International Online Multidisciplinary Journal
Review of Research
ISSN No : 2249-894X
Impact Factor 2.1002 (UIF) [Yr.2013]
3.1402 (UIF) [Yr.2014]
Save Tree, Save Paper, Save World

ISSN 2249-894X Impact Factor : 3.1402 (UIF) Volume - 5 | Issue - 4 | Jan - 2016

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THE MINIMUM NEIGHBOURHOOD ENERGY OF A GRAPH

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ABSTRACT :

In a graph $G \in (V, E)$, a subset $S \subseteq V$ is called a neighbourhood set of G,if G is the union of the subgraphs induced by the closed neighbourhood of the vertices of S. The neighbourhood number of G is the minimum cardinality among all minimal neighbourhood sets of G is denoted by $n(G)$. In this paper, we study the minimum neighbourhood energy, denoted by $E_N(G)$, of a graph G.We are computing the minimum neighbourhood energies of complete graph, complete bipartite graph, star graph, cocktail party graph and Friendship graph. Upper and lower bounds for $E_N(G)$ are established.

Mathematics Subject Classification: **05C50, 05C69**.

KEYWORDS: Neighbourhood set, neighbourhood number, minimum neighbourhood matrix, minimum neighbourhood eigenvalues, minimum neighbourhood energy.

1 INTRODUCTION

In this paper, by a graph $G = (V, E)$ we mean a simple graph that is finite, have no loops no multiple and directed edges. We denoted by $n = |V|$ and $m = |E|$ to the number of vertices and edges of G, respectively. For a vertex $v \in V$, the open neighbourhood of v in G, denoted $N(v)$, is the set of all vertices that are adjacent to v and the closed neighbourhood of v is $N[v] = N(v) \cup \{v\}$. The degree of vertex v in G, denoted by $d(v)$, is the number of its neighbors in G. We denoted by Δ and δ the maximum and minimum degree among the vertices of G, respectively. For non-empty subset $S \subseteq V$ and a vertex $v \in V$ we denote by $d_S(v)$ the degree of v has in S, i.e. $d_S(v) = |N(v) \cap S|$. A graph H is a subgraph of a graph G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$.We denoted by $\langle S \rangle$ the induced subgraph of G that is induced by vertex set $S \subseteq V$ such that $E(\langle S \rangle) = \{uv \in E(G) | u, v \in S\}$. The graph $W_n = K_1 + C_{n-1}, for\ n \geq 4$ is called the wheel graph of order n.We denoted by $\lceil x \rceil$ to the smallest integer number greater than or equals to x and $\lfloor x \rfloor$ to the greatest integer number smaller than or equals to x. For more terminologies and notations in graph theory do not define here, we refer the reader to books [Bondy et al. and Harary F.].

A subset $S \subseteq V$ is called a neighbourhood set of G, if $G = \bigcup_{v \in S} \langle N(v) \rangle$.The neighbourhood number of G is the minimum cardinality among all minimal neigh- borhood sets of G is denoted by $n(G)$. The concept of neighbourhood number of a graph was introduced by Sampathkumar et al. [18], there are more studies and details in this concept can be see it in [10, 11, 19] and the references therein.

The concept of energy of a graph was introduced by I. Gutman [7] in the year 1978. Let G be a graph with n vertices and m edges and let $A = (a_{ij})$ be the adjacency matrix of the graph. The eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n \, of \, A$, assumed in non increasing order, are the eigenvalues of the graph G. As A is real symmetric, the eigenvalues of G are real with sum equal to zero. Let $\lambda_1, \lambda_2, \ldots, \lambda_t \, for \, t \leq n$ be the distinct eigenvalues of G with multiplicity $m_1, m_2, \ldots, m_t$, respectively, the multiset of eigenvalues of $A(G)$ is called the spectrum of G and denoted by

$$S_{pec}(G) = \begin{pmatrix} \lambda_1 & \lambda_2 \ldots \ldots \ldots \ldots . \lambda_t \\ m_1 & m_2 \ldots \ldots \ldots \ldots . m_t \end{pmatrix}$$

The energy $E(G) \; of \; G$ is defined to be the sum of the absolute values of the eigen- values of G, i.e.

$$E(G) = \sum_{i=1}^{n} |\lambda_i|$$

For more details on the mathematical aspects of the theory of graph energy see [2, 8, 15]. The basic properties including various upper and lower bounds for energy of a graph have been established in [16, 17], and it has found remarkable chemical applications in the molecular orbital theory of conjugated molecules [5, 6].

