ISSN 2249-894X Impact Factor : 3.1402 (UIF) Volume - 5 | Issue - 12 | September - 2016

Review Of Research

QUASI K-IDEALS IN INTRA K-REGULAR Γ −SEMIRINGS

Alandkar S. J.
Head, Department of Mathematics,Walchand College of Arts & Science, Solapur.

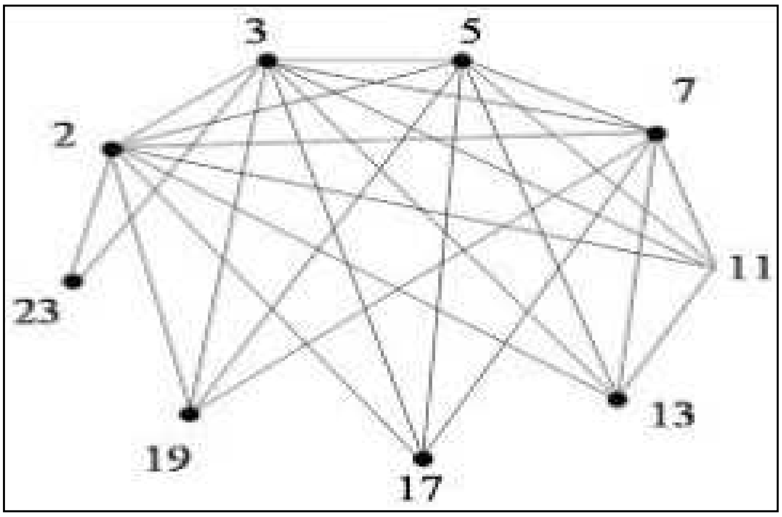
ABSTRACT. A Γ-semiring M whose additive reduct is a semilattice, is called an intra k-regular Γ-semiring if for each a ∈S there exists x ∈ M such that $a + x\alpha a\alpha a\alpha x = x\alpha a\alpha a\alpha x$, $\alpha \in \Gamma$.Here we introduce quasi k-ideals in Γ-semirings and characterize both the k-regular and intra k-regular Γ-semirings by their quasi k-ideals.

Mathematics Subject Classi cation(2010). 16Y60.

Keywords: Quasi k-ideals, k-ideals, k-regular Γ-semirings, intra k-regular Γ-semirings.

1 INTRODUCTION.

Otto Steinfeld [17], [18], [19], [20] introduced the notion of quasi ideals in rings and semigroups. Quasi ideals are generalizations of both left ideals and right ideals as well as a particular case of bi-ideals. S. Lajos [10] defined a generalization of this notion namely the (m,n)-quasi ideal. Lajos[11] also characterized the quasi ideals in regular semigroups. Kapp [8] observed that quasi ideal can also be obtained by an absorbant semigroup with 0 and every $\mathcal{H}$ -class together with 0. Many authors in [7], [9], [5] characterized the quasi ideals of different classes of semigroups and semirings. Von Neumann[12] defined a ring R to be regular if the multiplicative reduct (R, .) is a regular semigroup.



Semirings in SL+ by their different types of ideals are studied in [1]. Bourne [4] introduced the k-regular semirings as a generalization of regular rings. Further these semirings have been studied by Sen, Weinert, Bhuniya, Adhikari [1], [14], [15], [16]. For any semigroup F, the semiring P(F) of all subsets of F is a k-regular semiring if and only if F is a regular semigroup [16]. We have introduced the intra k-regular semirings as the class of semirings to which the semiring P(F) belong when F is an intra regular semiring [3]. Also a semiring S is intra k-regular if and only if every k-ideal of S is semiprime.

Rao M. K.[13] defined Γ-semiring as generalization of semiring and Γ-rings. Alandkar S. J.[2] studied and characterize the k-regular Γ-semirings by their quasi k-ideals. And studied the notion of quasi k-ideals in a semiring and characterize the k-regular semirings using quasi k- ideals. In this paper we used the quasi k-ideals to characterize the intra k-regular Γ -semirings and the Γ -semirings which are intra k-regular.

