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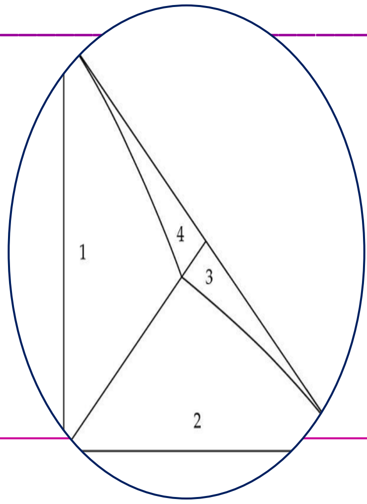
KANNAN FIXED POINT THEOREM OF SOME EXTENSION ON 2-METRIC SPACES



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ABSTRACT:

The fascinating nonlinear speculation of the established one of metric space is a 2-metric space. In this paper, we set up settled point hypotheses in 2-Metric Spaces by utilizing some new augmentations of Kannan settled point hypothesis got by Kannan. The outcome can be considered as an expansion and speculation of settled point hypotheses on 2-metric spaces and Kannan settled point hypotheses, given by some outstanding creators declared in the accessible writing.

KEYWORDS: *2-Metric Space, Extension, Fixed Point, Mapping, Kannan Fixed Point Theorem.*

INTRODUCTION:

The concept of a 2-metric space has been started and extensively created by Gä hler in a progression of papers and that's only the tip of the iceberg. Various settled point hypotheses has been demonstrated for 2-metric spaces. Iseki examined the settled point hypotheses in 2-metric spaces. Naidu and Prasad presented new settled point hypotheses on 2-metric space. The investigation was additionally improved by B.E. Rhoades, Miczko and Palezewski Imdad, Kumar and Khan expanded their work. After this few specialists utilized the concept of mappings. Hsiao examined a property of contractive sort mappings in 2-metric spaces. In addition Rhoades and other presented a few properties of 2-metric spaces and demonstrated some settled point and basic settled point hypotheses for contractive and extension mappings and furthermore have discovered some intriguing outcomes in 2-metric space. This concept is helpful in getting more far reaching settled point hypotheses. Kannan's hypothesis is free of the well-known Banach compression guideline and that it additionally portrays the metric culmination concept.

Definition 1: Suppose (X, d) are the matrix spaces and f be a mapping on X. The mapping f is called Kannan type mapping and has a unique fixed point if there exist $0 \leq \lambda$ and $<\frac{1}{2}$ such that,

$$d(fx, fy) \leq \lambda\, [d(x, fx) + d(y, fy)]\ \forall\ x, y \in X,\ \lambda \in [0, \tfrac{1}{1}]]$$

A metric space is a set X that has a concept of the distance d(x, y) between each twosome of points x, y ∈ X. A metricon a set is a function that fulfills the minimal properties we might except of a distance.

Definition 2: A metric d on a set X is a function $d : X \times X \rightarrow [0, \infty)$ such that for all x, y ∈ X:

1. $d(x, y) \geq 0$ and $d(x, y)= 0$, if $x = y$;
2. $d(x, y) = d(y, x)$, (symmetry)
3. $d(x, y) \leq d(x, z) + d(z, y)$ (triangle inequality).

A metric space (X, d) is a set X with a metric d defined on X

We can describe a wide range of metrics on a similar set, yet in the event that the metric on X is clear from the unique situation, we suggest to X as a metric space and exclude unequivocal specify of the metric d.

Definition 3: Consider X be a nonempty set. A real valued function d on X×X×X is said to be a 2-metric in X.

1. The distance point x, y in X in each pair. There exists a point $z \in X$ such that $d(x, y, z) \neq 0$.
2. $d(x, y, z) = 0$, when at least of x=y, x=z, and y= z are equal
3. $d(x, y, z) = d(y, x, z) = d(x, z, y)$
4. $d(x, y z) \leq d(x, y, w) + (x, w, z) + d(w, y, z)$ for all $x, y, z, w \in X$

When d is a 2-metric on X, then the methodical pair (X, d) is called 2-metric space.

