresponding population and economic police accordingly. We will be hopefuls that the study will be a powerful for booster to proper the Indian economy.

In conclusion the exponential Model predicted an growth rate of approximately 2% and predicted India's population to be 3.3491 Billion in the year 2050. The 165.7283. Population growth of any country depends on the vital coefficients. Here we found out that the vital coefficients a and b are a = 0.030 and b = 18.1019×10^{-5} respectively. Thus the population growth rate of India according to this model is approximately 5% per annum. It also predicted the population of India to be 6.1009 Billion in 2050. we can conclude that the Exponential Model gave a good forecasting result as compared to the Logistic model.

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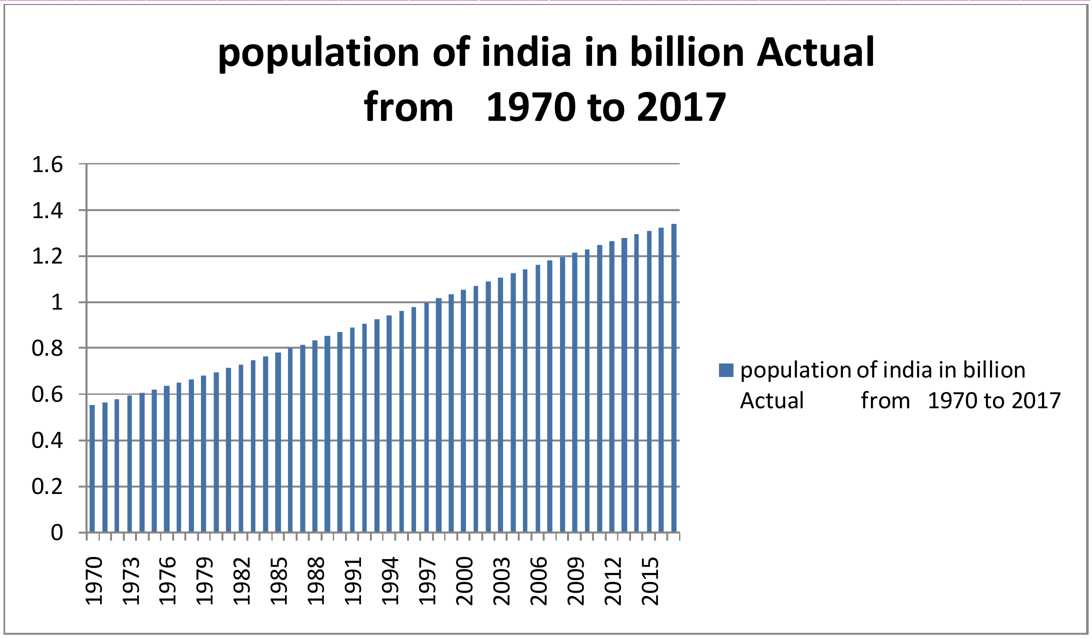
population of india in billion Actual from 1970 to 2017
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1970
1973
1976
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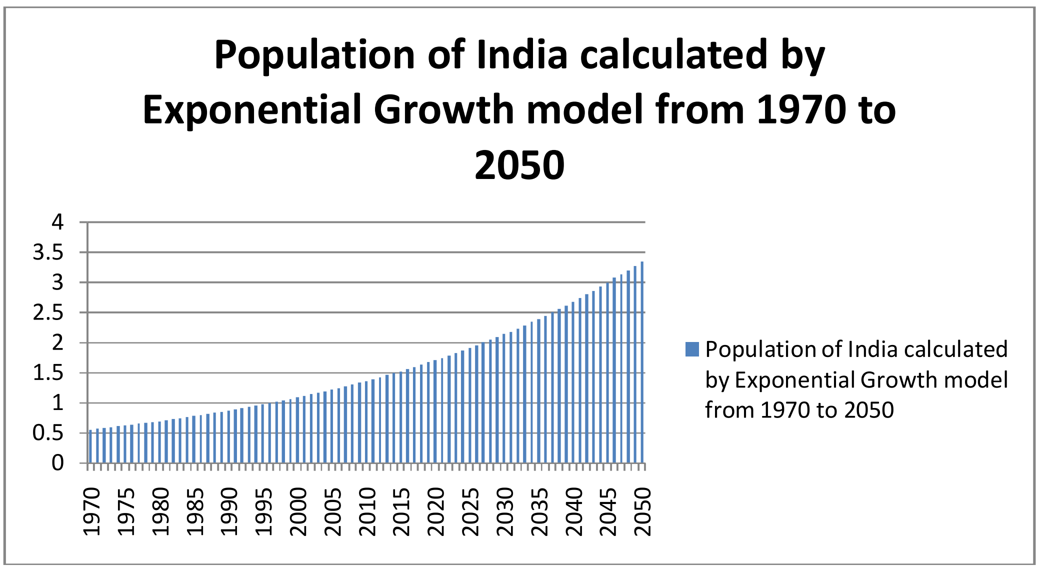
population of india in billion Actual from 1970 to 2017


Population of India calculated by Exponential Growth model from 1970 to 2050
4
3.5
3
2.5
2
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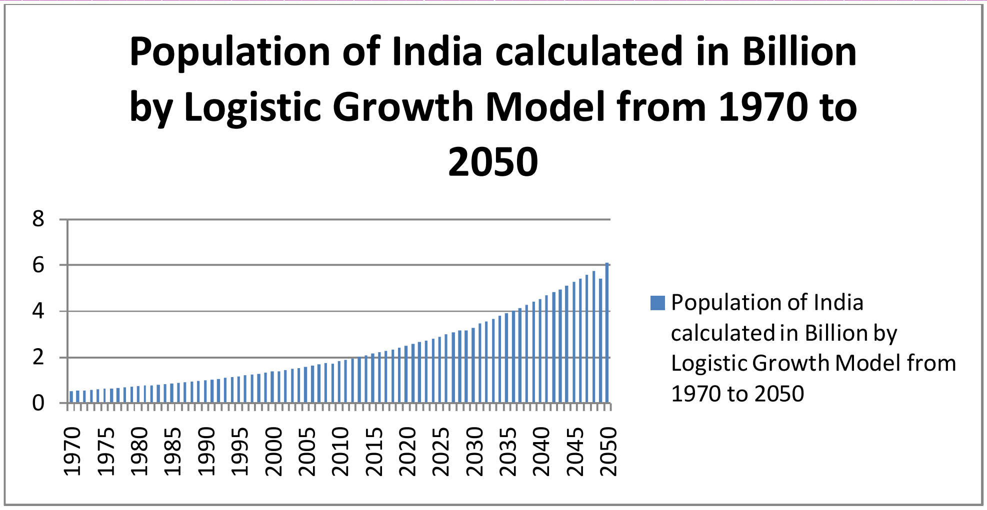
Population of India calculated by Exponential Growth model from 1970 to 2050


Population of India calculated in Billion by Logistic Growth Model from 1970 to 2050
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1970 1975 1980 1985 1990 1995 2000 2005 2010 2015 2020 2025 2030 2035 2040 2045 2050
Population of India calculated in Billion by Logistic Growth Model from 1970 to 2050