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NUMERICAL ANALESSIS OF NON-STEADY FLOW OF A CODUCTING BINGHAM FLUID IN AN ANNULUS IN PRESENCES OF TIME VARYING TOROIDAL PRESSURE GRADIENT

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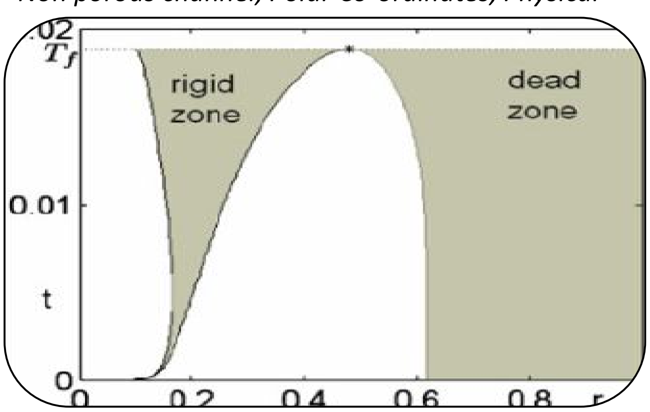
ABSTRACT:

In this paper concerned with the non-steady flow of a conducting Bingham plastic in an annulus under time varying toroidal pressure gradient. It has been assumed that a constant radial magnetic field is acting on the fluid and no applied or polarization voltage exist, i.e. no energy is being added or extracted by electrical means. It has been observed that the toroidal component of the velocity in Bingham fluid is grater then that of viscous fluid and it increases as the value of non-dimensional Bingham number increases. The present paper aims at extending the investigations of reference Singh [9] & Attia & Ahmed [10] to Bingham plastics. We have, however, considered a non-porous channel which is appropriate for fluid under discussion.

KEY-WORDS: *Newtonian fluids. Viscoelastic fluids, Non porous channel, Polar Co-ordinates, Physical components, Bingham fluids, Bingham plastic, Stress components, Dimensionless quantities, Bingham number, Pressure gradient, Laplace transform.*



INTRODUCTION:

Under the impetus of both academies curiosity and promotional necessity considerable efforts are being made in recent years to investigate unsteady flow problem of non-Newtonian fluids. The unsteady flow or Bingham plastic was initiated by Oldroyd [1]. Non-steady rotatory flow of such fluids has been discussed by Praia [2], Gireesha, Venkatesh &Bagewadi [3] and Ret west [4]. Kapur & Srivastava [5], Ghosh & Debnath [6] have discussed unsteady flow of viscous fluid in an annulus in presence of a time varying steroidal pressure radiant. Srivastava and Tendon [7] have extended this problem for viscoelastic fluids. Prakash [8] has discussed laminar flow in an annulus with arbitrary time varying pressure gradient and arbitrary initial velocity. Recently, Singh [9], Maryem &Azzouzi [11] have investigated the unsteady notion of viscous conducting fluid in an annulus between two porous coaxial oracular cylinders under a steroidal pressure gradient.

2. FORMULATION OF THE PROBLEM:

We take cylindrical polar coordinate system (γ', θ', z') with x-axis along the common axis of two infinitely long coaxial circular cylinders of radii dR and $R(d < 1)$. Due to axial symmetry, the velocity field is independent of θ. Therefore, the physical components of the velocity of a conducting Bingham fluid in absence of suction and injection are given by

$$v_r = 0\,, v_\theta = v^1(v,t)\,, v_z == 0 \tag{2.1}$$

Using the rheological equation $-S_{ij} = v_{ij} + 2\eta_0 e_{ij}$ for a Bingham plastic, the stress components for the present problem are given by

$$S_{rr} = S_{\theta\theta} = S_{zz} = S_{rz} = S_{\theta z} = 0 \tag{2.2}$$

$$S_{r\theta} = v + \eta_0 r \frac{\partial}{\partial r}, \left(\frac{v}{r}\right) \tag{2.3}$$

It is further assured that no applied or polarization voltages exist, i.e. $\vec{E} = \vec{O}$ which corresponds to the once when no energy is being added or extracted by electrical means. Under these assumption, the Lorentz's force in θ-direction is given by Hetsroni, Mosyak, Pogrebnyak, & Yarin [12]

$$F_\theta = \frac{\sigma He^2 v \mu e^2}{r^2} \tag{2.4}$$

Where H_e is the applied magnetic field.