Definition 4: A sequence $\{x_n\}$ in 2-metric space (X, d) is said to be convergent to an element $x \in X$ if $\lim_{n\to\infty} d(x_n, x, a)= 0$ for all $a \in X$.
It follows that if the sequence $\{x_n\}$ converges to x then $\lim_{n\to\infty} d(x_n, a,b)= d(x, a, b)$ for all $a\ b \in X$.

Definition 5: A sequence $\{x_n\}$ in a 2-metric space X is a Cauchy sequence if $d(x_m, x_n, a) = 0$ as $m, n\to\infty$ for alla $\in X$.

Definition 6: A 2-metric space (X, d) is said to be complete if every Cauchy sequence in X is convergent.

Definition 7: If a sequence $\{x_n\}$ in a 2-metric space converges to x then every subsequence of $\{x_n\}$ alsoconverges to the same limit x.

Definition 8: Let (X, d) be a metric space. A mapping $T:X\longrightarrow X$ is said sequentially convergent if wehave, for every sequence $\{y_n\}$, if $\{T\ y_n\}$ is convergence then $\{y_n\}$ also is convergence.

Definition 9: Let (X, d) be a metric space. A mapping $T: X\longrightarrow X$ is said sub sequentially convergent if wehave, for every sequence $\{y_n\}$, if $\{T\ y_n\}$ is convergence then $\{y_n\}$ has a convergent subsequence.

Definition 10: If (X, d) be a compact metric space, then every function $T: X\longrightarrow X$ is subsequentiallyconvergent and every continuous function $T: X\longrightarrow X$ is sequentially convergent.

Definition 11: A function $\phi: R \to R^+$ is said to be Φ function if it satisfies the following conditions.

1. $\phi(t) = 0$ if $t = 0$,
2. $\phi(t)$ is severely intonation increasing and $\phi(t) \to \infty$ as $t \to \infty$,
3. ϕ is left constant in $(0, \infty)$,
4. ϕ is constant at 0.

Mai Result:

Theorem 1: Let (X, d) be a complete 2-Metric Space, where f is a self-mapping which satisfies the following inequality for all $x, y, z \in X$, $\lambda \in [0,1)$, $F \in \Phi$.
$F(d(fx, fy, fz)) \leq \lambda\ [F(d(x, fx, fx+1)) + F(d(y, fy, fy+1)) + F(d(z, fz, fz+1))]$

Then f has a unique fixed point and for every $x_0\in X$, the sequence of iterates $\{x_n\}$ or $\{f^n x_0\}$ converges to thisfixed point.

Proof: Let $x_0 \in X$ be an arbitrary point. We can define the iterative sequence $\{x_n\}$ by $x_{n+1} = f\,x_n$ or $x_n = f\,x_n - 1$ for $n \in N$.
Now $F(d(x_n, x_{n+1}, x_{n+2})) = F(d(f\,x_{n-1}, f\,x_n, f\,x_{n+1}))$

$\leq \lambda\,[F(d(x_{n-1}, f\,x_{n-1}, f\,x_n)) + F(d(x_n, f\,x_n, f\,x_{n+1})) + F(d(x_{n+1}, f\,x_{n+1}, f\,x_{n+2}))]$

$\leq \lambda\,[F(d(x_{n-1}, x_n, x_{n+1})) + F(d(x_n, x_{n+1}, x_{n+2})) + F(d(x_{n+1}, x_{n+2}, x_{n+3})]$

In this manner
$\leq \frac{x}{1-\lambda}\,[[F(d(x_{n-1}, x_{n+1}, x_{n+3}))] \leq \left(\frac{\lambda}{1-\lambda}\right)^2 [F(d(xn-2, x_n, x_{n+2}))] \leq \ldots \leq \left(\frac{\lambda}{1-\lambda}\right)^n [F(d(x_0, x_1, x_2))]$

For every $l, m, n \in N$ such that $l > m > n$, we have

$F(d(xl, xm, xn)) = F(d(f\,xl-1, f\,xm-1, f\,xn-1))$

$\leq \lambda\,[F(d(x_{l-1}, f\,x_{l-1}, f\,x_l)) + F(d(x_{m-1}, f\,x_{m-1}, f\,x_m)) + F(d(x_{n-1}, f\,x_{n-1}, f\,x_n))]$