2. PRELIMINARIES.

Recall the definitions from[2]

Definition 2.1. Γ- semiring. Let (M, +) and (Γ, +) be commutative semigroups. Define the mapping $M\times \Gamma \times M \rightarrow M$ (image to be denoted by (x, α, y) →xαy) satisfying the following conditions:

i) xαy ∈ M,

ii) (x+y) αz = (xαz) + (yαz), x(α + β)z = xαz+ xβz, xα(y + z) = xαy+ xαz,

iii)(xαy)βz = xα(yβz), for all x, y, z ∈ M and all α, β∈ Γ.

Then M is a Γ- semiring.

Every Γ-ring is a Γ- semiring but every Γ-semiring need not be a Γ-ring. For this we consider the following Example.

Example 1. Let M= Γ = (Z^+, +) be the semigroup of all nonzero positive integers. Define the mapping $M\times \Gamma \times M \rightarrow M$ (image to be denoted by (x, α, y) → xαy) where xαy is the usual multiplication of the x, α and y for all x, y ∈ M and all α∈ Γ. Then M is a Γ-semiring but not a semiring.

Definition 2.2. Sub-Γ-semiring. Let M be a Γ-semiring. A nonempty subset S of M is a sub-Γ-semiring of M if S itself is a Γ-semiring with the same operations of Γ-semiring M.
Definition 2.3. Ideal of a Γ-semiring. A nonempty subset I of a Γ-semiring M is a left (resp. right) ideal of M if for x, y ∈ I and r ∈M we have x+y ∈ I and rαx ∈I(resp. xαr ∈I), where α ∈ Γ. If I is both left as well as right ideal then we say that I is an ideal of M.
Example 2. Consider the Example of the Γ-semiring M followed by the definition. Here I = $(2Z^+, +, \Gamma)$ is an ideal of M**.**
Following are the definitions introduced for generalization of algebraic structure semigroups, semilattice , semiring, reduct by considering set Γ of operations :
Definition 2.4. A band is a Γ-semigroup in which every element is an idempotent. A commutative band is called a Γ-semilattice. Throughout this paper, unless otherwise stated, M is always a Γ-semiring whose additive reduct is a Γ-semilattice and the variety of all such Γ-semirings is denoted by ML+.
Definition 2.5. A non-empty subset L of a Γ-semiring M is called a left ideal of M if L + L⊆ L and M ΓL ⊆ L. The right ideals are defined dually. A subset I of M is called an ideal of M if it is both a left and a right ideal of M. A non-empty subset A is called an interior ideal of M if A + A ⊆ A and M ΓAΓ M ⊆ A. A non-empty subset A of M is called semiprime if for a ∈ M, a^2=aαa ∈ A implies that a ∈ A.
Definition 2.6. Henriksen [6] defined an ideal (left, right) I of a semiring S to be a k-ideal (left, right) if for a; x ∈ S, a; a + x ∈ I) x ∈ I. We extend this concept to Γ-semiring
We define interior k-ideal similarly.
Definition 2.7. A non-empty subset A of M is called a k-subset of Γ-semiring M if for x ∈ M, a ∈ A; x + a ∈ A implies that x ∈ A.
Definition 2.8. The k-closure $\bar{A}$ of a non-empty subset A is given by,

$$\bar{A} = \{x \in S / \exists\ a, b \in A \text{ such that } x + a = b \}.$$

This is the smallest k-subset containing A. If A and B be two subsets of M such that A ⊆ B then it follows that $\bar{A} \subseteq \bar{B}$. Since the additive reduct (M, +) is a Γ-semilattice, it follows that an ideal (left, right) K of M is a k-ideal(left, right) if and only if $\bar{K}$ = K.
Definition 2.9. A sub Γ-semiring Q is called a quasi ideal of M if QΓM∩ MΓQ ⊆ Q. A quasi ideal Q is called a quasi k-ideal of M if Q = Q.
For examples of quasi k-ideals of a Γ-semiring we would like to explore the following natural connection between quasi ideals of a Γ-semigroup F and quasi k-ideals of the Γ-semiring P(F) of all subsets of F.
Definition 2.10. Let F be a Γ-semigroup and P(F) be the set of all subsets of F. Define addition and multiplication on P(F) by:

