



A PROPAGATION OF MICRO-POLAR FLUID USED AS BLOOD CONTAINED TWO PARALLEL PLATES

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ABSTRACT

The study of a propagation of micro-polar fluid used as blood contained two parallel plates When both plates are fixed and only pressure is acting then velocity profiles are negative for small Reynold's number. The velocity profile reduce with the increase of time, which shows that the micro-rotation become dominant as the time lapses and tries to retard the motion of the fluid. From the table it is clear that the micro-rotation change their way and get reversed as the time increases. It is observed that the micro-rotation decay in the vicinity of the wall but become active in the central region with lapse of time and micro-rotation effects are larger for large Reynolds number.

KEYWORDS: Micro-polar fluid, Lubricant, Flow behaviour, Micro-rotation,

INTRODUCTION

Several experimental studies show that effectiveness of lubricating oil can be improved on blending small amounts of long chained polymer additives with Newtonian lubricants. Most of the modern lubricant are mainly the polymer thicked oils or lubricants with additives. These lubricants become heavily contaminated with suspended metal particles and they start to exhibit non-Newtonian behaviour. Eringen's [1] micro-polar fluid theory is

the generalization of the classical theory of fluids, which accounts for polar effects. This theory accurately describes the rheological behaviour of lubricants with polymer additives.

The experimental result support the achievement of better lubricating effectiveness on blending small amount of long chained additives with the Newtonian lubricant. Micro polar fluids obtained from the general micro fluids by imposing the assumption of the skew symmetry of the gyration tensor. A number of theories of the micro continuum have been developed to explain the behaviour of the fluids as polymeric fluids. The study of the flow behavior using the

theory micro-polar lubrication was initiated with the problem of a two dimensional slider bearing. Eringen [1] suggested that the theory of micro-polar fluid, might serve as a satisfactory model for the description of the flow properties of polymeric fluids, fluid containing certain additives and in particular the animal blood.

The classical Navier-Stokes theory is inadequate especially in predicting the behaviour of such type of suspended particles or the moment of blood cells and the boundary layer flows of such fluids. Ranuka [3] has studied some flow problem of micro-polar fluid between two parallel plates and in a circular cylinder. Sukla et al [4] derived



the generalized Reynolds equation for micro-polar fluids with application to one dimensional slider bearing. Singh and Sinha[5] made a detailed order of the magnitude study and obtained the same form of Reynolds equations for the three-dimensional case. Laxmana Rao [2] studied the general solution of the field equations of the micro-polar fluids. As these fluids are capable of predicting the typical micro-rotational behaviour, they can play important role in engineering and biological sciences.

In this chapter we investigate the oscillatory flow behaviour of micro-polar fluid between two parallel plates. The upper plate is oscillating in the horizontal plane and lower plate is moving with a constant velocity.

FORMULATION OF THE PROBLEM

Proceeding with the usual assumptions of Stokesian flow, i.e. neglecting the velocity derivatives with respect to axial distance and leaving the inertia terms. We use the usual assumptions of lubrications theory which are valid for the present problems also. The governing equations of micro-polar fluids for the two dimensional flow between two parallel plates are as follows;

$$\frac{\partial u_x}{\partial x} = \frac{\partial v_x}{\partial y} = 0 \quad (1)$$

$$(\mu_v + k_v) \frac{\partial^2 u_x}{\partial y^2} - k_v \frac{\partial v_x}{\partial y} - \frac{\partial p_x}{\partial x} = \rho \frac{\partial u_x}{\partial t} \quad (2)$$

$$\partial_v \frac{\partial^2 \eta_x}{\partial y^2} - k_v \frac{\partial u_x}{\partial y} - 2k_v V_x = \rho J \frac{\partial v_x}{\partial t} \quad (3)$$

The equations (1), (.2) and (3) are solved under the following boundary conditions:

$$\left. \begin{array}{ll} u_x = k_1 U_0 \cos \omega t & \text{at } y = h \\ u_x = m U_0 & \text{at } y = -h \\ v_x = 0 & \text{at } y = \pm h \end{array} \right] \quad (4)$$

Where

U_0 the starting velocity and ω is the frequency of upper plate.

