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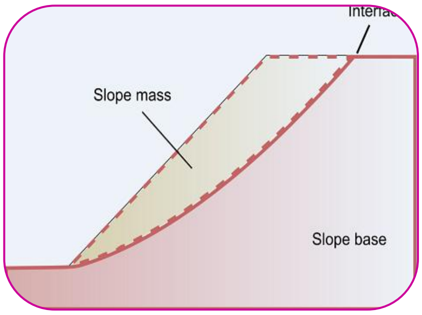
REVIEW OF RESEARCH

# APPLICATION OF DYNAMIC PROGRAMMING TO SLOPE STABILITY ANALYSIS

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**ABSTRACT:**

*The applicability of the dynamic programming method to two-dimensional slope stability analyses is studied. The critical slip surface is defined as the slip surface that yields the minimum value of an optimal function. The only assumption regarding the shape of the critical slip surface is that the surface is an assemblage of linear segments. Stresses acting along the critical slip surface are computed using a finite element stress analysis. Assumptions associated with limit equilibrium methods of slices related to the shape of the critical slip surface and the relationship between interslice forces are no longer required.*

**KEY WORDS:** *dynamic programming, slope stability, stress analysis, optimization theory, limit equilibrium methods of slices.*

## 1. INTRODUCTION

A conventional slope stability analysis involving limit equilibrium methods of slices consists of the calculation of the factor of safety for a specified slip surface of predetermined shape and the determination of the location of the critical slip surface with the lowest factor of safety. To render the inherently indeterminate analysis determinate, conventional limit equilibrium methods generally make use of assumptions regarding the relationship between the interslice forces. These assumptions become disadvantages to limit equilibrium methods, since the actual stresses acting along the slip surface are quite approximate and the location of the critical slip surface depends on the shape assumed by the analyst.

The assumptions related to the interslice force function in limit equilibrium methods are unnecessary when a finite element stress analysis is used to obtain the normal and shear stresses acting at the base of slices (Fredlund and Scoular 1999). A stress analysis provides normal and shear stresses through the use of the finite element numerical method with a *switch on* of the gravity forces. Subsequently, the equation for the factor of safety becomes linear. Assumptions regarding the uncertainty of the shape of the critical slip surface can be omitted when an appropriate optimization technique is introduced into the analysis.

Optimization techniques have been developed by several researchers for over two decades and have provided a variety of approaches to determine the shape and location of the critical slip surface (Celestino and Duncan 1981; Nguyen 1985; Chen and Shao 1988; Greco 1996). Each approach has its own advantages and shortcomings. The main shortcoming associated with these approaches, however, is that the actual stresses within a slope are quite approximate. This disregard for a more accurate assessment of the stresses can lead to inaccuracies in the computation of the factor of safety and an inability to analyze more complex problems.

The dynamic programming method can be combined with a finite element stress analysis to provide a more complete solution for the analysis of slope stability because the technique overcomes the primarily difficulties associated with limit equilibrium methods. The disadvantage of the dynamic programming approach is that there are more variables to specify for the analysis, such as Poisson's ratio and the elastic moduli of the soils involved.

The dynamic programming method for a slope stability analysis has not been widely used in engineering practice primarily because of the complexity of the formulation and the lack of verification of the computed results. Baker (1980) introduced an optimization procedure that utilized the algorithm of the dynamic programming method to determine the critical slip surface. In this approach, the associated factors of safety were calculated using the Spencer (1967) method of slices. Yamagami and Ueta (1988) enhanced Baker's approach by combining the dynamic programming method with a finite element stress analysis to more accurately calculate the factor of safety (Fig. 1). The critical slip surface was assumed to be a chain of linear segments connecting two *state* points located in two successive *stages*. The resisting and the actuating forces used to calculate an *auxiliary function* were determined from stresses interpolated from Gaussian points within the domain of the problem. Yamagami and Ueta analyzed two example problems to illustrate the proposed procedure.

Zou et al. (1995) proposed an improved dynamic programming technique that used essentially the same method as that introduced by Yamagami and Ueta (1988). The modification made by Zou et al. was that the critical slip surface might contain a segment connecting two *state* points located in the same *stage*. The stability of a trial dam in Nong Ngu Hao, Bangkok, Thailand, was analyzed as part of the study of the proposed procedure.