Under these circumstances, the momentum equation $-\rho \frac{D}{Dt} v = \rho_{ij}^{lij} + \in^{ikl} J_k B_l$ for θ-direction reduces to:

$$\frac{dv}{dt} = -\frac{1}{er}\frac{d\rho}{d\theta} + \frac{2v}{\rho r} + \frac{\eta_0}{\rho}\left[\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r}\right. \tag{2.5}$$

$$-\frac{v}{r^2} - \frac{\sigma v \mu_e{}^2 Hc^2}{\eta_0 r^2}$$

The imposed boundary and initial conditions are:

$$\left.\begin{matrix} v = 0\,, r = dR \\ v = 0\,, r = R \end{matrix}\right] \qquad t > 0 \tag{2.6}$$

$$v = 0 \qquad\qquad t < 0$$

Taking a content characteristic pressureP_e, it is now convenient to introduce the following dimensionless quantities:

$$r = \frac{r}{R}\,, t = \frac{\eta_0 t}{\rho R^2}\,, v = \frac{2\eta_0 v}{\rho R}\,,$$

$$B = \frac{4vl}{\rho_c}\,, M_n^2 = \frac{\sigma \mu^2 Hc^2}{\eta_0} \tag{2.7}$$

$$and \quad f(t) = -\frac{2}{Pc}\frac{\partial p}{\partial \theta}$$

Where

Mn= Magnetic field perimeter

B = Bingham number

$f(t)$ = Pressure gradient

In view of the relations (2.7) equations (2.5) and (2.6) yield respectively

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} + \frac{f(t)+B}{r} - \frac{N^2 v}{r^2} \tag{2.8}$$

Where

$$N^2 = 1 + M_n^2 \tag{2.9}$$

And

$$\left.\begin{array}{l} v = 0\,, r = d \\ v = 0\,, r = 1 \end{array}\right\} t \geq 0 \qquad v = 0 \qquad t \leq 0 \tag{2.10}$$

3. SOLUTION OF THE PROBLEM:

Multiplying the question (2.8) by e^{-st} and integrating is with respect to t from o to ∞, we obtain

$$\frac{d^2\bar{v}}{dr^2} + \frac{1}{r}\frac{d\bar{v}}{dr} - \left(\frac{N^2}{r^2} + s\right)\bar{v} = -\frac{\overline{f(s)}+B/s}{r} \tag{3.1}$$

Where

$$\bar{v} = \int_0^\infty v\ \bar{e}^{st}dt \tag{3.2}$$

And

$$\overline{f(s)} = \int_0^\infty f(t)\ \bar{e}^{st}dt \tag{3.3}$$

The boundary conditions on $\bar{v}$ may be written as

$$at\ R = d\,, \bar{v} = 0 \qquad at\ R = l\,, \bar{v} = 0 \tag{3.4}$$

Employing the technique of variation of premature for solving the question (3.1) subject to conditions (3.4), we have finally

$$\bar{v} = \frac{\overline{f(s)}+B/s}{s}\left[\frac{1}{r} - \frac{\bar{d}^2 T_N(r_1,\sqrt{s})T_N(d,r,\sqrt{s})}{T_N(d,1,\sqrt{s})}\right] \tag{3.5}$$

Where

$$T_N(x,y,\sqrt{s})\ I_N(x\sqrt{s})\ K_N(y\sqrt{s}) - I_N(y\sqrt{s})\ K_N(x\sqrt{s}) \tag{3.6}$$

And I_N and K_N are Bessel's functions of imaginary arguments of order N.