Recently C. Adiga et al [1] defined the minimum covering energy, $E_C(G)$ of a graph which depends on its particular minimum cover C. Further, minimum dom- inating energy, Laplacian minimum dominating energy and minimum dominating distance energy of a graph G can be found in [12, 13, 14].

Motivated by these papers, we study the minimum neighbourhood energy $E_N(G)$ of a graph G. We compute minimum global dominating energies of some standard graphs. Some properties of characteristic polynomial of a minimum monopoly dis- tance matrix of a graph G are obtained. Upper and lower bounds for $E_N(G)$ are established. It is possible that the upper dominating energy that we are consider- ing in this paper may be have some applications in chemistry as well as in other areas. The following is fundamental result which will be required for many of our arguments in this paper:

Proposition 1.1. [18]

- *For any graph G of order $n, n(G) = 1$, if and only if G has a vertex of degree $n - 1$.*
- *If G is a bipartite graph $K_{r,s}$, then $n(G) = min\{r, s\}$.*
- *For a cycle graph $C_n, n \geq 4, n(C_n) = \left\lceil \frac{n}{2} \right\rceil$*

2 The Minimum Neighbourhood Energy of a Graph

Let G be a graph of order n with vertex set $V(G) = \{v_1, v_2, \ldots, v_n\}$ and edge set $E(G)$. Let S be a neighbourhood set in G. The neighbourhood number $n(G)$ of G is the cardinality of a smallest neighbourhood set in G. Any neighbourhood set S in G with cardinality equals to $n(G)$ is called a minimum neighbourhood set of G. The minimum neighbourhood matrix of G is the $n \times n$ matrix, denoted by $A_N(G) = (a_{ij})$, Where

$$a_{ij} = \begin{cases} 1, & if \; v_i v_j \in E\,; \\ 1, & if \; i = j \; and \; v_i \in S\,; \\ 0, & otherwise \end{cases}$$

The characteristic polynomial of $A_N(G)$ is denoted by

$$f_n(G, \lambda) = det\,(\lambda I - A_N(G))\,.$$

The minimum neighbourhood eigenvalues of a graph G are the eigenvalues of $A_N(G)$. Since $A_N(G)$ is real and symmetric, its eigenvalues are real numbers and we label them in non-increasing order $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$. The minimum neighbourhood energy of G is defined as:

$$E_N(G) = \sum_{i=1}^{n} |\lambda_i|$$

We first compute the minimum neighbourhood energy of a graph in Figure 1.

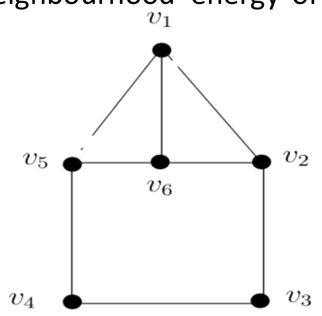

Figure 1

Let G be a graph in Figure 1, with vertices set $\{v_1, v_2, v_3, v_4, v_5, v_6\}$. Then G has more than one set as a minimum neighbourhood set. For example, $N_1 = \{v_1, v_3, v_4\}$.Then

$$A_{GD_1}(G) = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

The characteristic polynomial of $A_{N_1}(G)$ is

$$f_n(G, \lambda) = \lambda^6 - 3\lambda^5 - 5\lambda^4 + 12\lambda^3 + 9\lambda^2 - 9\lambda - 5.$$

Hence, the minimum neighbourhood eigenvalues are $\lambda_1 \approx 3.2263$, $\lambda_2 \approx 1.9102$, $\lambda_3 \approx 1$, $\lambda_4 \approx -0.4939$, $\lambda_5 \approx -1$, $\lambda_6 \approx -1.6426$.

Therefore the minimum neighbourhood energy of G is

$$E_{N_1}(G) \approx 9.2730.$$

But if we take another minimum neighbourhood set of G, namely $N_2 = \{v_2, v_5\}$, we get that

$$A_{GD_2}(G) = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

The characteristic polynomial of $A_{N_2}(G)$ is

$$f_n(G,\lambda) = (\lambda + 1)^2(\lambda^4 - 5\lambda^3 + 4\lambda^2 + 8\lambda).$$

The minimum neighbourhood eigenvalues are $\lambda_1 \approx 3.3089$, $\lambda_2 \approx 1.6734$, $\lambda_3 \approx 1.2838$, $\lambda_4 = \lambda_5 = -1$, $\lambda_6 \approx -1.2661$. Therefore the minimum neighbourhood energy of G is $E_{N_2}(G) \approx 9.5322$.

