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MEAN SQUARE ESTIMATION OF THE PARAMETER IN THE $U(\theta, 2\theta)$

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ABSTRACT: In the literature, an extensive work on sequential fixed width confidence interval for the parameter of U(θ, mθ) model, where m>1 is known, is available. In this article we develop and compare minimum mean square estimation procedures for estimating the parameter θ of $U(\theta, 2\theta)$.

Keywords: Fixed sample size (FSS) procedure; Maximum likelihood estimator (m.l.e); Sufficient statistics; Linear estimator; Minimum distance criteria.

Mathematics Subject Classification (2000): Primary 62L12.

1. INTRODUCTION

The problem of obtaining confidence intervals having a specified width for the parameter in the density U(θ, mθ), where m>1 is known and θ >0, has been considered by Patil and Rattihalli (2011) and Patil (2012, 2014, 2015). Govindarajulu (2000) has considered the problem of finding risk-efficient sequential estimation procedure for U(0, θ) density. For further details on sequential estimation, one may refer to Ghosh et.al (1997).

In this article we propose minimum mean square estimation procedures for estimating the parameter of U(θ, 2θ) density. Section 2 contains a linear estimator of θ by using m.l.e. In Sections 3, we propose a linear estimator using sufficient statistics and its modification based on minimum distance criteria and compares these methods.

2. LINEAR ESTIMATOR USING M.L.E.

Let $X_1, X_2, ...X_n$ be independent identical distribution (i.i.d) random variables with U(θ, 2θ) distribution. Let $X_{n(1)} = \min(X_1, X_2, ...X_n)$ and $X_{n(n)} = \max(X_1, X_2, ...X_n)$. Note that $X_{n(n)}/2$ is the m.l.e of θ. Let a linear estimator of θ be

$$T_1 = T(X) = \alpha X_{n(n)}/2 + (1-\alpha)X_{n(1)},\ 0 \le \alpha \le 1. \qquad (2.1)$$

Then mean squared error (MSE) of T_1 is

$$MSE(T_1) = E\left[\alpha X_{n(n)}/2 + (1-\alpha)X_{n(1)} - \theta\right]^2$$

$$= (\alpha^2/4)E(X^2_{n(n)}) + (1-\alpha)^2 E(X^2_{n(1)}) + \alpha(1-\alpha)E(X_{n(n)}X_{n(1)}) +$$

$$\theta^2 - \theta\alpha E(X_{n(n)}) - 2(1-\alpha)\theta E(X_{n(1)})$$

We know that

$$E(X_{n(n)}) = \theta + n\theta/(n+1) = \theta(2n+1)/(n+1)$$

$$E(X_{n(1)}) = 2\theta - n\theta/(n+1) = \theta(n+2)/(n+1)$$

$$E(X^2_{n(n)}) = \theta^2(4n^2+8n+2)/[(n+1)(n+2)]$$

$$E(X^2_{n(1)}) = \theta^2(n^2+5n+8)/[(n+1)(n+2)]$$

and $E(X_{n(1)}\, X_{n(n)}) = \theta^2(2n^2+7n+5)/[(n+1)(n+2)]$

Substituting these we get,

$$MSE(T_1) = \theta^2(7\alpha^2+10\alpha+4)/[2(n+1)(n+2)] \quad (2.2)$$

Note that $MSE(T_1)$ is an increasing function of θ and decreasing function of n. Further for fixed θ, $7\alpha^2+10\alpha+4$ is a convex and has minimum value at $\alpha = 5/7$. Hence $MSE(T_1)$ is minimum at $\alpha = 5/7$. Thus the minimum MSE linear estimator for θ is

$$T_1 = 5X_{n(n)}/4 + 2X_{n(1)}/7 \quad (2.3)$$

And $MSE\ (T_1) = 0.214\theta^2/(n+1)(n+2)$ (2.4)

3. LINEAR ESTIMATOR USING SUFFICIENT STATISTICS

Let a linear estimator for θ based on sufficient statistics $(X_{n(n)}, X_{n(1)})$ be

$$T_2 = \alpha_1 X_{n(n)} + \alpha_2 X_{n(1)} \quad (3.1)$$

Then the MSE of T_2 is

$$MSE(T_2) = E[\alpha_1 X_{n(n)} + \alpha_2 X_{n(1)} - \theta]^2$$

$$= \alpha_1^2\ E(X_{n(n)}^2) + \alpha_2^2\, E(X_{n(1)}^2) + 2\,\alpha_1\,\alpha_2\, E(X_{n(n)}\, X_{n(1)}) + \theta^2 - 2\theta\alpha_1 E(X_{n(n)}) - 2\theta\alpha_2 E(X_{n(1)})$$

Substituting the expectations obtained in Section 2 and taking derivative w.r.t α_1 we get,

$$0 = 2\alpha_1(4n^2+8n+2) + 2\alpha_2(2n^2+7n+5) - 2(2n^2+5n+2)$$

which implies $\alpha_1(4n^2+8n+2) + \alpha_2(2n^2+7n+5) = 2n^2+5n+2$ (3.2)

