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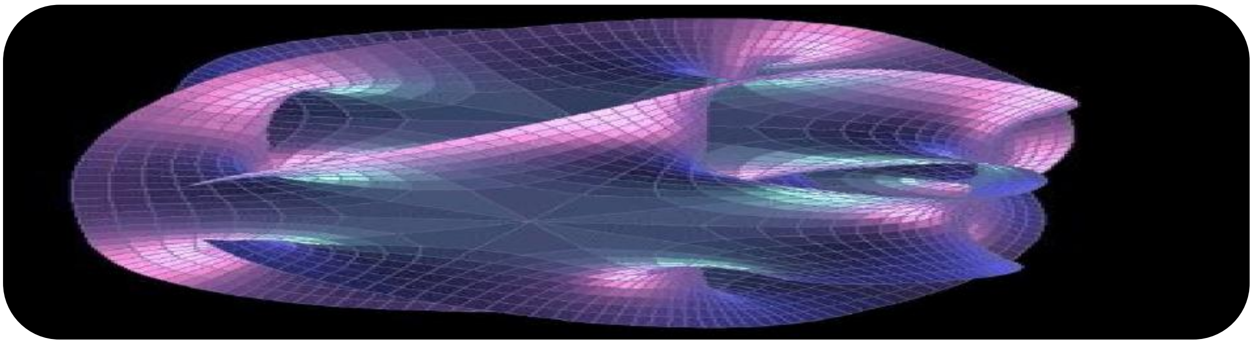

THE SINGULAR HYPERSURFACE METRIC

Dr. Amit Kumar Srivastava
Department of Physics , D.A-V. College, Kanpur , (U.P.), India.

1. ABSTRACT:

A solution of the Einstein equations is by definition cosmological if it can reproduce the (FLRW) metric by taking limiting values of arbitrary constants or functions. It has been a conventional wisdom in cosmology that the FLRW models successfully describe the large scale properties of our observed Universe, even since the 1930ies. At the same time, it has been a conventional wisdom in the general relativity theory that finding exact solutions of the Einstein equations is extremely difficult and possible only for exceptionally simple cases. These both views were challenged repeatedly by lonely rebels, but a few generations of physicists and astronomers have been educated with these conventional wisdoms solidly incorporated into their minds. As a result of it, a large number of literatures have come into existence in which exact solutions generalising FLRW were derived and applied to describe our observed Universe. There are various inhomogeneous cosmological models in literature and a great majority of them are based on L-T model [*Lemaitre (1933), Tolman (1934)*]. The arbitrary parameters of a solution (constants or functions) often enter several physical quantities, and forcing a certain limit upon one of the quantities may result in trivializing others at the same time.

KEY WORDS: thermalized, Locally rotationally symmetric (LRS), nonthermalized, magnetofluid, barotropic, Weyl tensor, hypersurfaces

3. INTRODUCTION

4. The Source of the Energy-momentum Tensor:

The physical significance of cosmological models, homogeneous and inhomogeneous, we propose to investigate the following cosmological models with heat flux i.e. exact solutions of Einstein's field equations with non thermalized perfect fluid as the source term:

(i) Plane symmetric inhomogeneous cosmological models with heat flux.

(ii) Locally rotationally symmetric (LRS) Bianchi space time filled with perfect fluid and heat flux.
(iii) A general class of inhomogeneous cosmological models which admits two spacelike commuting killing vectors and have separable metric coefficients with nonthermalized perfect fluid as the source term of energy momentum tensor.

In this way we shall investigate some new and old exact solutions of Einstein's field equations generalising the plane symmetric, locally rotationally symmetric Bianchi space time and cylindrically symmetric spacetimes with perfect fluid and heat flux as the source of the energy momentum tensor.

5. METHOD:

The Reimann tensor is defined for a covariant vector filed k_α so that

(1.1) $$k_{\alpha;\beta\gamma} - k_{\alpha;\gamma\beta} = k_\rho R^{\rho}_{\alpha;\beta\gamma}$$

where semicolon denotes covariant derivative, and comma stands for partial and ordinary derivatives respectively. The
Einstein equations are –

(2) $$G_{\alpha\beta} \equiv R_{\alpha\beta} - \tfrac{1}{2} g_{\alpha\beta} R,$$
$$= K\, T_{\alpha\beta} + \wedge\, g_{\alpha\beta}$$

(1.3) $$\kappa \equiv 8\Pi\, G/C^4$$

where G be the gravitational constant, $\wedge$ the cosmological constant, the energy momentum tensor may contain the following contributions :