The examples above illustrate that the minimum neighbourhood energy of a graph G depends on the choice of the minimum neighbourhood set. i.e. the minimum neighborhood energy is not a graph invariant.

In the following section, we introduce some properties of characteristic poly- nomial of minimum neighbourhood matrix of a graph G.

Theorem 2.1. Let G be a graph of order n, size m, neighbourhood number $n(G)$ and let $fn(G,\lambda) = c_0\lambda^n + c_1\lambda^{n-1} + c_2\lambda^{n-2} + \cdots \dots \dots + c_n$ be the characteristic polynomial of the minimum neighbourhood matrix of a graph G. Then

1. $c_0 = 1$.
2. $c_1 = -n(G)$
3. $c_2 = \binom{n(G)}{2} - m$

Proof. 1. From the definition of $f_n(G,\lambda)$.

2. Since the sum of diagonal elements of $A_N(G)$ is equal to $n(G)$. The sum of determinants of all 1×1 principal sub matrices of $A_N(G)$ is the trace of $A_N(G)$,which evidently is equal to $n(G)$.Thus, $(-1)^1 c_1 = |D|$.
3. $(-1)^2 c_2$ is equal to the sum of determinants of all 2×2 principal sub matrices of $A_N(G)$, that is

$$c_2 = \sum_{1\le i<j\le n} \begin{vmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{vmatrix}$$
$$= \sum_{1\le i<j\le n} (a_{ii}a_{jj} - a_{ij}a_{ji})$$
$$= \sum_{1\le i<j\le n} a_{ii}a_{jj} - \sum_{1\le i<j\le n} a_{ij}{}^2$$
$$= \binom{n(G)}{2} - m$$

Theorem 2.2. *Let G be a graph of order n. Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigenvalues of $A_N(G)$. Then*

(i) $\sum_i^n \lambda_i = n(G)$

(ii) $\sum_i^n \lambda_i{}^2 = n(G) + 2m$

Proof. (i) Since the sum of the eigenvalues of $A_N(G)$ is the trace of $A_N(G)$, then

$$\sum_i^n \lambda_i = \sum_i^n a_{ii} = n(G)$$

(ii) Similarly the sum of squares eigenvalues of $A_N(G)$ is the trace of $(A_N(G))^2$. Then $\sum_i^n \lambda_i{}^2 = \sum_{i=1}^n \sum_{j=1}^n a_{ij}a_{ji}$

$$= \sum_i^n a_{ii}{}^2 + \sum_{i\neq j}^n a_{ij}a_{ji}$$
$$= \sum_i^n a_{ii}{}^2 + 2 = \sum_{i<j}^n a_{ij}{}^2$$
$$= n(G) + 2m.$$

Bapat and S. Pati [3], proved that if the graph energy is a rational number then it is an even integer. Similar result for minimum neighbourhood energy is given in the following theorem.

Theorem 2.3. *Let G be a graph with a neighbourhood number $n(G)$. If the minimum neighbourhood energy $E_N(G)$ of G is a rational number, then*

$$E_N(G) \equiv n(G) \ (mod\ 2).$$

Proof. Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the minimum neighbourhood eigenvalues of a graph G of which $\lambda_1, \lambda_2, \ldots, \lambda_r$ are positive and the rest are non-positive, then

$$\sum_{i=1}^{n} |\lambda_i| = (\lambda_1 + \lambda_2 + \ldots + \lambda_r) - (\lambda_{r+1} + \lambda_{r+2} + \ldots + \lambda_n).$$

$$= 2(\lambda_1 + \lambda_2 + \ldots + \lambda_r) - (\lambda_1 + \lambda_2 + \ldots + \lambda_n)$$

$$= 2q - n(G). \ where\ q = \lambda_1 + \lambda_2 + \ldots + \lambda_r.$$

Since $\lambda_1, \lambda_2, \ldots, \lambda_r$ are algebraic integers, so is their sum.

Hence $(\lambda_1 + \lambda_2 + \ldots + \lambda_r)$ must be an integer if $E_N(G)$ is rational. Hence the theorem is hold.

3 The Minimum neighbourhood Energy of Some Graphs

In this section, we investigate the exact values of the minimum neighbourhood energy of some standard graphs.