Assuming $\overline{f(s)}$ to be of the form $f_1(s)/f_2(s)$ where both $f_1(s)$ and $f_2(s)$ are single valued and following Singh [9], we get from the equation (3.5) by the inversion theorem of Laplace transform

$$v = -\sum \frac{e^{-ket} f_1(-ke)}{k_e f_2(-ke)} \left[\frac{1}{r} - \frac{\bar{d} T_N\left(r, 1, \sqrt{ke}\right) + T_N\left(d, r, \sqrt{Ke}\right)}{T_N\left(d, 1, \sqrt{ke}\right.} \right]$$

$$+ \sum \frac{f_1(-\alpha_1^2 \bar{e}^{\alpha_i^2 t}}{f_2\left(-\alpha_i^2\right)} \left[\frac{\bar{d} J_N(\alpha_i) + J_N(d\alpha_i) - {J_N}^2(d\alpha_i)}{{J_N}^2(d\alpha_i) - {J_N}^2(\alpha_i)} T_N(r, 1, \alpha_i) \right]$$

$$+ B \sum \frac{(1 - \bar{e}^{\alpha_i^2 t}}{(\alpha_i^2)} \left[\frac{\bar{d} J_N(\alpha_i) + J_N(d\alpha i) - {J_N}^2(d\alpha_i)}{{J_N}^2(d\alpha_i) - {J_N}^2(\alpha_i)} T_N(r, 1, \alpha_i) \right] \tag{3.7}$$

Where -K_e is a sere of $Sf_2(s) = 0$ and $S = -\propto_i^2$ are zeros of$T_N\left(d, 1, \sqrt{s}\right) = 0$. The expression for the transverse velocity in the absences of magnetic field is obtained by setting $Mn(Megnetic\ field\ perametr) = 0$ in the equation (3.7) and is given by

$$v = -\sum \frac{e^{-ket} f_1(-ke)}{k_e f_2(-ke)} \left[\frac{1}{r} \frac{\bar{d} T_1\left(r, 1, \sqrt{ke}\right) + T_1\left(d, r, \sqrt{Ke}\right)}{T_1\left(d, 1, \sqrt{ke}\right)} \right]$$

$$+ \sum \frac{f_1(-\alpha_1^2 \bar{e}^{\alpha_i^2 t})}{f_2\left(-\alpha_i^2\right)} \left[\frac{\bar{d} J_1(\alpha_i) + J_1(d\alpha_i) - {J_1}^2(d\alpha_i)}{{J_1}^2(d\alpha_i) - {J_1}^2(\alpha_i)} T_1(r, 1, \alpha_i) \right]$$

$$+ B \sum \frac{(1 - \bar{e}^{\alpha_i^2 t})}{(\alpha_i^2)} \left[\frac{\bar{d} J_1(\alpha_i) + J_1(d\alpha_i) - {J_1}^2(d\alpha_i)}{{J_1}^2(d\alpha_i) - {J_1}^2(\alpha_i)} T_1(r, 1, \alpha_i) \right] \tag{3.8}$$

This case has been discussed by Kapur & Srivastava [5] by using Hansel transform for viscous Newtonian fluid (i.e. B=0)

4. PARTICULAR- CASES:

Case - [I] if the pressure gradient is given by Hwang & Kim [13]

$$f(t) = D\bar{e}^{ct} \tag{4.1}$$

Then we have

$$\overline{f(s)} = \frac{D}{s+c} \tag{4.2}$$

Consequently, the expression for velocity yields

$$v = \frac{-D\bar{e}^{ct}}{c}\left[\frac{1}{r} - \frac{\bar{d}^{1}T_N\left(r,1,\sqrt{c}\right) + T_N\left(d,r,\sqrt{c}\right)}{T_N\left(d,1,\sqrt{c}\right)}\right]$$

$$+D\sum\frac{\bar{e}^{\alpha_i^2 t}}{C-\alpha_i^2}\left[\frac{\bar{d}^{1}J_N(\alpha_i)\times J_N(d\alpha_i) - {J_N}^2(d\alpha_i)}{{J_N}^2(d\alpha_i) - {J_N}^2(\alpha_i)}T_N(r,1,\alpha_i)\right]$$

$$+B\sum\frac{(1-\bar{e}^{\alpha_i^2 t})}{\alpha_i^2}\left[\frac{\bar{d}^{1}J_N(\alpha_i)+J_1(d\alpha i)-{J_N}^2(d\alpha i)}{{J_N}^2(d\alpha_i)-{J_N}^2(\alpha_i)}T_N(r,1,\alpha_i)\right] \tag{4.3}$$