U + V = U ∪ V and
U Γ V = {aαb / a ∈ U; b ∈ V, α ∈ Γ} }, for all U, V ∈ P(F),

Then (P(F); +; α) , α ∈ Γ is a Γ-semiring whose additive reduct is a Γ-semilattice. Then we have the following result.
Theorem 2.11 Let F be a semigroup. Then Q is a quasi k-ideal of P(F) if and only if Q = P(P) for some quasi-ideal P of F.[2]
Lemma 2.12. Let S be a semiring. Then for all right k-ideal R and left k-ideal L of S, R ∩ L is a quasi k-ideal of S.[2]
Lemma 2.13. Let M be a Γ-semiring and a ∈ M.
1.Then the principal left k-ideal of M generated by a is given by $L_k(a)$ = {u ∈ M / u+a + sαa = a + sαa, for some s∈ M, α ∈ Γ }.
2.Then the principal right k-ideal of M generated by a is given by $R_k(a)$ = {u ∈M / u + a + aαs = a + aαs,for some s ∈ M, α ∈ Γ }.

Bourne [3] defi ned a Γ-semiring M to be regular if for each a∈ M there exist x, y∈ M such that a + aαxαa = aαyαa, for α ∈ Γ. If a Γ-semiring M happens to be a ring then the Von Neumann regularity and the Bourne regularity are equivalent. This is not true in a Γ-semiring in general (For counter example we refer [12]). Adhikari, Sen and Weinert [1] renamed the Bourne regularity of a Γ-semiring as k-regularity to distinguish from the notion of Von Neumann regularity.
Definition 2.14. A Γ-semiring M is called a k-regular Γ-semiring if for each a ∈ M there exist x, y ∈ M such that a + aαxαa = aαyαa, α ∈ Γ.

Since (M, +) is a semilattice,
we have a + aαxαa = aαyαa ⟹ a + aαxαa + (aαxαa + aαyαa) = aαyαa + (aαxαa +aαyαa)
⟹ a + aα(x+y)αa = aα(x+ y)αa .
Thus, a Γ-semiring M is k-regular if and only if for all a∈ M there exists x∈ M such that a + aαxαa = aαxαa..
Let M be a k-regular Γ-semiring and a∈ M. Then there exists x ∈ M such that a + aαxαa = aαxαa. Then we have
a + aαxαa = aαxαa ⟹ a + aαxα(a + aαxαa) = aαxα(a + aαxαa)
⟹ a + aαxαaαxαa = aαxαaαxαa.

Thus, a Γ-semiring M is k-regular if and only if for all a ∈ M there exists x ∈ M such that

$$a + a\alpha x\alpha a\alpha x\alpha a = a\alpha x\alpha a\alpha x\alpha a \quad \ldots \quad (1)$$

For examples and properties of k-regular Γ-semiring s we refer [2], [14], [15], [16].
We observe that the proof of this result can be made signicantly simpler when the Γ-semiring M is taken from ML+.
Theorem 2.15.[2] Let M be a Γ-semiring . Then M is k-regular if and only if $\overline{R\Gamma L}$ = R∩L for any right k-ideal R and left k-ideal L of M.
Theorem 2.16.[2] Let M be a k-ideal of M if and only if k-regular Γ-semiring and A be a non-empty subset of M. Then A is a quasi A = $\overline{R\Gamma L}$, where R is a right k-ideal and L is a left k-ideal of M.

Theorem 2.17.[2] For a Γ-semiring M the following conditions are equivalent:
1. M is k-regular.
2. Q = $\overline{Q\Gamma M\Gamma Q}$ for every quasi k-ideal Q of M.