$\leq \lambda\,[F(d(x_{l-1}, x_l, x_{l+1})) + F(d(x_{m-1}, x_m, x_{m+1})) + F(d(x_{n-1}, x_n, x_{n+1})]$

$\leq \lambda\left[\left(\frac{\lambda}{1-\lambda}\right)^{l-1} + \left(\frac{\lambda}{1-\lambda}\right)^{m-1}\left(\frac{\lambda}{1-\lambda}\right)^{n-1}\right] F(d(x_0, x_1, x_2))$

Letting $l, m, n \to \infty$, we can get $\lim_{l,mn\to\infty} F(d(x_l, x_m, x_n)) = 0$

And it is already declared that $F \in \Phi$, hence $\lim_{l,mn}\longrightarrow\infty\; d(x_l, x_m, x_n) = 0$

As (X, d) be a complete 2-Metric Space, i.e. $\{x_n\}$ is a Cauchy sequence and convergent such that $\lim_{n\to\infty} x_n = u$, where $u \in X$.

Hence $F(d(f\,u, x_{n+1}, x_{n+2})) = F(d(f\,u, f\,x_n, f\,x_{n+1}))$

$\leq \lambda\,[F(d(u, f\,u, f\,u+1)) + F(d(x_n, f\,x_n, f\,x_{n+1})) + F(d(x_{n+1}, f\,x_{n+1}, f\,x_{n+2}))]$

$\leq \lambda\,[F(d(u, f\,u, f\,u+1)) + F(d(x_n, x_{n+1}, x_{n+2})) + F(d(x_{n+1}, x_{n+2}, x_{n+3}))]$

Due to the property of Φ function, F is continuous, letting $n \to \infty$
$F(d(f\,u, u, u)) \leq \lambda\,[F(d(u, f\,u, f\,u+1)) + F(0) + F(0)]$
$F \in \Phi$ so that $F(0) = 0$ and therefore $f\,u = u$.
It is clear that f has a fixed point.

Theorem 2: Consider (X, d) be a complete 2-Metric Space, where f, g are self-mapping such that g is continuous,one-one and sub sequentially convergent. For all $x, y, z \in X$, $\lambda \in [0,1/2)$

$d(g\,f\,x, g\,f\,y, g\,f\,z)) \leq \lambda\,[d(g\,x, g\,f\,x, g\,f\,x+1) + d(g\,y, g\,f\,y, g\,f\,y+1) + d(g\,z, g\,f\,z, g\,f\,z+1)]$

Then f has a unique fixed point. If g is sub sequentially convergent then the sequence of iterates $\{x_n\}$ or $\{f^n x_0\}$ converges to this fixed point.

Proof: Let $x0 \in X$ be an arbitrary point. We can define the iterative sequence $\{x_n\}$ by $x_{n+1} = f\,x_n$ or $x_n = f\,x_{n-1}$ for $n \in N$.
Now $d(g\,x_n, g\,x_{n+1}, g\,x_{n+2}) = d(g\,f\,x_{n-1}, g\,f\,x_n, g\,f\,x_{n+1})$

$\leq \lambda$ [d(g x_{n-1}, g f x_{n-1}, g f x_n) + d(g x_n, g f x_n, g f x_{n+1}) + d(g x_{n+1}, g f x_{n+1}, g f x_{n+2})]

$\leq \lambda$ [d(g x_{n-1}, g x_n, g x_{n+1}) + d(g x_n, g x_{n+1}, g x_{n+2}) + d(g x_{n+1}, g x_{n+2}, g x_{n+3})]

Using induction,

$\leq \frac{\lambda}{1-\lambda}$ [d (g x_{n-1}, g x_{n+1}, g x_{n+3})] $\leq \left(\frac{\lambda}{1-\lambda}\right)^2$ [d (g x_{n-2}, g x_n, g x_{n+2})] $\leq \ldots \leq \left(\frac{\lambda}{1-\lambda}\right)^n$ d (g x_0, g x_1, g x_2)]

