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HISTORY OF MATHEMATICS IN INDIA: SPECIAL REFERENCE ARYABHATA & BHĀSKARACHARYA

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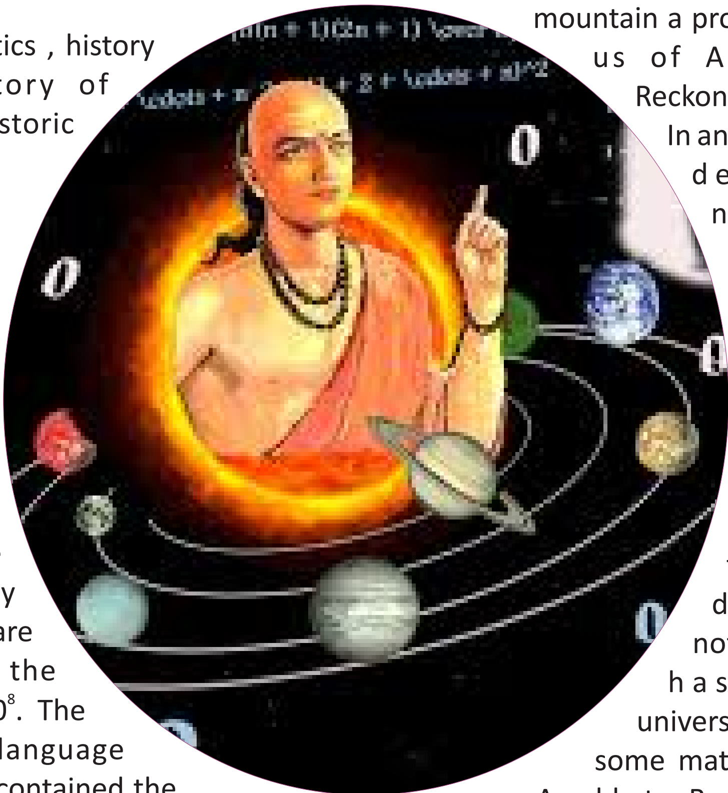
ABSTRACT:

The history of science begins with the history of technology. The non-civilized man was very observant. He was hunted. Moreover, hunters and traders needed to count. Therefore, it was in prehistoric times that man acquired the first notions of Mathematics, Astronomy, Zoology, Botany.

KEY WORDS:
History Of Mathematics , history of science , history of technology , prehistoric times .



INTRODUCTION:

Mathematics In India:

In India there are no special Vedic or Brahmanic mathe- matical texts. But the Vedic language shows that large numbers were in fairly common use. There are special terms for the powers of ten up to 10^8. The classical Sanskrit language went still further. It contained the Vedic series as far as 10^{23}. The oldest extant mathematical records from India are the Sulba Sutras (dated variously between the 8th century BC and the 2nd century AD), appendices to religious texts which give simple rules for constructing altars of various shapes, such as squares, rectangles, parallelograms, and others.

Mathematics played an important role in classical Indian astronomy. There are references to astronomical speculations about the remarkable numbers27, 10800 and 4,32,000. A Buddhist text the Lalitavistara, hints to a method of assessing the number of grains of sand on a mountain a problem which reminds us of Archimedes Sand-Reckoner.

In ancient India before the decimal system, number symbols were used. These symbols were officially adopted on Ashoka inscrip- tions from the middle of the third century B.C. India was the first country to have used the complete decimal system of notation. This system has now become universal. In Ancient period some mathe- maticians were Aryabhata, Brahmagupta, Shridhra-chaya, Mhavir, Bhāskara II (Bhaskara- charya).

Aryabhata

Aryabhata was born in 476. Aryabhata provides no information about his place of birth. He settled in the region between the Narmada and Godavari rivers in central India.

Education: Aryabhata went to Kusumapura (Patana) for advanced studies and lived there for some time. A verse mentions that Aryabhata was the head of an institution (*Kulapa*) at Kusumapura, it is speculated that Aryabhata might have been the head of the Nalanda university as well. Aryabhata is also reputed to have set up an observatory at the Sun temple in Taregana, Bihar.