The objective of this research program is to study the use of the dynamic programming method in solving practical slope stability problems. The analytical procedure behind the dynamic programming method is mainly based on the research of Yamagami and Ueta (1988). A computer program named DYNPROG was developed to interface with a general partial differential equation solver known as FlexPDE (PDE Solutions Inc. 2001) to determine the stress states in the soil mass and then determine the shape and location of the critical slip surface and the corresponding factor of safety. Numerous example problems have been solved using DYNPROG. Examples studied include homogeneous slopes, layered slopes, and a case history. The results obtained from the analyses were compared with results from several wellknown limit equilibrium methods of slices (Fredlund and Krahn 1977).

## 2. BACKGROUND

Bellman (1957) introduced a mathematical method called the dynamic programming method. One of the objectives of the dynamic programming method was to maximize or minimize a function. The dynamic programming method has been widely used in various fields other than geotechnical engineering. Baker (1980) appears to be the first to apply the optimization technique in the analysis of the stability of slopes.

## 2.1 Definition of the factor of safety

For an arbitrary slip surface AB, as shown in Fig. 2, the equation for the factor of safety can be defined as

$$F_s = \frac{\int_A^B \tau_f \, dL}{\int_A^B \tau \, dL} \qquad [1]$$

where τ is the mobilized shear stress along the slip surface, $\tau_f$ is the shear strength of the soil, and d$L$ is an increment of length along the slip surface. It is assumed that the critical slip surface can be approximated by an assemblage of linear segments. Each linear segment connects two *state* points located in two successive *stages*. The *stage–state* system forms a grid consisting of rectangular elements called the *search grid*. The rectangular elements formed by the search grid are called *grid elements*. In this discretized from, the overall factor of safety for the slip surface AB is defined as follows:

$$F_s = \frac{\sum_{i=1}^{n} \tau_{f_i} \Delta L_i}{\sum_{i=1}^{n} \tau_i \Delta L_i} \qquad [2]$$

where $n$ is the number of discrete segments, $\tau_i$ is the shear stress actuated, $\tau_{fi}$ is the shear strength, and $\Delta L_i$ is the length of the segment.

## 2.2 Theory of the dynamic programming method

A minimization is necessary for the value of the factor of safety, $F_s$, in eq. [2]. It was shown by Baker (1980) that the minimum of $F$s in eq. [2] can be found by using an *auxiliary function G*. The auxiliary function is also known as the *return function*, and it can be defined as follows (Fig. 3):