Theorem 2.18.[2] For a Γ-semiring M the following conditions are equivalent:
1. M is k-regular.
2.Q ∩ J = $\overline{Q\Gamma J\Gamma Q}$ for every quasi k-ideal Q and every k-ideal J of M.
3.Q ∩ I = $\overline{Q\Gamma I\Gamma Q}$ Q for every quasi k-ideal Q and every interior k-ideal I of M.
Theorem 2.19.[2] For a Γ-semiring M the following conditions are equivalent:
1. M is k-regular.
2. R∩L ⊆ $\overline{R\Gamma L}$ for every right k-ideal R and every left k-ideal L of M.
3. Q∩L ⊆ $\overline{Q\Gamma L}$ for every quasi k-ideal Q and every left k-ideal L of M.
Theorem 2.20.[2] For a Γ-semiring M the following conditions are equivalent:
1. M is k-regular.
2.Q ∩ R ⊆ $\overline{R\Gamma Q}$ for every quasi k-ideal Q and every right k-ideal R of M.
Theorem 2.21.[2] For a Γ-semiring M, the following conditions are equivalent:
1. M is k-regular.
2.R ∩ Q ∩ L ⊆ $\overline{R\Gamma Q\Gamma L}$ for every right k-ideal R, every quasi k-ideal Q and every left k-ideal L of M.

3. QUASI IDEALS IN INTRA K-REGULAR Γ-SEMIRINGS.

In this section we characterize intra-k-regular Γ-semirings using quasi-k-ideals. For a semigroup F, the Γ-semiring P(F) is intra k-regular if and only if F is an intra regular semigroup [3]. Again the Theorem 2.2 shows that the quasi k-ideals are natural analogue in Γ-semiring s of the notion of quasi ideals of a semigroup. Thus it is natural to extend the results, characterising the intra regular semigroups by quasi ideals to Γ-semirings.
Definition 3.1. A Γ-semiring M is called an intra k-regular Γ -semiring if for each a ∈ M, $\alpha \in \Gamma$, a ∈ $\overline{M\Gamma a^2 \Gamma M}$. It is easy to check that a Γ -semiring M is intra k-regular if and only if for each a ∈ M there exists x ∈ M such that

$$a + x\alpha a\alpha a\alpha x \in x = x\alpha a\alpha a\alpha x \in x,\ \alpha \in \Gamma.\ \text{Where}\quad a\alpha a = a^2 \qquad (2)$$

