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AN INVESTIGATION OF COMPLETE ORDERED FIELD

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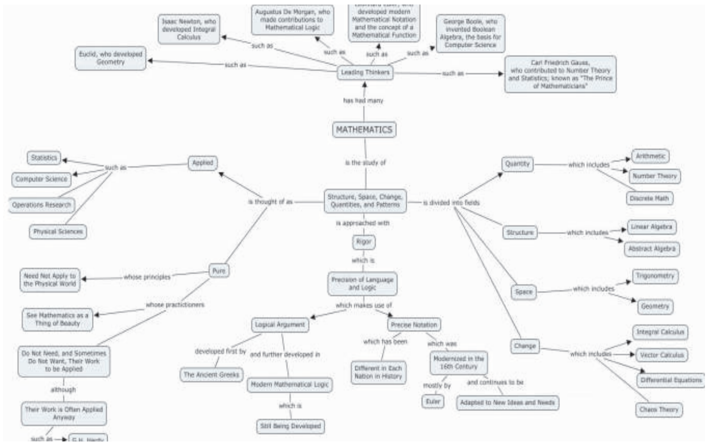
ABSTRACT

In this paper we will investigate which of the Number system say (N, I, R, Q, R-Q, C) perform field and if it is field then either it is complete ordered field or not. For this we will use field structure, ordered structure, complete set, bounded set, bounded above and bounded below.

Here, N stands for set of Natural Numbers.
I stands for set of Integers.
Q stands for set of Rational Numbers.
R-Q stands for set of Irrational Numbers.
R stands for set of Real Numbers.
C stands for set of Complex Numbers.



KEYWORDS : *Complete Ordered Field , ordered structure, complete set, bounded set.*

INTRODUCTION :

In this paper we examine the field which is complete ordered field. Before that we give some definitions like algebraic structure, field structure, ordered structure etc.

Algebraic Structure: A number system is said to be an algebraic structure if it is closed w.r.to addition and multiplication.

Table 1.1 reported the Number system which are Algebraic structure or not.

Table 1.1: Representation of Number System

Number System	Closed w.r.t. Addition	Closed w.r.t. X	Algebraic Structure
Natural Number [N]	√	√	√
Integer [I]	√	√	√
Rational Number [Q]	√	√	√
Irrational Number [R-Q]	√	X	X
Real Number [R]	√	√	√
Complex Number [C]	√	√	√

- Set of Irrational numbers cannot be Algebraic structure because of

if√ 2√, 2? R-Q

then √ 2 * √ 2 = 2 not ?R-Q

not closed w.r.t. multiplication.

- **Field structure:-** A number system is perform a field structure if it satisfy the following properties

If R is a number system then

- a+b ?R V a, b ?R [closure property]
- a+[b+c] = [a+b]+c V a,b,c ?R [associative property]
- V a ?R there exist an element say 0 ?R such that
- a+0 =a = 0+a [existence of additive identity]
- For each a ?R and a unique element b ?R such that
- a+b=0=b+a [existence of additive inverse]i.e. b=-a
- Va,b ?R we have a+b = b+a [commutative property]
- V a,b ?R, a*b ?R [closure property w.r.t. multiplication]
- V a,b,c ?R, a*[b*c] = [a*b]*c [associative property w.r.t. multiplication]
- For each Va ?R and 1 ?R such thata*1 = 1*a = a [existence of multiplicative identity]
- For each 0 ≠a ?R and b ?R such that a-b=1=b-a [existence of multiplicative inverse]
- V a,b ?R we have a*b = b*a [commutative property w.r.t. multiplication]
- a*[b+c] = a*b +a*c V a,b,c ?R

Some examples of Number system which are field structurei.e field structure contains three properties mainly

- Additive abelian group.
- All non-zero numbers perform abelian group w.r.t. multiplication.
- Distributive property

Ordered Structure:A Number system is said to perform ordered structure if it satisfy given below four properties.

If A is any number system

- Law of Trichotomy: If a,b ?A then either a<b or b<a or a=b.
- For example: 2,3 ?I then 2<3

- Law of Transitivity: If a,b,c ?A and a<b and b<c then a should be less than c.
- Ordered compatibility w.r.to multiplication:- If a,b,c ? A then if a<b and c>0

Then ac<bc.
Table 1.2 reported the Number system which is field structure or ordered structure.

Table 1.2: Number System with different structure

Number System	Additive Abelian group	Multiplicative Abelian Group	Field Structure	Ordered Structure
Natural Number [N]	X (No natural number has additive inverse)	X (Multiplicative identity not exist)	X	√
Integer [I]	√	X(No non-zero integer has multiplicative inverse)	X	√
Q	√	√	√	√
R-Q	√	X(Not closed w.r.t. multiplication)	X	√
Real Number [R]	√	√	√	√
C	√	√	√	X (Complex number are not compatible)

- Bounded above Set- A set B is bounded above if there exist a real number x Ɛ R such that

a ≤x V aƐ B.
- A set is said to be unbounded above if no such x exist.
- A set B is said to be bounded below if there exist a real number x ? R such that a ≥x V a
- B.
- If no such x exists then B is unbounded below.
- Least upper bound: An upper bound is said to be least upper bound if it is smallest in all upper bound.

Greatest Lower bound: A lower bound is said to be greatest lower bound if it is greatest in all lower bounds.
- Least upper bound of a set also known as supremem of the set and the greatest lower bound also known by infimum of the set.

Table 1.3 reported the Number system bounded above/bounded below.

Table 1.3: Number System Bounded Above/Bounded Below

Number System	Bounded Above	Bounded Below
N	X	√
I	X	X
Q	X	X
R-Q	X	X
R	X	X
Finite Set	√	√

Bounded set: A set is bounded if it is bounded above as well as bound below.Complete structure: A number system is said to be complete if every subset of the system which is bounded above/ below has its sup/inf in the system.

Examples of Number systems which are complete.
Q – Set of Rational numbers is not complete.
For example if we take subset
A= {x e Q/ 1<x<e} of Q Then its sup is 'e' which is not Rational number.
R-Q: Set of Irrational numbers also not complete because if we take
A={x e R-Q\ 1<x<2} Sup is 2 which is not irrational number.
N: Set of Natural number is complete.

For example:
A={x e N\ $\sqrt{\ }$ 2x≤π^2 }
Then Inf is – 2
Supremum- 9
Both are natural numbers.Same as set of integers is complete.
Set of complex numbers is not complete because it is not compatible.
Set of Real numbers every non-empty subset which is bounded above has the supremum or bounded below has the infimum the number system R.
Hence, R is also complete.
Hence, from Table 1.2 and set of Real numbers i.e R is a ordered field
And also R is a complete number system.
Hence R is the only complete ordered field.

CONCLUSION

As complete ordered field is that Number system which is field, ordered set and complete set. But N, I, R-Q, Q are not field. So, they are not complete ordered field. Q is a field, ordered set but not complete hence not complete ordered structure. C is not compatible so not a ordered structure. Hence not a complete ordered structure. R is a field, ordered set which is also complete. Therefore, R is a complete ordered set.

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