(1.4) $$T = T^f + T^h + T^v + T^n + T^s + T^e + T^s + T^m$$

where:

(i) T^f denotes the perfect fluid distribution with

(1.5) $$T^f_{\alpha\beta} = (\in + p)\, u_\alpha u_\beta - p\, g_{\alpha\beta}$$

$\in$ being the energy density, p be the pressure and u_α the velocity field of the fluid. In some cases, the fluid will obey the barotropic equitation of state

(1.6) $$f(\in, p) = 0$$

The velocity is normalized

$$u_\alpha u^\alpha = 1$$

and its covariant derivative may be decomposed as

(1.8) $$u_{\alpha;\beta} = \acute{\upsilon}_\alpha u_\beta + \sigma_{\alpha\beta} + w_{\alpha\beta} + \frac{1}{3}\theta(g_{\alpha\beta} - u_\alpha u_\beta)$$

where

(1.9) $\acute{u}_{\alpha} = u_{\alpha\,;\,\rho}\, u^{\rho}$

be the acceleration vector obeying $\acute{u}_{\alpha}\, u^{\alpha} = 0$
(when $\acute{u}_{\alpha} = 0$ the flow lines of the fluid are geodesics)

(1.10) $\theta = u^{\alpha}{}_{;\,\alpha}$
be the expansion scalar

(1.11) $\sigma_{\alpha\beta} = u_{(\alpha;\beta)} - \acute{u}_{(\alpha}\, u_{\beta)} - \frac{1}{3}\theta\,(g_{\alpha\beta} - u_{\alpha} u_{\beta})$

be the shear tensor and is symmetric in its indices and obeys $\sigma^{\alpha}{}_{\alpha} = 0 = \sigma_{\alpha\beta}\, u^{\beta}$. The round brackets around indices denote symmetrisation and

(1.12) $w_{\alpha\beta} = u_{[\alpha;\beta]} - \acute{u}_{[\alpha}\, u_{\beta]}$

be the rotation tensor and is antisymmetric and obeys

(1.13) $w_{\alpha\beta}\, u^{\beta} = 0$

The square bracket around the indices denotes anti- symmetrization. The shear tensor is zero if and only if the shear scalar σ defined by $\sigma^2 = ½\, \sigma_{\alpha\beta}\, \sigma^{\alpha\beta}$ is zero. The rotation tensor is zero if and only if the rotation scalar w defined by $w^2 = ½\, w_{\alpha\beta}\, w^{\alpha\beta}$ is zero. Often the special perfect fluid with p = 0 (dust matter) is considered as source. If the dust is the only source, then necessarily $\acute{u}^{\alpha} = 0$.

(ii) T^h denotes the heat flow contribution

(1.14) $T^h_{\alpha\beta} = q_{\alpha}\, u_{\beta} + q_{\beta}\, u_{\alpha}$

where q^{α} be the heat flow vector obeying $q_{\alpha}\, u^{\alpha} = 0$. It defines the direction in which energy is transported across the flow lines. One has to assume in addition that the number density of the fluid particles, n is conserved.

(1.15) $(nu^{\alpha})\,;\alpha = 0,$

such that the first law of thermodynamics holds

$Td\,(S/n) = d\,(\in /n) + pd\,(1/n)$

where T be the temperature and S be the entropy density and that

(1.17) $q_{\alpha} = -q\,[\,(\delta^{\beta}_{\alpha} - u_{\alpha}\, u^{\beta})\, T_{;\beta} + T\, \acute{u}_{\alpha}\,]$

where q be the coefficient of thermal conductivity

(1.18) $$S^{\alpha}{}_{;\alpha} \geq o$$

where

$$S^{\alpha} - Su^{\alpha} + q^{\alpha} / T$$

In an inhomogeneous cosmological model, spatial variations of temperature are expected. It is then natural to suppose that heat transfer between fluid particles occurs. It is the main motivation for considering cosmological solution with heat-flow.

(iii) T^{v} denotes contribution of viscosity.

(1.20) $$T^{v}{}_{\alpha\beta} = \eta\, \sigma_{\alpha\beta}$$

when η be the coefficient of viscosity.

(iv) T^{n} denotes the energy momentum tensor of a null fluid.