Works : Aryabhata is the author of several treatises on mathematics and astronomy, some of which are lost. The mathematical part of the *Aryabhatiya* covers arithmetic, algebra, plane trigonometry, and spherical trigonometry. It also contains continued fractions, quadratic equations, sums-of-power series, and a table of sines.

The *Arya-siddhanta*, a lost work on astronomical computations, is known through the writings of Aryabhata's contemporary. This work appears to be based on the older Surya Siddhanta and uses the midnight-day reckoning, as opposed to sunrise in *Aryabhatiya*. It also contained a description of several astronomical instruments: the gnomon (*Shanku-Yantra*), a shadow instrument (*Chhaya-Yantra*), possibly angle-measuring devices, semicircular and circular (*Dhanur-Yantra / Chakra-Yantra*), a cylindrical *Stickyasti-Yantra*, an umbrella-shaped device called the *Chhatra-Yantra*, and water clocks of at least two types, bow-shaped and cylindrical. A third text, which may have survived in the Arabic translation, is Al ntf or Al-nanf.

Aryabhatiya :

The name "Aryabhatiya" is due to later commentators. Aryabhata himself may not have given it a name. His disciple Bhaskara I calls it *Ashmakatantra* (or the treatise from the Ashmaka) The text consists of the 108 verses and 13 introductory verses, and is divided into four *padas* or chapters:

1.*Gitikapada*: (13 verses): large units of time—*kalpa, manvantra*, and *yuga*—which present a cosmology different from earlier texts such as Lagadha's *Vedanga Jyotisha* (B.C.1st century). There is also a table of sines (jya), given in a single verse. The duration of the planetary revolutions during a *mahayuga* is given as 4.32 million years.

2.*Ganitapada* (33 verses): covering mensuration (*k?etra vyavahara*), arithmetic and geometric progressions, gnomon / shadows (*shanku-chhAyA*), simple, quadratic, simultaneous, and indeterminate equations

3.*Kalakriyapada* (25 verses): different units of time and a method for determining the positions of planets for a given day, calculations concerning the intercalary month (*adhikamAsa*), *kShaya-tithis*, and a seven-day week with names for the days of week.

4.*Golapada* (50 verses): Geometric/trigonometric aspects of the celestial sphere, features of the ecliptic, celestial equator, node, shape of the earth, cause of day and night, rising of zodiacal signs on horizon, etc. In addition, some versions cite a few colophons added at the end, extolling the virtues of the work, etc.

MATHEMATICS :

Place value system and zero : The place-value system, first seen in the 3rd-century Bakhshali Manuscript, was clearly in place in his work. While he did not use a symbol for zero, the French mathematician Georges Ifrah argues that knowledge of zero was implicit in Aryabhata's place-value system as a place holder for the powers of ten with null coefficients.

Approximation of π : Aryabhata worked on the approximation for pi (p), and may have come to the conclusion that is irrational. In the second part of the Aryabhatiyam (ga?itapāda 10), he writes:

Caturadhikam Satama??agu?am Dva?a??statha Sahasra?am
Ayutadvayavi?kambhasyasanno V?ttapari?aha?.

"Add four to 100, multiply by eight, and then add 62,000. By this rule the circumference of a circle with a diameter of 20,000 can be approached."

This implies that the ratio of the circumference to the diameter is ((4 + 100) × 8 + 62000)/20000 = 62832/20000 = 3.1416, which is accurate to five significant figures.

It is speculated that Aryabhata used the word *asanna* (approaching), to mean that not only is this an approximation but that the value is incommensurable (or irrational). If this is correct, it is quite a sophisticated insight, because the irrationality of pi was proved in Europe only in 1761 by Lambert. After Aryabhatiya was translated into Arabic (B.C. 820) this approximation was mentioned in Al-Khwarizmi's book on algebra.

Trigonometry

In Ganitapada 6, Aryabhata gives the area of a triangle as
Tribhujasya Phalashariram Samadalakoti Bhujardhasamvargah
that translates to: "for a triangle, the result of a perpendicular with the half-side is the area."