The corresponding expression for the non-hydro magnetic case is given by (setting Mn=0),

$$v = \frac{-D\bar{e}^{ct}}{c}\left[\frac{1}{r} - \frac{\bar{d}^{1}T_1\left(r,1,\sqrt{c}\right) + T_1\left(d,r,\sqrt{c}\right)}{T_1\left(d,1,\sqrt{c}\right)}\right]$$
$$+D\sum\frac{\bar{e}^{\alpha_i^2 t}}{C-\alpha_i^2}\left[\frac{\bar{d}J_1(\alpha_i)J_1(d\alpha_i) - {J_1}^2(d\alpha_i)}{{J_1}^2(d\alpha_i) - {J_1}^2(\alpha_i)}T_1(r,1,\alpha_i)\right]$$
$$+B\sum\frac{(1-\bar{e}^{\alpha_i^2 t})}{\alpha_i^2}\left[\frac{\bar{d}J_N(\alpha_i)+J_N(d\alpha_i)-{J_1}^2(d\alpha_i)}{{J_1}^2(d\alpha_i)-{J_1}^2(\alpha_i)}T_1(r,1,\alpha_i)\right] \tag{4.4}$$

If the pressure gradient is constant (set C= 0), the corresponding expression for the velocity is given by Avinash [14]

$$v = -D\sum\frac{e^{-\alpha^{2it}}}{\alpha_i^2}\left[\frac{\bar{d}^{1}J_1(\alpha_i) + J_1(d\alpha_i) - {J_1}^2(d\alpha_i)}{{J_1}^2(d\alpha_i) - {J_1}^2(\alpha_i)}T_N(r,1,\alpha_i)\right]$$

$$+B\sum\frac{(1-\bar{e}^{\alpha_i^2 t})}{\alpha_i^2}\left[\frac{\bar{d}^{1}J_1(\alpha_i) + J_1(d\alpha_i) - {J_1}^2(d\alpha_i)}{{J_1}^2(d\alpha_i) - {J_1}^2(\alpha_i)}T_1(r,1,\alpha_i)\right]$$

$$+\frac{D}{2}\left[r\log r + \frac{d^2(1-r^2)\log d}{r(d^2-1)}\right] \tag{4.5}$$

From the above equation (4.5), in the limiting case, we obtain the velocity for steady flow for viscous fluid under constant toroidal pressure gradient as

$$v = -\frac{D}{2}\left[r\log r + \frac{d^2(1-r^2)\log d}{r(d^2-1)}\right] \tag{4.6}$$

This is the same result as obtained by Goldstein [15].

Case - [II]

Let the pressure gradient be periodic so that

$$f(t) = -re^{i\beta t} \tag{4.7}$$

Then the expression for velocity acquires the form

$$v = \frac{ire^{i\beta t}}{\beta}\left[\frac{1}{r} - \frac{\bar{d}^1 T_N\left(r, 1, \sqrt{-i\beta}\right) + T_N(d, r, -i\beta}{T_N(d, 1, \sqrt{-i\beta})}\right]$$

$$+r\sum \frac{\bar{e}^{\alpha_i^2 t}}{i\beta + \alpha_i^2}\left[\frac{\bar{d}^1 J_N(\alpha_i) + J_1(d\alpha_i) - {J_N}^2(d\alpha_i)}{{J_N}^2(d\alpha_i) - {J_N}^2(\alpha_i)} T_N(r, 1, \alpha_i)\right]$$

$$+B\sum \frac{(1-\bar{e}^{\alpha_i^2 t})}{\alpha_i^2}\left[\frac{\bar{d}^1 J_N(d\alpha_i) + J_N(\alpha_i) - {J_N}^2(d\alpha_i)}{{J_N}^2(d\alpha_i) - {J_N}^2(\alpha_i)} T_1(r, 1, \alpha_i)\right] \tag{4.8}$$