We introduce the following non-dimensional quantities;

$$\left. \begin{aligned}
 u_x &= U_0 u & , & & y &= h_0 y \\
 v_x &= \frac{v U_0}{h_0} & , & & y &= h x \\
 n_2 &= \frac{\mu_v}{\gamma_v} h^2 & , & & n_3 &= \frac{k_v}{\gamma_v} h^2 \\
 R_0 &= \frac{\rho h U_0}{\mu_v + k_v} & , & & J &= J h^2 \\
 \tau &= \frac{U_0 t}{h} & , & & \omega &= \frac{U_0}{h} \\
 p_x &= \frac{U_0 (\mu_v + k_v)}{h} p
 \end{aligned} \right\} \tag{5}$$

The equation (2) and (3) reduce the following form;

$$\frac{\partial^2 u}{\partial y^{*2}} + \frac{n_3}{n_2 + n_3} \frac{\partial v}{\partial y^*} - F(\tau) = R_0 \frac{\partial u}{\partial \tau} \tag{6}$$

$$\frac{\partial^2 u}{\partial y^2} - n_3 \frac{\partial u}{\partial y} - 2n_3 v = J_0 (n_2 + n_3) R_0 \frac{\partial v}{\partial t} \tag{7}$$

Where $\frac{\partial p}{\partial x} = F(\tau)$ (8)

The boundary conditions are transformed as;

$$\left. \begin{aligned}
 u &= k_1 \cos \tau & \text{at } \dot{y} &= 1 \\
 u &= m & \text{at } \dot{y} &= -1 \\
 v &= 0 & \text{at } \dot{y} &= \pm 1
 \end{aligned} \right\} \tag{9}$$

SOLUTION OF THE PROBLEM

Taking the Laplace transformation of the equation (6) and (7) we obtain;

$$\frac{\partial^2 \bar{u}}{\partial y^2} + \frac{n_3}{n_2 + n_3} \frac{\partial \bar{v}}{\partial y} - \Psi(s) = R_0 s \bar{u} \tag{10}$$

$$\frac{\partial^2 \bar{v}}{\partial y^2} - n_3 \frac{\partial \bar{u}}{\partial y} - 2n_3 \bar{v} = J_0(n_2 + n_3) R_0 s \bar{v} \tag{11}$$

Where

$$\left. \begin{aligned} \bar{u} &= \int_0^\infty e^{-s\tau} u d\tau \\ \bar{v} &= \int_0^\infty e^{-s\tau} v d\tau \\ \text{And} \\ \Psi(s) &= \int_0^\infty e^{-s\tau} F(\tau) d\tau \end{aligned} \right\} \tag{12}$$

The boundary condition transform to:

$$\left. \begin{aligned} \bar{u} &= \frac{k_1 s}{1 + s} & \text{at } \dot{y} = 1 \\ \bar{u} &= \frac{m}{s} & \text{at } \dot{y} = -1 \\ \text{And } \bar{v} &= 0 & \text{at } \dot{y} = \pm 1 \end{aligned} \right\} \tag{13}$$

Eliminating \bar{u} among the equation (10) and (11) we get;

$$(D^2 - I_1^2)(D^2 - I_2^2) \bar{v} = 0 \tag{14}$$

Where

$$\left. \begin{aligned} I_1^2 + I_2^2 &= \frac{n_3^2}{n_2 + n_3} + \frac{2n_2 n_3}{n_2 + n_3} + 2n_3 + R_0 \{ J_0(n_2 + n_3) + 1 \} s \\ \text{And} \\ I_1^2 + I_2^2 &= 2R_0 n_3 s + J_0(n_2 + n_3) R_0^2 s^2 \end{aligned} \right\} \tag{15}$$

We consider at present the case for large times i.e. s to be very small and weak coupling and n_3 is small, are comparison to n_2 . with these assumptions we obtain the approximate value of I_1 and I_2 as follows.

$$\begin{aligned}
 I_1 &= \pm \left[\frac{n_3(2n_2 + n_3)}{(n_2 + n_3)} + J_0(n_2 + n_3)R_0s \right]^{\frac{1}{2}} \\
 \text{And } I_2 &= \pm (R_0s)^{\frac{1}{2}}
 \end{aligned} \tag{16}$$

Thus the solution of equation (14) is given by:

$$\bar{v} = A_{11} \cos hI_1\dot{y} + B_{11} \sin hI_1\dot{y} + C_{11} \cos hI_2\dot{y} + D_{11} \sin I_2\dot{y} \tag{17}$$

Where A_{11} , B_{11} , C_{11} and D_{11} are arbitrary constants.