Theorem 3.1. *For $n \geq 2$, the minimum* neighbourhood energy of *the complete graph K_n, is equal to.* $(n-2) + \sqrt{n^2 - 2n + 5}$.

Proof. For the complete graph K_n, by Proposition 1.1,the neighbourhood number is $n(K_n) = 1$.Hence, for K_n the minimum neighbourhood matrix is same as minimum dominating matrix [14], therefore the minimum neighbourhood energy is equal to minimum dominating energy i.e. $E_N(K_n) = (n-2) + \sqrt{n^2 - 2n + 5}$

Theorem 3.2. For $n \geq 2$, *the minimum neighbourhood energy of a star graph $K_{1,n-1}$ is equal to* $\sqrt{4n-3}$.

Proof. Let $K_{1,n-1}$ be a star graph with vertex set $V = \{v_0, v_1, v_2, \cdots, v_{n-1}\}$, where, v_0 is the central vertex. Then by proposition 1.1 $n(K_{n-1}) = 1$, where the minimum neighbourhood set of K_{n-1} is $\{v_0\}$. Hence, for $K_{1,n-1}$ the minimum neighbourhood matrix is same as minimum covering matrix [1], therefore the minimum neighbourhood energy is equal to minimum covering energy. i.e. $E_N(K_{1,n-1}) = \sqrt{4n-3}$.

Theorem 3.3. *For the complete bipartite graph $K_{r,s}$, for $r \geq s$, the minimum neighbourhood energy is equal to* $(r-1) + \sqrt{4rs+1}$.

Proof. For the complete bipartite graph $K_{r,s}$ with vertex set $V = (V_1, V_2)$ where V_1 *and* V_2 are the partite sets of its, $V_1 = \{v_1, v_2, \cdots, v_r\}$ *and* $V_2 = \{u_1, u_2, \cdots, u_s\}$. By Proposition 1.1, the neighbourhood number is $n(K_{r,s}) = r$, then, the minimum neighbourhood set is $N = V_1$. Hence, for $K_{r,s}$ the minimum neighbourhood matrix is same as minimum covering matrix [1], therefore the minimum neighbourhood energy is equal to minimum covering energy.

i.e. $E_N(K_{r,s}) = (r-1) + \sqrt{4rs+1}.$

Definition 3.4. *The cocktail party graph, denoted by* $K_{2\times p}$, *is a graph having vertex set* $V(K_{2\times p}) = \cup_{i=1}^{p}\{u_i, v_i\}$ *and edge set* $E(K_{2\times p}) = \{u_iu_j, v_iv_j, u_iv_j, v_iu_j : 1 \le i < j \le p\}$ *i.e.,* $n = 2p, m = \frac{p^2-3p}{2}$ *and for every* $v \in V(K_{2\times p}), d(v) = 2p-2.$

Theorem 3.5. *For the cocktail party graph* $K_{2\times p}$ *of order 2p, for p* $\geq$ *3,the minimum neighbourhood energy is equal to* (2p-3)+ $\sqrt{4n^2-4n+9}$.

Proof. For cocktail party graphs $K_{2\times p}$ with vertex set $V = \cup_{i-1}^{p}\{u_i, v_i\}$,the minimum neighbourhood set is same as minimum dominating matrix [14], therefore the minimum neighbourhood energy is equal to minimum dominating energy. i.e., $E_N(K_{r,s}) = (2n-1) + \sqrt{4n^2-4n+9}$.

Definition 3.6. *The Friendship graph* F_n *for* $n \geq 2$, *is the graph constructed by joining n copies of* K_3 *graph with a common vertex.*

Theorem 3.7. *For the Friendship graph* $F_n, n \geq 2$, *the minimum neighbourhood energy is equal to* $(2n-1) + 2\sqrt{2n}$.

Proof. For the Friendship graph F_n *for* $n \geq 2$, with vertex set $V = \{v_0, v_1, v_2, \cdots, v_{2n}\}$, where v_0 is the central vertex. Then by proposition 1.1,$n(F_n) = 1$, where the minimum neighbourhood set of F_n *is* $\{v_0\}$. Hence, for F_n the minimum neighbourhood matrix is

$$A_N(F_n) = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \end{pmatrix}_{(2n+1)\, X\, (2n+1)}$$

