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PROPERTIES OF THE METRIC IN 5-DIMENSIONS

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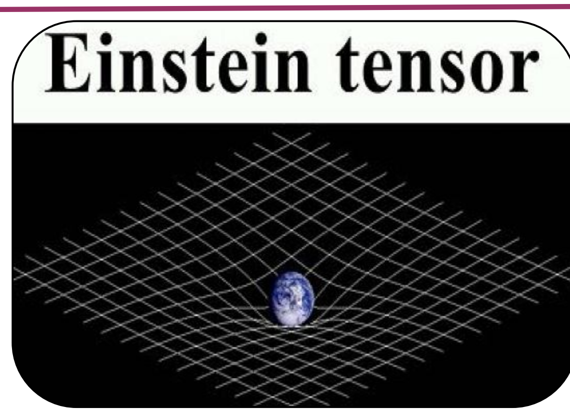
ABSTRACT:

We present some properties of the metric $ds^2 = \bar{g}_{AB} dx^A dx^B$ and Einstein tensor $\bar{G}_{AB}$ in 5-dimensions. The 5-dimensional coordiante systemis made of one time, 3-spatial and one background component, i.e. (t, x^i, ξ). the object in that reference frame will experience a force. Such reference frames are termed as non-inertial reference frames, and the Mach's principle is well satisfied. Hence, we have explored some of the features of the metric and Einstein tensor $\bar{G}_{AB}$ in 5-dimensions.

KEY WORD: Einstein tensor, Newtonian gravity, Spacetime.

1. INTRODUCTION:



Newtonian gravity may provide a very suitable description of gravity for week gravitational field, and not time varying. Several new theories are postulated to explain the galactic rotation curves without any form of dark matter such as (*Milgrom 1983*), (*Milgrom 2011*). But it violates many other physical laws including the law of momentum conservation. On the other hand, theories like modified gravity i.e. (*Moffat 2006*), (*Brownstein and Moffat 2006*), (*Moffat and Toth 2008*). Though, these theories explain many observational phenomena without any form of dark matter, but they may not taken seriously. (*Das 2012*) proposed new theory of gravity based on the Mach's principle. In this report we have presented some properties of the metric and Einstein tensor $\bar{G}_{AB}$ in 5-dimensions.

2.METHOD

The Metric and Some Properties in 5-dimensions:

Let us consider the 5-dimensional coordinate system which is composed of one time, 3-spatial and one background component i.e. (t, x^i, ξ). The line element reads as

$$ds^2 = \bar{g}_{AB} dx^A dx^B, \tag{1}$$

where $\bar{g}_{AB}$ is the 5-dimensional metric. The indices A,B,C ..... run from 0 to 4. The indices α, β, γ .... run from 0 to 3 and indices i, j, k .... i.e. spatial indices run from 1 to 3. The fifth dimension ξ as a space-like coordinate. The line element remains invariant under any deformatioin of $\bar{g}_{AB}$.

If all the g_{A4} are constants then one may expect to obtain the 4-dimensional general theory of relativity, and along fifth dimension, the metric will remain flat. Again, if g_{A4} are not constant and are functions of $\left(t, x^i, \xi\right)$ then there will be some contribution in the 4-dimension at curvature tensor from the fifth dimension. The 5-dimensional metric $\bar{g}_{AB}$ in 4+1 dimensional form reads

$$\bar{g}_{AB} = \begin{pmatrix} g_{\alpha\beta} + K^2\varphi^2 A_\alpha A_\beta & K\varphi A_\alpha \\ K\varphi^2 A_\beta & \varphi^2 \end{pmatrix}, \tag{2}$$

And

$$\bar{g}_{AB} = \begin{pmatrix} g^{\alpha\beta} & -KA^\alpha \\ -KA^\beta & \frac{1}{\varphi^2} + K^2 A^\mu A_\mu \end{pmatrix}, \tag{3}$$

where $g_{\alpha\beta}$ is a 4-dimensional metric, A_α as a 4-dimensional vector and φ as a scalar.

The 5-dimensional Einstein tensor projects itself in the 4-dimensional space. In view of $\bar{G}_{AB} = 0,$ and eqs. (2) and (3) and also using Kaluza-Klein technique without making the fifth dimension compactified, one obtains

$$\bar{G}_{AB} = \frac{K^2\varphi^2}{2}\left(\frac{1}{4} g_{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta} - F_\alpha^\gamma F_{\beta\gamma}\right) - \frac{1}{\varphi}\left[\nabla_\alpha\left(\partial_\beta \varphi\right) - g_{\alpha\beta} \Box \varphi\right] + p_{\alpha\beta} \tag{4}$$

$$\nabla^\alpha F_{\alpha\beta} = -\frac{3\partial^\alpha \varphi}{\varphi} F_{\alpha\beta} + Q_\beta \tag{5}$$

$$\Box \varphi = \frac{K^2 \varphi^3}{4} F_{\alpha\beta} F^{\alpha\beta} + U \tag{6}$$

where

$$F_{\alpha\beta} = A_{\alpha;\beta} - A_{\beta;\alpha}, \qquad (7)$$

and $P_{\alpha\beta}, Q_{\beta}$ and U be the terms containing the derivatives of the metric with respect to ξ.

It is obvious that left hand side of eq. (4) is a tensor and so the entire right hand side is also a tensor in 4-dimensions, but the individual terms are not tensor. One of the nice feature of eq. (4) is that left hand side is same as the Einstein tensor in general theory of relativity. It shows that the fifth dimension may curve the spacetime in view of Mach's principle.

3. CONCLUSION:

If the acceleration of a particle is measured with respect to the background provided the background is fixed with respect to some reference frame then no object will experience any force in the reference frame. In such a case the terms in the right hand side of eq. (4) will become zero, i.e. $G_{\alpha\beta} = 0.$ However, if in some coordinate system the background object starts accelerating then the terms in the right hand side of eq. (4) will become nonzero i.e. $G_{\alpha\beta} \neq 0.$ Hence, the object in that reference frame will experience a force. Such reference frames are termed as non-inertial reference frames, and the Mach's principle is well satisfied. Hence, we have explored some of the features of the metric and Einstein tensor $\bar{G}_{AB}$ in 5-dimensions.

REFERENCES

1. Milgrom, M. (1983), A.J. vol. 270, 365.
2. Milgrom, M. (2011), arXiv.org/abs/1101.5122.
3. Moffat, J.W. (2006), arXiv: gr-qc/0506021.
4. Brownstein, J.R. and Moffat, J.W. (2005), arXiv: astro-ph/0506370.
5. Brownstein, J.R. and Moffat, J.W. (2005), arXiv: astro-ph/0507222.
6. Moffat, J.W. and Toth, V.T. (2008), arXiv: 0710.0364.
7. Das, S. (2012), arXiv: 1206.6755.