In the following theorem we characterized the intra k-regular Γ-semirings by their left and right k-ideals.
Theorem 3.2 ([3]) Let (M, +, Γ) be a Γ-semiring . Then the following conditions are equivalent:
1. M is intra k-regular.
2. L ∩ R ⊆ $\overline{L\Gamma R}$ for every left k-ideal L and every right k-ideal R of M.
We left it to check to the readers that a Γ-semiring M is both k-regular and intra k-regular if and only if for each a ∈ M there exists z ∈ M such that $a + a\alpha z\alpha a^2 z\alpha a = a\alpha z\alpha a^2 z\, a$, $\alpha \in \Gamma$.
Theorem 3.3 For a Γ-semiring M, the following conditions are equivalent:
1. M is both k-regular and intra k-regular.
2. Q = $\overline{Q^2}$. for every quasi k-ideal Q of M.
Proof. (1) ⇒ (2) : Let Q be a quasi k-ideal of M and a ∈ Q. Since M is both k-regular and intra k-regular there exists x ∈ M such that $a + a\alpha x\alpha a^2\alpha x\alpha a = a\alpha x\alpha a^2\alpha x\alpha a$, $\alpha \in \Gamma$. However αxαa ∈ Q Γ MΓQ ⊆ QΓM ∩MΓQ ⊆ Q implies that a ∈ Q^2. Hence Q ⊆ $\overline{Q^2}$. Again, since Q is a k-sub-Γ-semiring , it follows that Q^2⊆ Q, whence $\overline{Q^2}$. ⊆ Q. Thus Q = $\overline{Q^2}$.
(2) ⇒ (1) : Let L and R be a left k-ideal and a right k-ideal of M respectively. Then Q = R ∩ L is a quasi k-ideal of M. Then R ∩ L = $\overline{(R \cap L)^2}$ = $\overline{(R \cap L)\Gamma(R \cap L)}$ ⊆ $\overline{R\Gamma L}$ ∩ $\overline{L\Gamma R}$. Thus M is both k-regular and intra k-regular, by Theorem 3.6 in [1] and Theorem 3.2.
Theorem 3.4. For a Γ-semiring M, the following conditions are equivalent:
1. M is k-regular and intra k-regular.
2. P ∩ Q ⊆ $\overline{P\Gamma Q}$ for every quasi k-ideals P and Q of M.
3. B ∩ Q ⊆ $\overline{B\Gamma Q}$ for every k-bi-ideal B and every quasi k-ideal Q of M.
4. P ∩ B ⊆ $\overline{P\Gamma B}$ for every quasi k-ideal P and every k-bi-ideal B of M.
5. G ∩Q ⊆ $\overline{G\Gamma Q}$ for every generalized k-bi-ideal G and every quasi k-ideal Q of M.
6. P ∩ G ⊆ $\overline{P\Gamma G}$ for every quasi k-ideal P and every generalized k-bi-ideal G of M.
Proof. It is clear that (6) ⇒ (4) ⇒ (2) and (5) ⇒ (3) ⇒ (2). So we have to prove (1) ⇒ (6), (1) ⇒ (5) and (2) ⇒ (1).
(1) ⇒ (6) : Let P and G be a quasi k-ideal and a generalized k-bi-ideal of M respectively. Let a ∈ P ∩G. Since M is both k-regular and intra k-regular, there exists x ∈ M, $\alpha \in \Gamma$ such that $a + a\alpha x\alpha a^2\alpha x = a\alpha x\alpha a^2\alpha x$, by(3). Then $a\alpha x\alpha a$ ∈ P ΓM ∩ M ΓP ⊆ P and $a\alpha x\alpha a$ ∈ G ΓMΓG ⊆ G implies that $a\alpha x\alpha a^2\alpha x$ ∈ PΓG and so a ∈ PΓG. Thus P ∩ G ⊆ $\overline{P\Gamma G}$.
(1) ⇒ (5) :Similar to (1) ⇒ (6).
(2) ⇒(1) :Let Q be a quasi k-ideal. Then Q∩Q ⊆ $\overline{Q\Gamma Q}$ Also $\overline{Q\Gamma Q}$⊆ Q, since Q is a sub Γ-semi ring and k-set. Thus Q = $\overline{Q^2}$ and so M is both k-regular and intra k-regular, by Theorem 3.3.
Theorem 3.5 For a Γ-semiring M, the following conditions are equivalent:
1.M is k-regular and intra k-regular.

2.$L \cap R \subseteq \overline{L\Gamma R} \cap \overline{R\Gamma L}$ for every left k-ideal L and every right k-ideal R of M.
3.$L \cap Q \subseteq \overline{L\Gamma Q} \cap \overline{R\Gamma L}$ for every left k-ideal L and every quasi k-ideal Q of M.
4.$R \cap Q \subseteq \overline{R\Gamma Q} \cap \overline{Q\Gamma R}$ for every right k-ideal R and every quasi k-ideal Q of M.
5.$P \cap Q \subseteq \overline{Q\Gamma P} \cap \overline{P\Gamma Q}$ for all quasi k-ideals P and Q of M.
6.$Q \cap B \subseteq \overline{Q\Gamma B} \cap \overline{B\Gamma Q}$ for every quasi k-ideal Q and every k-bi-ideal B of M.
7.$Q \cap G \subseteq \overline{Q\Gamma G} \cap \overline{G\Gamma Q}$ for every quasi k-ideal Q and every generalized k-bi-ideal G of M.