The characteristic polynomial of $A_N(F_n)$ is

$$f_n(F_n, \lambda) = \begin{vmatrix} \lambda-1 & -1 & -1 & -1 & -1 & \cdots & -1 & -1 \\ -1 & \lambda & -1 & 0 & 0 & \cdots & 0 & 0 \\ -1 & -1 & \lambda & 0 & 0 & \cdots & 0 & 0 \\ -1 & 0 & 0 & \lambda & -1 & \cdots & 0 & 0 \\ -1 & 0 & 0 & -1 & \lambda & \cdots & 0 & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ -1 & 0 & 0 & 0 & 0 & \cdots & \lambda & -1 \\ -1 & 0 & 0 & 0 & 0 & \cdots & -1 & \lambda \end{vmatrix}_{(2n-1)\, X\, (2n+1)}$$

$$= (\lambda+1)^n(\lambda-1)^{n-1}(\lambda^2 - 2\lambda - (2n-1)).$$

and the minimum neighbourhood spectrum of F_n is

$$MN\, S_{pec}(F_n) = \begin{pmatrix} -1 & 1 & 1+\sqrt{2n} & 1-\sqrt{2n} \\ n & n-1 & 1 & 1 \end{pmatrix}$$

Therefore, the minimum neighbourhood energy of F_n is

$$E_M(F_n) = (2n-1) + 2\sqrt{2n}.$$

4 Bounds for Minimum global Domination Energy of a Graph

In this section we shall investigate with some bounds for minimum neighbourhood energy of a graph.

Theorem 4.1. *Let G be a graph of order n and size m. Then*

$$\sqrt{2m + n(G)} \leq E_N(G) \leq \sqrt{n(2m + n(G)}$$

Proof. Consider the Couchy-Schwartiz inequality

$$(\sum_{i=1}^{n} a_i b_i)^2 \leq (\sum_{i=1}^{n} a_i^2)(\sum_{i=1}^{n} b_i^2)$$

By choose $a_i = 1$ *and* $b_i = |\lambda_i|$, we get

$$\begin{aligned}(E_N(G))^2 = (\sum_{i=1}^{n}|\lambda_i|)^2 &\leq (\sum_{i=1}^{n} 1)(\sum_{i=1}^{n} \lambda_i^2)\\ &\leq n(2m + |S|)\\ &\leq n(2m + n(G)).\end{aligned}$$

Therefore, the upper bound is hold. For the lower bound, since

$$(\sum_{i=1}^{n}|\lambda_i|)^2 \geq \sum_{i=1}^{n} \lambda_i^2$$

Then $\quad (E_N(G))^2 \geq \sum_{i=1}^{n} \lambda_i^2 = 2m + |S| = 2m + n(G)$

Therefore,

$$E_N(G) \geq \sqrt{2m + n(G)}$$

Similar to McClellands [17] bounds for energy of a graph, bounds for $E_N(G)$ are given in the following theorem.

Theorem 4.2. *Let G be a graph of order and size n and m, respectively.*

If P =det(A_N(G)), then $E_N(G) \geq \sqrt{2m + n(G) + n(n-1)P^{\frac{2}{n}}}.$

Proof. Since

$$(E_N(G))^2 = \left(\sum_{i=1}^{n}|\lambda_i|\right)^2 = \left(\sum_{i=1}^{n}|\lambda_i|\right)\left(\sum_{i=1}^{n}|\lambda_i|\right) = \sum_{i=1}^{n}|\lambda_i|^2 + 2\sum_{i\neq j}|\lambda_i||\lambda_j|.$$

Employing the inequality between the arithmetic and geometric means, we get

$$\frac{1}{n(n-1)}\sum_{i\neq j}|\lambda_i||\lambda_j| \geq (\prod_{i\neq j}|\lambda_i||\lambda_j|)^{\frac{1}{[n(n-1)]}}.$$

Thus

$$\begin{aligned}(E_N(G))^2 &\geq \sum_{i=1}^{n}|\lambda_i|^2 + n(n-1)(\prod_{i\neq j}|\lambda_i||\lambda_j|)^{\frac{1}{[n(n-1)]}}\\ &\geq \sum_{i=1}^{n}|\lambda_i|^2 + n(n-1)(\prod_{i\neq j}|\lambda_i|^{2(n-1)})^{\frac{1}{[n(n-1)]}}\\ &= \sum_{i=1}^{n}|\lambda_i|^2 + n(n-1)\left|\prod_{i\neq j}\lambda_i\right|^{\frac{2}{n}}\\ &= 2m + n(G) + n(n-1)P^{\frac{2}{n}}.\end{aligned}$$

This completes the proof.

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