Using the value of \bar{v} from (15) in (14) and showing we get;

$$\begin{aligned}
 \bar{u} &= \frac{1}{I_1n_3} \left[\{I_1^2 - (2n_3 + J_0(n_2 - n_3)R_0s)\} (A_{11} \sin hI_1\dot{y} + B_{11} \cos hI_1\dot{y}) \right] \\
 &+ \frac{1}{I_2n_3} \left[\{I_2^2 - (2n_3 + J_0(n_2 + n_3)R_0s)\} (C_{11} \sin hI_2\dot{y} + D_{11} \cos hI_2\dot{y}) \right] + E_{11}
 \end{aligned} \tag{18}$$

Finally the constant E_{11} is eliminated by putting the value of \bar{u} and \bar{v} is equation (10) we get;

$$\bar{u} = \frac{\left[R_0 \{1 - J_0(n_2 + n_3)\} s - n_3 \right]}{n_3 I_2} (C_{11} \sin hI_2\dot{y} + D_{11} \cos hI_2\dot{y}) - \frac{\Psi(s)}{s} \tag{19}$$

Where the constants A_{11} , B_{11} , C_{11} and D_{11} are given as follows;

$$\left. \begin{aligned}
 A_{11} &= \frac{-\{(k_1 - m)s^2 - m\} n_3 I_2 \cos hI_2}{2s(1 + s^2) \left[R_0 \{1 - J_0(n_2 + n_3)\} s - n_3 \right] \sin h2 \cosh I_1} \\
 B_{11} &= \frac{-[k_1 s^2 + m + 2\Psi(s)] n_3 I_2 \sinh I_2}{2s(1 + s^2) \left[R_0 \{1 - J_0(n_2 + n_3)\} s - n_3 \right] \sin hI_1 \cosh I_2} \\
 C_{11} &= \frac{\{(k_1 - m)s^2 - m\}}{2s(1 + s^2) \left[R_0 \{1 - J_0(n_2 + n_3)\} s - n_3 \right] \sin h2} \\
 D_{11} &= \frac{\{k_1 + m + 2\Psi(s)\}}{2s(1 + s^2) \left[R_0 \{1 - J_0(n_2 + n_3)\} s - n_3 \right] \sin hI_1}
 \end{aligned} \right\} \tag{20}$$

For small s, expanding all the terms in ascending powers of s and retaining only the first order terms except the singularities in the denominator. Then we get the expression \bar{u} and \bar{v} as follows,

$$\bar{u} = \frac{1}{2} \left[\left(\frac{k_1 s}{1 + s^2} - \frac{m}{s} \right) \frac{\sin hI_2 \dot{y}}{\sin hI_2} + \left(\frac{k_1 s}{1 + s^2} + \frac{m + 2\Psi(s)}{R_0 s} \right) \frac{\cos hI_2 \dot{y}}{\cos hI_2} \right] - \frac{\Psi(s)}{R_0 s} \tag{21}$$

And

$$\begin{aligned}
 \bar{v} &= \frac{n_3}{2a} \left[\frac{k_1(1 - R\dot{y}^2)}{s - \frac{n_3}{a}(1 + s^2)} + \frac{R_0(k_1 - m)\dot{y}^2}{\left(s - \frac{n_3}{a}\right)} - \frac{m}{s\left(s - \frac{n_3}{a}\right)} + \left\{ \frac{k_1}{\left(s - \frac{n_3}{a}\right)\left(s + \frac{1}{R}\right)} - \frac{k_1}{\left(s - \frac{n_3}{a}\right)\left(s + \frac{1}{R_0}\right)(1 + s^2)} + \frac{m + 2\Psi(s)}{R_0\left(s - \frac{n_3}{a}\right)\left(s + \frac{1}{R_0}\right)} \right\} \dot{y} - \right. \\
 &\left. \left\{ \frac{k_1(1 - R_0)}{\left(s - \frac{n_3}{a}\right)(1 + s^2)} + \frac{R_0(k_1 - m)}{\left(s - \frac{n_3}{a}\right)} - \frac{m}{s\left(s - \frac{n_3}{a}\right)} \right\} \frac{\cos hI\dot{y}}{\cos hI_1} - \left\{ \frac{k_1}{\left(s - \frac{n_3}{a}\right)\left(s + \frac{1}{R_0}\right)} - \frac{k_1}{\left(s - \frac{n_3}{a}\right)\left(s + \frac{1}{R}\right)(1 + s^2)} + \right. \right. \\
 &\left. \left. \frac{m + 2\Psi(s)}{R_0\left(s - \frac{n_3}{a}\right)\left(s + \frac{1}{R_0}\right)} \right\} \frac{\sin hI_1 \dot{y}}{\sin hI_1} \right] \tag{22}
 \end{aligned}$$