Proof. It is clear that(7) ⇒ (6) ⇒ (5) ⇒ (4) ⇒ (2)and (5) ⇒ (3) ⇒ (2). So we are to prove (1) ⇒ (7) and (2) ⇒ (1) only.
(1) ⇒ (7) : Let G be a two generalized k-bi-ideal and Q be a quasi k -ideal of M. Let $a \in G \cap Q$. Since M is both k-regular and intra k-regular, there exists $x \in$ M such that $a + a\alpha x\alpha a^2\alpha x\alpha a = a\alpha x\alpha a^2\alpha x\alpha a$. This can be written $\alpha \in \Gamma$ $a\alpha s\alpha a + (a\alpha x\alpha a)\alpha(a\alpha x\alpha a) = (a\alpha x\alpha a)\alpha(a\alpha x\alpha a)$. Now $a\alpha x\alpha a \in Q\Gamma M\Gamma Q \subseteq Q\Gamma M \cap M\Gamma Q \subseteq Q$ and $a\alpha x\alpha a \in G$. Then $\alpha \in \Gamma$ $(a\alpha x\alpha a)\alpha(a\alpha x\alpha a) \in Q\Gamma G$ and $(a\alpha x\alpha a)\alpha(a\alpha x\alpha a) \in G\Gamma Q$. Thus $a \in \overline{Q\Gamma G}$ and $a \in \overline{G\Gamma Q}$. Therefore $a \in \overline{Q\Gamma G} \cap \overline{G\Gamma Q}$. Hence $G \cap Q \subseteq \overline{G\Gamma Q} \cap \overline{Q\Gamma G}$.
(2) ⇒ (1) : Let $L \cap R \subseteq \overline{L\Gamma R} \cap \overline{R\Gamma L}$. Then $L \cap R \subseteq \overline{L\Gamma R}$ and $L \cap R \subseteq \overline{R\Gamma L}$. Thus M is both k-regular and intra-k-regular by Theorem 3.6 of [1] and Theorem 3.2.

Theorem 3.6 For a Γ-semiring M, the following conditions are equivalent:
1. M is k-regular and intra k-regular.
2.$Q \cap L \subseteq \overline{Q\Gamma L\Gamma Q}$for every quasi k-ideal Q and every left k-ideal L of M.
3.$Q \cap R \subseteq \overline{Q\Gamma R\Gamma Q}$ for every quasi k-ideal Q and every right k-ideal R of M.
4.$Q \cap P \subseteq \overline{Q\Gamma P\Gamma Q}$ for all quasi k-ideals Q and P of M.
5.$Q \cap B \subseteq \overline{Q\Gamma B\Gamma Q}$ for every quasi k-ideal Q and every k-bi-ideal B of M.
6.$Q \cap G \subseteq \overline{Q\Gamma G\Gamma Q}$ for every quasi k-ideal Q and every generalized k-bi-ideal G of M.
7.$B \cap Q \subseteq \overline{B\Gamma Q\Gamma Q}$ for every k-bi-ideal B and every quasi k-ideal Q of M.
8.$G \cap Q \subseteq \overline{G\Gamma Q\Gamma G}$ for every generalized k-bi-ideal G and every quasi k-ideal Q of M.