$$G = \sum_{i=1}^{n} (R_i - F_s S_i) \qquad [3]$$

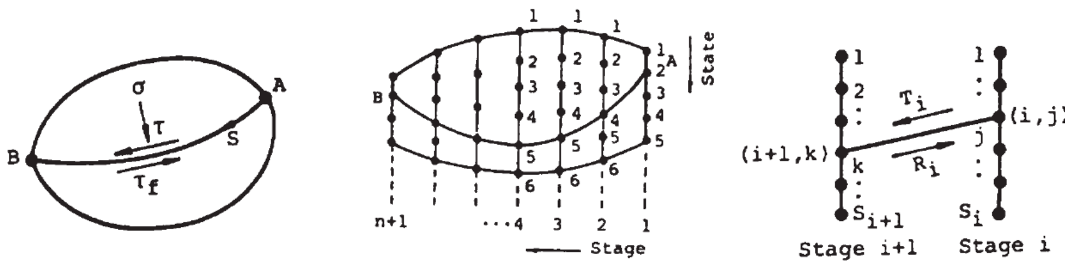


**Fig. 1. Search for the critical slip surface based on the dynamic programming method (after Yamagami and Ueta 1988).**

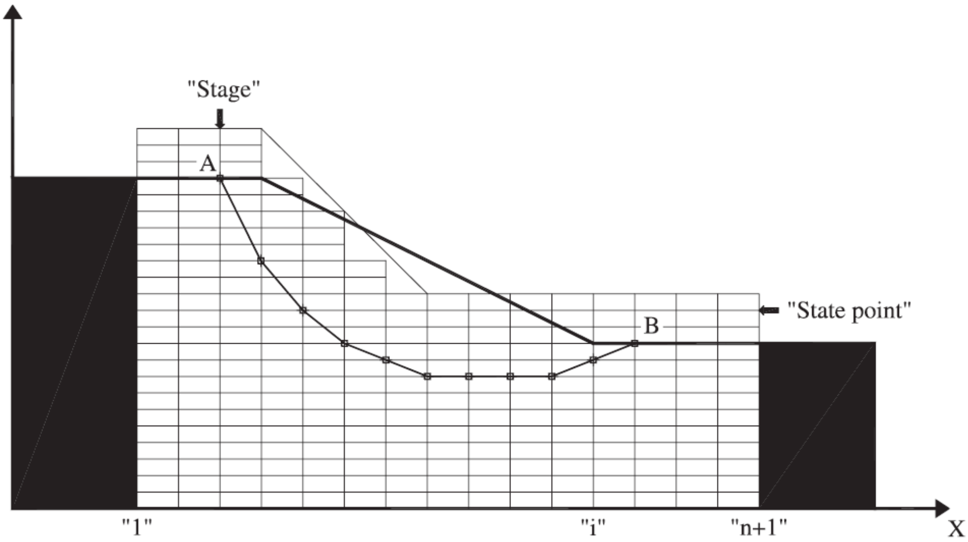


**Fig. 2. An arbitrary surface AB in a discretized form.**

where $S_i$ are actuating forces acting on the $i$th segment of the slip surface, $R_i$ are resisting forces acting on the $i$th segment of the slip surface, and $n$ is the total number of discrete segments making up the slip surface.

The minimum value of the auxiliary function is $G_{\mathrm{m}}$ and is defined as

$$G = \min \sum_{i=1}^{n} (R_i - F_s S_i) \qquad [4]$$

Along the $i$th segment, the shear strength for a saturated– unsaturated soil can be calculated using the following equation (Fredlund and Rahardjo 1993):

$$\tau_{f\,i} = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \quad [5]$$

where c', ϕ' and $\phi^b$ are the shear strength parameters of a saturated–unsaturated soil; $(\sigma_{\mathrm{n}} - u_{\mathrm{a}})$ is the net normal stress acting on the $i$th segment; and $(u_{\mathrm{a}} - u_{\mathrm{w}})$ is the matric suction.

The normal and shear stresses acting on the $i$th segment can be computed from a stress analysis as follows:

$$\sigma_n = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - \tau_{xy} \sin 2\theta \qquad [6]$$

$$\tau_n = \tau_{xy}(\sin^2\theta + \cos^2\theta) - \frac{(\sigma_y - \sigma_x)}{2} \sin 2\theta \qquad [7]$$

where $\sigma_{\mathrm{n}}$ and $\tau_{\mathrm{n}}$ are the normal and shear stresses acting on the $i$th segment, respectively; θ is the inclined angle of the $i$th segment with the horizontal direction; and $\sigma_x$, $\sigma_y$, and $\tau_{xy}$ are the normal and shear stresses acting in the $x$- and $y$-coordinate directions. These stresses can be determined using a finite element stress analysis that uses any particular soil behaviour model. If the density of the search grid is sufficiently fine, it can be assumed that stresses are constant within a small grid element. These constant stresses are signified by stresses at the centre points of the grid element. Consequently, the resisting and actuating forces acting on the *i*th segment of a slip surface can be calculated as follows (Fig. 3):

$$R_i = \sum_{ij=1}^{ne} R_{ij} = \sum_{ij=1}^{ne} \tau_{f_{ij}} l_{ij} = \sum_{ij=1}^{ne} [c'_{ij} + (\sigma_n^{ij} + u_a)\tan\phi'_{ij} + (u_a + u_w)\tan\phi_{ij}^b] l_{ij} \quad [8]$$