Case taking $\Psi(s) = \frac{k_2}{s}$ or $f(\tau) = k_2$ (constant)

i.e. a constant pressure gradient applied. Put this value of $\Psi(s)$ in equation (11) and (12) and taking inverse Laplace transform, we get;

$$\begin{aligned}
 u = \frac{1}{2} & \left[k_1 \left(\frac{\sin \sqrt{iR_0} y}{\sinh \sqrt{iR_0}} e^{-i\tau} + \frac{\sinh \sqrt{iR_0} y}{\sinh \sqrt{iR_0}} e^{i\tau} \right) - m\dot{y} + k \left(\frac{\cos \sqrt{iR_0} y}{\cosh \sqrt{iR_0} I_1} e^{-i\tau} + \frac{\cosh \sqrt{iR_0} y}{\cosh \sqrt{iR_0} I_1} e^{+i\tau} \right) + m \right. \\
 & \left. + k_2 (y^2 - 1) \right] \\
 & \sum_{n=1}^{\infty} \frac{k_1 2n^2 \pi^3 R_0}{n^4 \pi^4 - R_0^2} (-1)^n \sin n \pi y^* e^{-\frac{n^2 \pi^2 \tau}{R_0}} + \sum_{n=1}^{\infty} \left(\frac{4(2n+1)^3 \pi^3 k_1}{(2n+1)^4 \pi^4 - R_0^2} + \frac{8m}{(2n+1)\pi} \right) \\
 & (-1)^n \cos \left(\frac{2n+1}{2} \right) \pi y^* e^{-\frac{(2n+1)^2 \pi^2 \tau}{2 R_0}} + \sum_{n=1}^{\infty} \frac{4k_2 R_0^2 (-1)^n}{\pi^3 \left(\frac{2n-1}{2} \right)^3} \cos \left(\frac{2n+1}{2} \right) \pi y^* e^{-\frac{(2n+1)^2 \pi^2 \tau}{2 R_0}} \quad (23)
 \end{aligned}$$

And

$$\begin{aligned}
 v = \frac{n_3}{2a} & \left[\left\{ (a_{11} + a_{14}) \left(1 - \frac{\cos ha^* y^*}{\cos ha^*} \right) - (a_{12} - a_{13}) \left(y^{*2} - \frac{\cos ha^* y^*}{\cos ha^*} \right) \right. \right. \\
 & \left. \left. + (a_{17} + a_{18}) \left(y - \frac{\sin h^\alpha y^\alpha}{\sin ha^\alpha} \right) \right\} e^{\frac{n_3 \tau}{a}} + a_{14} \left(\frac{1 - \cos ha_1 y^*}{\cos ha^*} \right) \right. \\
 & \left. - a_{22} \left(y^* - \frac{\sin hb^* y^*}{\sin hb^*} \right) - e^{-\frac{\tau}{R_0}} - a_{23} \left(y^* - \frac{in ha_1 y^*}{\sin ha_1} \right) \right. \\
 & \left. - \left\{ (b_{11} (1 - R_0 - y^{*2}) + b_{12} y^* - b_{16} \cos ha_1 y^* - b_{19} \sin ha_1 y^* \sin b_1 y^* \right. \right. \\
 & \left. \left. - d_{17} \sin ha_1 y^* \cos b_1 y^* - d_{18} \sin ha_1 y^* \cos b_1 y^* + d_{19} \cos ha_1 y^* \cos b_1 y^* \right\} \sin \tau \right. \\
 & \left. - e_{17} \cos ha_1 y^* \sin b_1 y^* - e_{18} \cos ha_1 y^* \sin b_1 y^* - e_{19} \cos ha_1 y^* \cos b_1 y^* \right) \cos \tau \\
 & \left. - 2\pi \sum_{n=0}^{\infty} \left\{ Q_1 (-1)^n \cos Q_2 y^* e^{(-Q+iQ_2)\tau} + (Q_2 + Q_{35}) (-1)^n \cos Q_3 y^* \right\} e^{(-Q+iQ_3)\tau} \right] \quad (24)
 \end{aligned}$$

RESULTS AND CONCLUSIONS :

The results have been given in table 5.1 to 5.5 and some of them have been represented numerically. Thus, the import conclusion of these results are as follows.

1. When both plates are fixed and only pressure is acting then velocity profiles are negative for small Reynold's number.
2. The velocity profile reduce with the increase of time, which shows that the micro-rotation become dominant as the time lapses and tries to retard the motion of the fluid.
3. From the table it is clear that the micro-rotation change their way and get reversed as the time increases.
4. It is observed that the micro-rotation decay in the vicinity of the wall but become active in the central region with lapse of time and micro-rotation effects are larger for large Reynolds number.

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