Proof. It is clear that (8) ⇒ (7) ⇒ (4) ⇒ (3)and(6) ⇒ (5) ⇒ (4) ⇒ (2). So it is sufficient to prove (1) ⇒ (8), (1) ⇒ (6), (3) ⇒ (1) and (2) ⇒ (1).
⇒ (8) : Let G be a generalize k-bi-ideal and Q be a quasi k-ideal of M. Let $a \in G \cap Q \subseteq$ M. Since M is both k-regular and intra k-regular, there exists $x \in M$, $\alpha \in \Gamma$ such that
$a + a\alpha x\alpha a^2\alpha x\alpha a = a\alpha x\alpha a^2\alpha x\alpha a$
$\Rightarrow a + a\alpha x\alpha a^2\alpha x\alpha(a + a\alpha x\alpha a^2\alpha x\alpha a) = a\alpha x\alpha a^2\alpha x\alpha a\,(a + a\alpha x\alpha a^2\alpha x\alpha a)$
$\Rightarrow a + a\alpha x\alpha a^2\alpha x\alpha a = a\alpha x\alpha a^2\alpha x\alpha a^2\alpha x\alpha a,\ \alpha \in \Gamma$
$\Rightarrow a + (a\alpha x\alpha a)\alpha(a\alpha x\alpha a\alpha x\alpha a)\alpha(a\alpha x\alpha a) = (a\alpha x\alpha a)\alpha(a\alpha x\alpha a\alpha x\alpha a)\alpha(a\alpha x\alpha a)$.
Now $(a\alpha x\alpha a) \in G$ and $(a\alpha x\alpha a\alpha x\alpha a) \in Q\Gamma M\Gamma Q \subseteq Q\Gamma M \cap M\Gamma Q \subseteq Q$. Then $(a\alpha x\alpha a)\alpha(a\alpha x\alpha a\alpha x\alpha a)\alpha(a\alpha x\alpha a) \in GQG$. Thus $a \in \overline{G\Gamma Q\Gamma G}$. Hence $G \cap Q \subseteq \overline{G\Gamma Q\Gamma G}$.
(1) ⇒ (6) : Proceeding as above we can similarly prove that $Q \cap G \subseteq \overline{G\Gamma Q\Gamma G}$
(2) ⇒ (1) : Let L and R be a left k-ideal and a right k-ideal of M respectively. Then $Q = R \cap L$ is a quasi k-ideal of M. Therefore $Q \cap L \subseteq \overline{Q\Gamma L\Gamma Q}$ implies that $R \cap L = R \cap L \cap L = \overline{(R \cap L)\Gamma L\Gamma(R \cap L)} \subseteq \overline{R\Gamma L\Gamma(R \cap L)} \subseteq \overline{R\Gamma(L\Gamma L)} \subseteq \overline{R\Gamma L}$. Similarly $R \cap L = R \cap L \cap L = (R \cap L)\,\Gamma L\Gamma\,(R \cap L) \subseteq (R \cap L)\,\Gamma L\Gamma R \subseteq R\Gamma L\Gamma R \subseteq L\Gamma R$. Thus $L \cap R \subseteq \overline{L\Gamma R\Gamma L} \cap \overline{R\Gamma L}$. Hence M is both k-regular and intra k-regular, by Theorem 3.5.
(3) ⇒ (1) : Similar to (2) ⇒ (1) .

Theorem 3.7 For a Γ-semiring (M, +, Γ) , the following conditions are equivalent:
1. M is k-regular and intra k-regular.
2. $Q \cap R \cap L = \overline{Q\Gamma R\Gamma L}$ for every quasi k-ideal Q,every right k-ideal R and every left k-ideal L of M.

Proof. (1) ⇒ (2) : Assume that M is a k-regular and intra k -regular Γ-semiring . Let R, Q and L be a right k-ideal, a quasi k-ideal and a left k-ideal of M respectively. Let $a \in Q \cap R \cap L$. Since M is k-regular there exist $x \in M$ such that $a + a\alpha x\alpha a^2\alpha x\alpha a = a\alpha x\alpha a^2\alpha x\alpha a$, $\alpha \in \Gamma$. This can be written as $a + (a\alpha x\alpha a)\,\alpha(a\alpha x)\alpha a = (a\alpha x\alpha a)\alpha(a\alpha x)\alpha a$. Since R is a right k-ideal and Q is a quasi k-ideal of M, so $a\alpha x \in R$ and $a\alpha x\alpha a \in Q\,\Gamma M\,\Gamma Q \subseteq Q\,\Gamma M \cap M\,\Gamma Q \subseteq Q$. Thus $(a\alpha x\alpha a)\,\alpha\,(a\alpha x)\alpha a \in Q\Gamma R\Gamma L$. Then $a \in \overline{Q\Gamma R\Gamma L}$. Hence $Q \cap R \cap L \subseteq \overline{Q\Gamma R\Gamma L}$.
(2) ⇒ (1) : Let L and R be a left k-ideal and a right k-ideal of M respectively. As L and R are also quasi k-ideal of M, we have $L \cap R = R \cap R \cap L \subseteq \overline{R\Gamma R\Gamma L} \subseteq \overline{R\Gamma L}$ and $L \cap R = L \cap R \cap L \subseteq \overline{L\Gamma R\Gamma L} \subseteq \overline{L\Gamma R}$.Thus $L \cap R \subseteq \overline{R\Gamma L} \cap \overline{L\Gamma R}$. Hence M is both k-regular and intra k-regular, by Theorem 3:5.