$$S_i = \sum_{ij=1}^{ne} S_{ij} = \sum_{ij=1}^{ne} \tau_{ij} l_{ij} \quad [9]$$

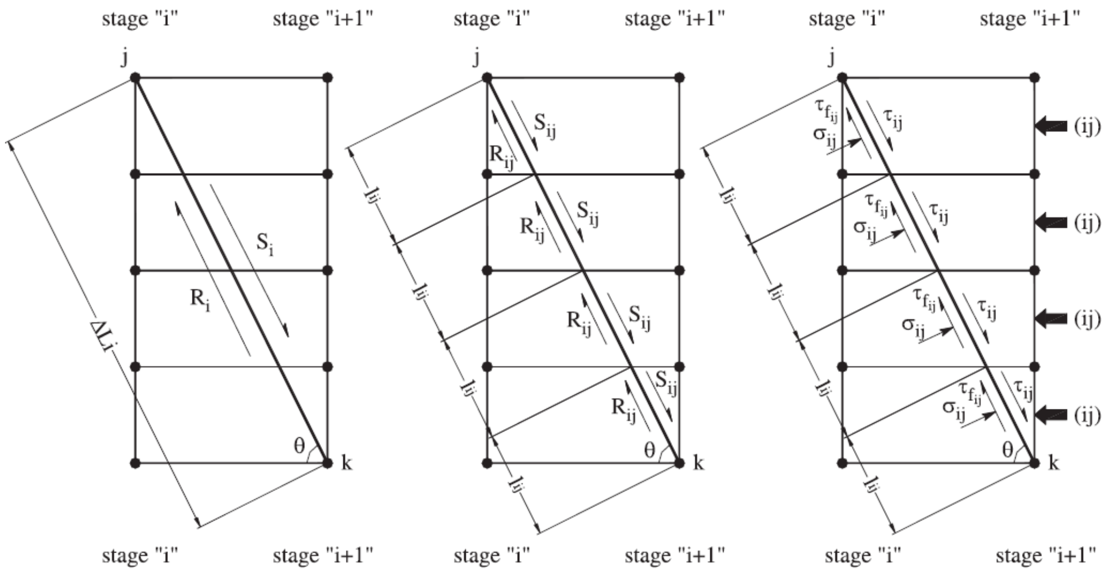


**Fig. 3. Actuating and resisting forces acting on the *i*th segment.**

where (*ij*) is a grid element travelled by the *i*th segment; $\tau_{fij}$ and $\tau_{ij}$ and $\tau_{ij}$ are the shear strength and shear stress actuated at the centre point of (*ij*), respectively; $c'_{ij}$, $\phi'_{ij}$, and $\phi^b_{ij}$ are the strength parameters of the saturated–unsaturated soil within (*ij*); ne is the number of (*ij*); and $l_{ij}$ is the length of the *i*th segment limited by the boundary of (*ij*).

An *optimal function*, $H_i(j)$, obtained at state point {*j*} located in stage [*i*] is introduced. The optimal function, $H_i(j)$, is defined as the minimum of the *return function*, *G*, calculated from a state point for the initial stage to state point {*j*} located in stage [*i*]. According to *the principle of optimality* (Bellman 1957), the *optimal function*, $H_{i+1}(k)$, obtained at state point {*k*} located in stage [*i* + *1*] is defined as

$$H_{i=1}(k) = H_i(j) + G_i(\overline{j,k}) \quad [10]$$

where $G_i(\overline{j,k})$ is the *return function* calculated from state point {*j*} of stage [*i*] to state point {*k*} of stage [*i* + *1*].

The *optimal point* in the final stage is defined as the state point at which the calculated optimal function is a minimum. From the optimal state point {*k*} found in the final stage, the optimal state point {$j$} located in the previous stage is also determined. The *optimal path* defined by connecting optimal state points located in every stage is eventually found by tracing back from the final stage to the initial stage. This optimal path defines the *critical slip surface*.

The value of the overall factor of safety, $F_s$, in eq. [3] has not been defined in advance and therefore an initial value must be assumed. The trial value of $F_s$ is updated using the value of $F_s$ evaluated after each trial of the search. The optimization process will stop when a predefined convergence is reached.

## 2.3 Finite element stress analysis using FlexPDE

The general partial differential equation solver known as FlexPDE is a flexible computer program that can be used to solve single or coupled sets of partial differential equations. FlexPDE allows the user to pose a problem in a compact problem-oriented form and proceed directly to a graphical presentation of the solution, without digressing to program the finite element method.