5. ASYMPTOTIC SOLUTION:

Considering constant toroidal pressure gradient D, the equation (3.5) yields by Hsich, Tsai, Lin, Huang & Chien [16]

$$\bar{v} = \frac{D+B}{S^2}\left[\frac{1}{r} - \frac{\bar{d}^1 T_N(r,1,\sqrt{s}) + T_N(d,r,-\sqrt{s})}{T_N(d,1,\sqrt{s})}\right] \tag{5.1}$$

In what follows, we shall make use of the asymptote expansions for $I_N(\sqrt{s}\times)$ and $k_n(\sqrt{s}\times)$ Vis.

$$I_N(\times\sqrt{s}) = \frac{e^{\sqrt{s}\times}}{(2\pi\times\sqrt{s})^{v_2}}\left[1 - \frac{4N^2-1}{8\times\sqrt{s}} + \ \dots\dots\dots\dots..\right] \tag{5.2}$$

$$K_N(X\sqrt{s}\ ) = e^{\sqrt{s}X}\left(\frac{\pi}{2\times\sqrt{s}}\right)^{1/2}\left[1 + \frac{4N^2-1}{8\times\sqrt{s}} + \ \dots\dots\dots\dots..\right] \tag{5.3}$$

And the formula

$$Sinh \times cosh \times = \frac{1}{2}e^{\times} \tag{5.4}$$

For large X.

Since motion for only small values of time is of interest, we have expanded for large values of S by Datta & Dalal [17]. Substituting the various asymptotic expansions in equation (5.1) and retaining term containing $\frac{1}{S^2}$, we obtain

$$\bar{v} = \frac{(D+B)}{s^2}\left[\frac{1}{r} + \frac{\bar{e}^{\sqrt{s}(r-d)}}{\sqrt{rd}} + \frac{\bar{e}^{\sqrt{s}(i-r)}}{\sqrt{r}}\right] \tag{5.5}$$

Employing the Lapse-inversion theorem, we get

$$v^* = (1 + {B^*}_m)\left[\frac{t}{r} + \frac{1}{\sqrt{rd}}\left\{t + \frac{(r-d)^2}{2}\right\} erfc\left(\frac{r-d}{2\sqrt{t}}\right)\right.$$

$$-\sqrt{\frac{t}{\pi r d}}(r-d)\bar{e}^{\frac{(r-d)^2}{4t}}+\frac{1}{\sqrt{r}}\left\{t+\frac{(1-r)^2}{2}\right\}erfc\left(\frac{1-r}{2\sqrt{t}}\right)$$

$$\left.-\sqrt{\frac{t}{\pi r}}(1-r)\,\bar{e}^{\frac{(1-r)^2}{4t}}\right] \quad (5.6)$$

Where

$$B^*{}_m=\frac{B}{D}\ \ and\ v^*=\frac{v}{D} \quad (5.7)$$

6. CONCLUSIONS:

Tables A and B have been prepared form the expression (5.6) for the velocity in θ-direction. It is observed from these tables that the velocity at particular instant and place increases as non-dimensional Bingham number increases and the velocity are greater in Bingham fluid than that of a viscous fluid, Erdelyi. & Associates [18]

TABLE-A

Variation of velocity profile (asymptotic) for various values of r $and\ B^*$ when $t=\ .5$

r/B^*	0.0	0.2	0.4
.6	1.9353	2.3223	2.7094
.7	1.6828	2.0395	2.3559
.8	1.5044	1.8053	2.1062
.9	1.3799	1.6559	1.9319

TABLE-B

Variation of velocity profile (asymptotic) for various values of $t\ and\ B^*$ when $r=\ .6$

t/B^*	0.0	0.2	0.4
0.5	1.9353	2.3223	2.7094
1.0	4.0992	4.9191	2.7389
1.5	6.3151	7.5782	0.8412
2.0	8.5903	10.2966	11.9816

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