REFERENCES

[1] M. R. Adhikari, M. K. Sen and H. J. Weinert, On k-regular semirings,Bull. Calcutta Math. Soc., 88 (1996) 141-144.
[2] S. J. Alandkar ,Quasi k-ideals in k-regular Γ-semirings, Golden Research Thought, Vol. 6, Issue 3, Sep 2016.
[3] A. K. Bhuniya and K. Jana, Bi-ideals in k-regular and intra k-regular semirings, Discuss. Math. Gen. Algebra Appl., 31 (2011) 5 23.
[4] S. Bourne,The Jacobson radical of a semiring, Proc. Natl. Acad. Sci. USA,37 (1951) 163 170.
[5] C. Donges,On quasi ideals of semirings, Int. J. Math. Math. Sci., 17 (1994) 47 58.
[6] M. Henricksen, Ideals in semirings with commutative addition, Notices Amer. Math. Soc., 5 (1958) 321.

[7]K. M. Kapp, On bi-ideals and quasi-ideals in semigroups, Publ. Math. Debrecen, 1 (1969) 179 185.
[8] K. M. Kapp, Green's relations and quasi ideals, Czechoslovak Math. J., 19(94) (1969) 80 85.
[9] Y. Kemprasit,Quasi-ideals and bi-ideals in semigroups and rings, Proceedings of the Inter- national Conference on Algebra and its Applications, 2002, 30 46.
[10]S. Lajos, Generalized ideals in semigroups, Acta. Sci. Math. (Szeged), 22 (1961) 217 222.
[11]S. Lajos, On quasi-ideals of regular rings, Proc. Japan Acad. Ser. A Math. Sci., 38 (1962) 210 211.
[12] J. V. Neumann,On regular rings, Proc. Natl. Acad. Sci. USA,22 (1936) 707 713.
[13] M. K. Rao, Γ- semiring I, Southeast Asian Bulletin of Maths., 19(1995) 49-54.
[14] M. K. Sen and A. K. Bhuniya,Completely k-regular semirings,Bull. Calcutta Math. Soc., 97 (2005) 455 466.
[15] M. K. Sen and A. K. Bhuniya,On Additive Idempotent k-Cli ord Semirings, Southeast Asian Bull. Math., 32 (2008) 1149 1159.
[16] M. K. Sen and A. K. Bhuniya,On semirings whose additive reduct is a semilattice, Semi- group Forum, 82 (2011) 131-140.
[17] O. Steinfeld,Über die Quasiideale von Halbgruppen, Publ. Math. Debrecen,4 (1956) 262 275.
[18] O. Steinfeld,Über die Quasiideale von Ringen, Acta Sci. Math. (Szeged), 17 (1956) 170 180.
[19] O. Steinfeld,Über die Quasiideale von Haibgruppen mit eigentlichem Suschkewitsch-Kern, Acta Sci. Math. (Szeged), 18 (1957) 235 242.
[20]O. Steinfeld, Quasi-ideals in rings and regular semigroups, Akademiai Kiado, Budapest, 1978.

Alandkar S. J.
Head, Department of Mathematics,Walchand College of Arts & Science, Solapur.