For the plane strain condition (i.e., strain in the $z$-coordinate direction $\varepsilon_z = 0$), a soil element subjected to its body forces has partial differential equations representing the stress balance defined as follows:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + F_x = 0$$

$$\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \sigma_y}{\partial x} + F_y = 0$$

where $\sigma_x$ and $\sigma_y$ are normal stresses in the Cartesian $x$- and $y$-coordinate directions, respectively; $\tau_{xy}$ is the shear stress in the $xy$ plane; and $F_x$ and $F_y$ are body forces in the $x$- and $y$–coordinate directions, respectively.

Partial differential eqs. [13] and [14] can be solved using FlexPDE along with specified boundary conditions. The domain of the problem is automatically divided by the computer program, FlexPDE, into triangular elements. The variables are presented by a simple polynomial equation over the problem domain. FlexPDE uses a Galerkin finite element model, with quadratic- or cubic-based functions involving nodal values of system variables.

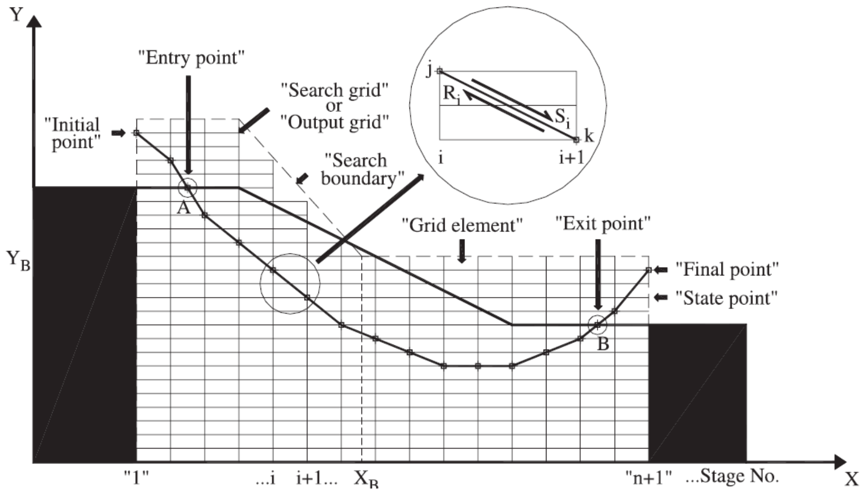


**Fig. 4. The analytical scheme of the dynamic programming method in slope stability analyses.**

The stresses are evaluated and stored at Gaussian points over the domain of the problem when solving eqs. [13] and [14] for a specific problem. FlexPDE can interpolate and export stresses from Gaussian nodes to the nodes of any arbitrary grid defined by the user. The stresses at the centre point of each grid element are interpolated using the interpolation shape functions. The stress-interpolation process is done prior to the performance of the dynamic programming search.

### 2.4 Description of the computer program DYNPROG

The analytical scheme of the dynamic programming method for performing a slope stability analysis is illustrated in Fig. 4. The computer program DYNPROG was developed to solve a slope stability problem using the following steps. (1) Input the geometry data and soil properties of the problem. (2) Import the output grid with corresponding nodal stresses from FlexPDE. (3) Define a search boundary using the output grid imported from FlexPDE as the search grid. (4) Interpolate stresses at the centre point of each grid element from nodal stresses. (5) Assume an initial factor of safety, $F$s (i.e., $F$s = 1). (6) Launch the search from all state points located in the initial stage. (7) Generate the first trial segment of the slip surface by connecting all state points of the initial stage to all state points located in the second stage. (8) Calculate the values of the optimal function obtained at all state points of the second stage using eqs. [10] and [11] and the assumed factor of safety, $F$s. The number of optimal functions to be calculated at one state point of the second stage is equal to the number of state points located in the initial stage. (9) Determine the minimum value of the optimal function at each state point in the second stage. The corresponding state point in the previous stage (i.e., the initial stage for the first segment) is identified. (10) Proceed to the next stage with the same routine until the final stage is reached. (11) Compare the values of the optimal functions obtained at all state points of the final stage and determine the state point at which the corresponding value of the optimal function is a minimum. The determined state point will be the first optimal point of the optimal path. (12) Trace back to the previous stage to find the corresponding state point with the first optimal point. This corresponding state point will be the second optimal point of the optimal path. (13) Keep tracing back to the initial stage to determine the entire optimal path. (14) Evaluate the actual factor of safety corresponding to the optimal path obtained from step 13 using eq. [2-14]. A new value for the factor of safety is calculated based on the initially assumed and the actual factors of safety. (15) Repeat the procedure until the difference between the assumed and the actual factor of safety is within the convergence criterion, $\delta$, defined prior to the performance of the optimization process. (16) Define the actual critical slip surface by determining the *entry* and *exit* points of the critical slip surface. These points are found at the intersections of the optimal path with the physical boundary of the slope.