Similarly by differentiation w.r.t α_2 we get,

$$\alpha_1(2n^2+7n+5) + \alpha_2(n^2+5n+8) = n^2+4n+4 \quad (3.3)$$

Now solving (3.2) and (3.3) we get,

$$\alpha_1 = 2(n+2)/(5n+9) \text{ and } \alpha_2 = (n+2)/(5n+9) \qquad (3.4)$$

Hence the minimum MSE linear estimator of θ is

$$T_2 = \frac{n+2}{5n+9}[2X_{n(n)} + X_{n(1)}] \qquad (3.5)$$

And

$$MSE(T_2) = \theta^2/(n+1)(5n+9) \qquad (3.6)$$

Note that the MSE of a m.l.e $X_{n(n)}/2$ is $\theta^2/(n+1)(2n+4)$ and MSE of an unbiased estimator $(X_{n(n)} + X_{n(1)})/3$ is $\theta^2/(n+1)(4.5n+9)$. Thus it is clear that for fixed n, the linear estimator T_2 is better than the m.l.e, unbiased estimator and T_1. Further T_2 can be written as,

$$T_2 = \frac{4n+8}{5n+9}(X_{n(n)}/2) + \frac{n+2}{5n+9}X_{n(1)}$$

$$= \frac{5n+10}{5n+9}\left[\frac{4n+8}{5n+10} + \frac{n+2}{5n+10}X_{n(1)}\right]$$

$$> X_{n(n)}/2$$

Hence we propose another estimator T_2^* based on minimum distance criteria given by,

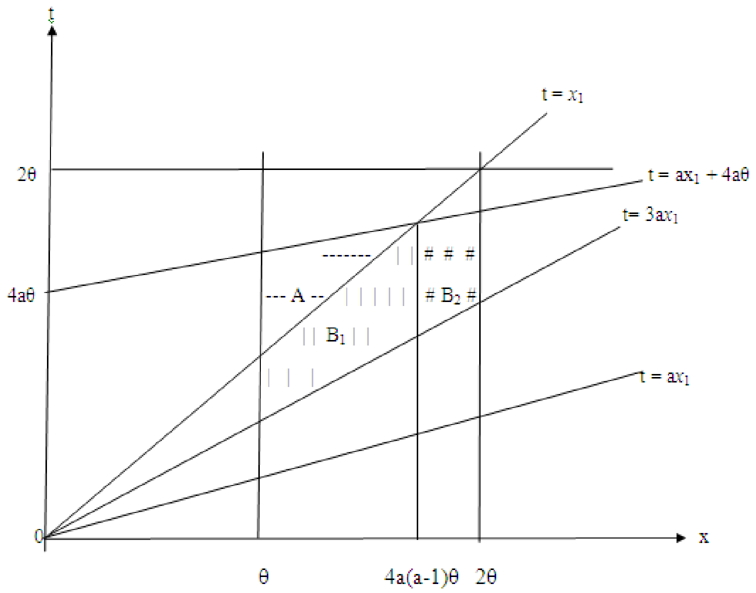
$$T_2^* = \begin{cases} X_{n(1)} & \text{if } T_2 > X_{n(1)} \\ T_2 & \text{if } X_{n(n)}/2 < T_2 < X_{n(1)} \end{cases} \qquad (3.7)$$

Let a = (n+2)/(5n+9) < 1/3, $T_2 = aX_{n(1)} + 2aX_{n(n)}$. Now the joint distribution of $X_{n(1)}$ and T_2 will be,

$$f(x_1, t) = \frac{n(n-1)}{\theta^n (2a)^{n-1}}[t - 3ax_1]^2, \text{ for } \theta < x_1 < 2\theta \text{ and } 3ax_1 < t < ax_1 + 4a\theta \qquad (3.8)$$

This can be shown as below.

Figure 3.1: Joint distribution of $X_{n(1)}$ and T_2



Note that $T_2^* = X_{n(1)}$ is in the region A and $T_2^* = T_2$ is in the region $B_1 U B_2$. Hence from Figure 3.1, it is clear that,

$$\text{MSE}(T_2) - \text{MSE}(T_2^*) = \int_{\theta}^{4a\theta a\theta/-a)} \int_{x_1}^{ax_1+4a\theta} [(t-\theta)^2 - (x_1-\theta)^2]\, f(x_1, t)\, dt\, dx_1 \tag{3.9}$$

The integrand on the region A is $[(t - \theta)^2 - (x_1 - \theta)^2] = t^2 - x_1^2 - 2\theta(t - x_1) = (t - x_1)[(t + x_1) - 2\theta] > 0$, since on region A, $t > x_1$ and $(t + x_1) - 2\theta > 2x_1 - 2\theta = 2(x_1 - \theta) > 0$. Hence the right hand side of (3.9) is positive. Thus T_2^* is uniformly better than T_2.

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