(1.21) $$T^{n}{}_{\alpha\beta} = \square\, k_{\alpha}\, k_{\beta}$$

where $k_{\alpha}\, k^{\alpha} = 0$ and the vector filed k_{α} defines the direction of flow of the null fluid. When the null fluid is present together with a perfect fluid then $k_{\alpha}\, u^{\alpha} = 1$ is assumed in addition. If $T = T^{n}$ i.e. if all the other contributions to energy momentum vanishes, then the equations of motion $T^{\alpha\beta}{}_{;\,\beta} = 0$ imply that $\underline{k}$ is a geodesic vector. The energy momentum tensor may be generated by a null electromagnetic field.

(v) T^{s} denotes the energy momentum tensor of a scalar field.

(1.22) $$T^{s}{}_{\alpha\beta} = \phi_{,\alpha}\, \phi_{,\beta} - \tfrac{1}{2}\, g_{\alpha\beta}\, (\phi_{,\rho}\, \phi^{,\rho} + m\phi^{2})$$

where ϕ be the scalar field, obeying

(1.23) $$g^{\alpha\beta}\, \phi_{;\alpha\beta} - m\phi = 0$$

where m is constant, interpreted as the mass of the field carrier. In most papers, the mass is assumed zero, and then the energy momentum tensor may be interpreted as due to the stiff perfect fluid with equation of state $\in = p$, where

(1.24) $$\in = p = \tfrac{1}{2}\, \phi_{,\rho}\, \phi^{,\rho}$$

(1.25) $$u_{\alpha} = \phi_{,\alpha} \,/\, (\phi_{,\rho}\, \phi^{,\rho})^{1/2}$$

The massive scalar field maybe interpreted as a perfect fluid, too with

(1.26) $$\in = \tfrac{1}{2}\, (\phi_{,\rho}\, \phi^{,\rho} - m\phi^{2})$$

(1.27) $p = ½ (\phi,_{\rho} \phi'^{\rho} + m\phi^{2})$

and u^{α} given by eq. (1.25) but the equation of state is undetermined when $m \neq 0$.

(iv) T^{e} denotes the energy momentum of the electromagnetic filed.

(1.28) $T^{e}_{\alpha\beta} = \frac{1}{4\pi} (F_{\alpha}{}^{\mu} F_{\mu\beta} + ¼ g_{\alpha\beta} F_{\mu\nu} F^{\mu\nu})$

where the electromagnetic field tensor F obeys the Maxwell relations.

(1.29) $F^{\alpha\beta}{}_{;\beta} = \frac{4\pi}{c} j^{\alpha}$

(1.30) $F [_{\alpha\beta;\gamma}] = 0$

j^{α} being current vector. The field is known null when

(1.31) $F_{\mu\nu} F^{\mu\nu} = 0$

(1.32) $F_{[\alpha\beta} F_{\gamma ?]} = 0$

Then, vector field k and w exist such that

$k_{\alpha} k^{\alpha} = k_{\alpha} w^{\alpha} = 0, \quad w_{\alpha} w^{\alpha} = -1$

$F_{\alpha\beta} = \lambda (w_{\alpha} k_{\beta} - w_{\beta} k_{\alpha})$

and the energy momentum tensor of the field assumes the form

(1.35) $T^{n}_{\alpha\beta} = ? \; k_{\alpha} k_{\beta}$

With

(1.36) $? = \lambda^{2} / (4\pi)$

For the combined solutions with the perfect fluid/ electromagnetic field, it is usually assumed that

(1.37) $j^{\alpha} = \rho u^{\alpha}$

where ρ be the electric charge density i.e. the electromagnetic field is produced by charges on the matter particles.

(vii) $T^{\$}$ denotes the energy momentum tensor associated with a cloud of strings with particles attached to them

(1.38) $T^{\$} = \in u_{\alpha} u_{\beta} - \lambda w_{\alpha} w_{\beta}$

with

(1.39) $u_\alpha \; u^\alpha = - w_\alpha \; w^\alpha = 1$

(1.40) $w^\alpha \; u_\alpha = 0$

where $\in$ and λ denote the energy density and the string tension density of the string cloud by the relation

(1.41) $\in \; = \; \in_p + \lambda$

where $\in_p$ is the particle density in the string cloud. The energy conditions imply $\in > o$, leaving the sign of the string tension density λ unrestricted. The unit time like vector u^α be the flow vector of the matter and space like vector w^α specifies the string direction in the cloud.

(viii) T^m denotes the energy momentum tensor of the perfect magnetofluid

(1.42) $T^m_{\alpha\beta} = (W + P) \; u_\alpha \; u_\beta - P \, g_{\alpha\beta} - h_\alpha \; h_\beta$

(1.43) $P = p + ½ \; |\underline{h}|^2$

(1.44) $W = \in + ½ \, \mu \; |\underline{h}|^2$

with

(1.45) $u_\alpha \; h^\alpha = 0$

(1.46) $h_\alpha \; h^\alpha = - |\underline{h}|^2$

where h_α is magnetic field and being constant.