For every l, m, n ∈ N such that l > m > n, we have

d(g x_l, g x_m, g x_n) $\leq \lambda$ [d(g x_l, g x_m, g x_{m-1}) + d(g x_{m-1}, g x_{m-2}, g x_{m-3}) + .+ d(g x_{m+1}, g x_m, g x_n))]

$\leq \left[\left(\frac{\lambda}{1-\lambda}\right)^{m-1} + \left(\frac{\lambda}{1-\lambda}\right)^{m-1} + \ldots + \left(\frac{\lambda}{1-\lambda}\right)^{n}\right]$d($x_0$, x_1, x_2)

$\leq \left[\left(\frac{\lambda}{1-\lambda}\right)^{n} + \left(\frac{\lambda}{1-\lambda}\right)^{n+1} + \ldots + \left(\frac{\lambda}{1-\lambda}\right)^{n+4}\right]$d($x_0$, x_1, x_2)

$\leq \left(\frac{\lambda}{1-\lambda}\right)^{n}\left[1 + \left(\frac{\lambda}{1-\lambda}\right)^{2} + \left(\frac{\lambda}{1-\lambda}\right)^{4} + \cdots\right]$d($x_0$, x_1, x_2)

$\leq \left(\frac{\lambda}{1-\lambda}\right)^{n}\left[\frac{1}{\left(1-\frac{\lambda}{1-\lambda}\right)^2}\right]$d($x_0$, x_1, x_2)

Since (X, d) is a complete 2-metric space and $\{x_n\}$ is a cauchy sequence. Where g is a sub sequentiallyconvergent and $\{x_n\}$ has a convergent subsequence. So there exist $\lim_{k\to\infty} x_{n(k)}$)= u such that u ∈ X and k ∈ (1,∞).

Because g is continuous, we conclude that $\lim_{k\to\infty}$ g $x_{n(k)}$= g u. So,

d(g f u, u+1, g u) $\leq$ d(g f u, $x_{n(k)}$, $x_{n\,k+1}$) + d($x_{n\,k+1}$, $x_{n\,k+2}$, $x_{n\,k+3}$) + d($x_{n\,k+3}$, $x_{n\,k+4}$, g u)

$\leq \lambda$[d(g u, g f u, u+1) + d(g $x_{n(k-1)}$, g $x_{n(k)}$, g $x_{n(k+1)}$) + d(g $x_{n(k)}$, g $x_{n(k+1)}$, g $x_{n(k+2)}$] + $\left(\frac{\lambda}{1-\lambda}\right)^{n(k)+1}$d(g x_0, g x_1, g x_2) + d($x_{n(k)+3}$, $x_{n(k+4)}$, g u)

$\leq \lambda$ d(g u, g f u, u+1) + $\lambda\left(\frac{\lambda}{1-\lambda}\right)^{n(k)-1}$ + d(g x_0, g x_1, g x_2)+ $\lambda\left(\frac{\lambda}{1-\lambda}\right)^{n(k)}$ + d (g x_0, g x_1, g x_2) + $\left(\frac{\lambda}{1-\lambda}\right)^{n(k)+1}$d(g x_0, g x_1, g x_2) + d($x_{n(k)+3}$, $x_{n(k+4)}$, g u)

$\leq \left(\frac{\lambda}{1-\lambda}\right)^{n(k)}$d(g x_0, g x_1, g x_2) + $\left(\frac{\lambda}{1-\lambda}\right)^{n(k)+1}$d(g x_0, g x_1, g x_2) + $\frac{\lambda}{1-\lambda}\left(\frac{\lambda}{1-\lambda}\right)^{n(k)+1}$d(g x_0, g x_1, g x_2) + $\left(\frac{\lambda}{1-\lambda}\right)$ d($x_{n(k)+3}$, $x_{n(k)+4}$, g u)

Letting k → ∞, we get d(g f u, u+1, g u) → 0

If g is sequentially convergent, by replacing {n} with {n(k)}, we get $\lim_{n\to\infty} x_n$= u and this shows that $\{x_n\}$converges to the fixed point of f. Also, as g is one-one and f u = u, we can conclude that f has a fixed point.

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