### 2.5 Restriction applied to the shape of the critical slip surface

The shape of the critical slip surface must be kinematically admissible. Baker (1980) assumed that the critical slip surface must be concave. Therefore, the condition applied to the shape of the critical slip surface proposed by Baker was that the first derivative calculated from the crest to the toe of the curve that represents the critical slip surface must be greater than or at least equal to zero. Kinematic restriction conditions were not mentioned in Yamagami and Ueta (1988). Zou et al. (1995) stated that a check must be made to assure that the critical slip surface is kinematically admissible. There was no further comment regarding how this “check” should be applied, however. The authors of this article suggest that kinematical restrictions play an important role in the applicability of the dynamic programming method in slope stability analysis. Using appropriate kinematical restrictions prevents the shape of the critical slip surface from being unreasonable.

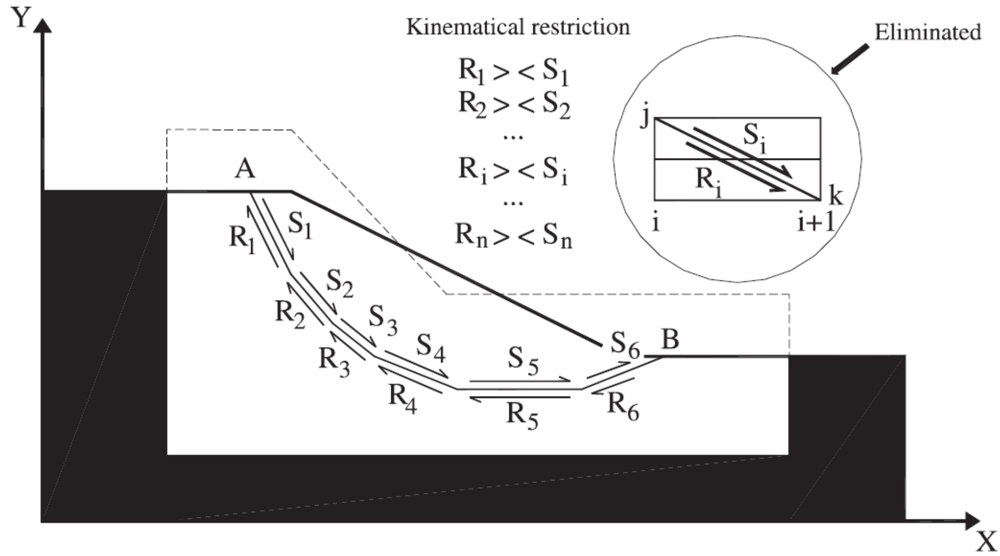


**Fig. 5. Kinematical restrictions applied to the shape of the critical slip surface.**

Theoretically, when failure takes place the resisting force and the actuating force along the slip surface must be in contrary directions. The resisting force must always act in the direction opposite to the mass movement. At the same time, the actuating force must be in the same direction as the movement (Fig. 5).

The kinematical restriction applied to the shape of the critical slip surface in this study is that if the actuating force calculated is in a contrary direction to the anticipated direction of mass movement, then the entire trial segment in which the actuating force is being calculated will be eliminated from the search. In other words, a trial segment will be eliminated from the optimization search if the actuating and resisting forces are found having the same sign. Applying this condition to the optimization procedure will eliminate all trial segments that constitute *kinky*-shaped slip surfaces.

## CONCLUSIONS

The dynamic programming method combined with a finite element stress analysis can be a viable and valuable tool for practical slope stability analyses. With the use of the finite element stress analysis, the present method provides a solution of greater flexibility compared with those produced by conventional limit equilibrium methods of slices.

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