6. CRITERIA FOR A FLRW LIMIT

A given metric has a FLRW limit is one of the sets of necessary and sufficient conditions may be imposed on it, leading to a nontrivial and nonsingular result. Those conditions are not always easy to apply. It is more practical to apply several necessary conditions in succession until the solution investigated is either reduced to a FLRW limit or proven not to have it. The necessary conditions are:

(i)the source must be perfect fluid. If the energy momentum tensor of the solution investigated has more components, e.g. heat flow, then the additional quantities must be set to zero. It is to be noted that a pure scalar field source and dust are special cases of perfect fluid and are compatible with the FLRW geometry. Pure null fluid and pure electromagnetic field are not, such solutions have no FLRW limit. Also solutions with tachyon fluid source, for which the energy momentum tensor has the form

(1.47) $T^f_{\alpha\beta} = (\in + p) \; u_\alpha \; u_\beta - p \, g_{\alpha\beta}$

but the vector field u is spacelike, have no FLRW limit.

(ii) The acceleration must be zero. In comoving and synchronous coordinates in which, if they exist.

(1.48) $u^\alpha = \delta^\alpha_o$

(1.49) $g_{oi} = 0, \; i = 1, 2, 3,$

The condition has the simple form

(1.50) $g_{oo',i} = 0, \quad i = 1, 2, 3$

(iii)The rotation must be zero i.e. w = o, which is equivalent to the existence of coordinates that are simultaneously comoving and synchronous.
(iv) The shear must be zero i.e. $\sigma = o$ which is equivalent to

(1.51) $$\mathring{u}\, g_{ij,o} = \frac{2}{3}\theta \, g_{ij}, \quad i, j = 1, 2, 3,$$

and for every pair of nonzero components (g_{ij}, g_{kl}) it may be written as

(1.52) $$(\ln g_{ij})_{,t} = (\ln g_{kl})_{,t} = \frac{2}{3}\theta \, g^{½}_{oo}$$

If any $(\ln g_{ij})_{,t}$ is zero, then $\theta = 0$ automatically in the $\sigma = 0$ limit, and the metric has no FLRW limit. If the coordinates are not commoving, but the velocity field is tangent to 2-surfaces S_2 with the coordinates (t, x^1) and the space-time is orthogonally transitive so that $g_{tA} = g_{1A} = 0$ where A = 2, 3, then $\sigma = 0$ implies

(1.53) $$u(\ln g_{AB})_{,\rho} = u^{\rho}(\ln g_{CD})_{,\rho} = 2/3\ \theta$$

for every pair of nonzero (g_{AB},g_{CD}), A,B,C,D = 2,3. Again, if any $u^{\rho}(\ln g_{AB})_{,\rho} = 0$, then $\theta = 0$ in the $\sigma = 0$ limit, and no FLRW limit exists.

(v) The gradient of pressure must be colinear with the velocity field

(1.54) $$u_{[\alpha\ p,\beta]} = 0$$

For a solution with a pure perfect fluid, it is equivalent to

(1.55) $$\acute{u}^{\alpha} = 0$$

(vi)The gradients of matter density and of the expansion scalar must be collinear with velocity
(vii)The barotropic equation of state

(1.56) $$\in_{[\alpha\ p,\beta]} = 0$$

must hold.

(viii)The Weyl tensor must vanish.
(ix) The hypersurfaces orthogonal to the velocity field must have constant curvature.

The arbitrary parameters of a solution (constants or functions) often enter several physical quantities, and forcing a certain limit upon one of the quantities may result in trivializing others at the same time. For example, the limit

(1.57) $$\sigma = 0$$

may automatically imply $\theta = 0$ (a static solution) or $\in = 0$ (a vacuum solution) or $\in + P = 0$ (then with a pure perfect fluid source, the Bianchi identities imply $\in = -P$ constant, an the spacetime is a vacuum with cosmological constant Λ). In such case no FLRWE limit exists.

7. CONCLUSION:

The gradient of pressure must be colinear with the velocity field. The gradients of matter density and of the expansion scalar must be collinear with velocity.

The hypersurfaces orthogonal to the velocity field must have constant curvature. The arbitrary parameters of a solution (constants or functions) often enter several physical quantities, and forcing a certain limit upon one of the quantities may result in trivializing others at the same time.

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