Aryabhata discussed the concept of *sine* in his work by the name of *ardha-jya*, which literally means "half-chord". For simplicity, people started calling it jya. Latin counterpart, *sinus*, which means "cove" or "bay"; thence comes the English *sine*. Alphabetic code has been used by him to define a set of increments. If we use Aryabhata's table and calculate the value of sin(30) (corresponding to hasjha) which is 1719/3438 = 0.5; the value is correct. His alphabetic code is commonly known as the Aryabhata cipher.

INDETERMINATE EQUATIONS

A problem of great interest to Indian mathematicians since ancient times has been to find integer solutions to Diophantine equations that have the form ax + by = c. This is an example from Bhaskara's commentary on Aryabhatiya:

Find the number which gives 5 as the remainder when divided by 8, 4 as the remainder when divided by 9, and 1 as the remainder when divided by 7

That is, find N = 8x+5 = 9y+4 = 7z+1. It turns out that the smallest value for N is 85. In general, diophantine equations, such as this, can be notoriously difficult. They were discussed extensively in ancient Vedic text Sulba Sutras, whose more ancient parts might date to 800 BCE. Aryabhata's method of solving such problems is called the *ku??aka* method.

Algebra

In *Aryabhatiya,* Aryabhata provided elegant results for the summation of series of squares and cubes:

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$ and

$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$$ (see squared triangular number)

ASTRONOMY :

Aryabhata's system of astronomy was called the *audAyaka* system, in which days are reckoned from *uday*, dawn at *lanka* or "equator". Some of his later writings on astronomy, which apparently proposed a second model (or *Ardha-Ratrika*, midnight) are lost but can be partly reconstructed from the discussion in Brahmagupta's *khanDakhAdyaka*. In some texts, he seems to ascribe the apparent motions of the heavens to the Earth's rotation. He may have believed that the planet's orbits as elliptical rather than circular.

CONCLUSION :

The best introduction to the genius of past is seen in the words of Bhaskara I who said, *“Aryabhatta is the master who, after reaching the furthest shores and plumbing the inmost depths of the sea of ultimate knowledge of mathematics, kinematics and spherics, handed over the three sciences to the learned world”.* Aryabhata's work was of great influence in the Indian astronomical tradition and influenced several neighbouring cultures through translations. It was largely due to his efforts that astronomy was recognized as a separate discipline from mathematics. The Arabic translation during the Islamic Golden Age (B.C. 820), was particularly influential. Some of his results are cited by Al-Khwarizmi and in the 10th century Al-Biruni stated that Aryabhata's followers believed that the Earth rotated on its axis.

His definitions of sine (*jya*), cosine (*kojya*), versine (*utkrama-jya*), and inverse sine (*otkram jya*) influenced the birth of trigonometry. He was also the first to specify sine and versine (1 - cos x) tables, in 3.75° intervals from 0° to 90°, to an accuracy of 4 decimal places.

In fact, modern names "sine" and "cosine" are mistranscriptions of the words *jya* and *kojya* as introduced by Aryabhata. As mentioned, they were translated as *jiba* and *kojiba* in Arabic and then misunderstood by Gerard of Cremona while translating an Arabic geometry text to Latin. He assumed that *jiba* was the Arabic word *jaib*, which means "fold in a garment", L. *sinus* . Aryabhata's astronomical calculation methods were also very influential. Along with the trigonometric tables, they came to be widely used in the Islamic world and used to compute many Arabic astronomical tables (zijes).

Some of the works of Aryabhatta include :

1)Aryabhatta worked out the value of pi.

2)He worked out the area of a triangle. His exact words were, *“ribhujasya phalashariram samadalakoti bhujardhasamvargah”* which translates “for a triangle, the result of a perpendicular with the half side is the area”.

3)He discussed the idea of sin.

4)He worked on the summation of series of squares and cubes (square-root and cube-root).

5)He talks about the “rule of three” which is to find the value of x when three numbers a, b and c is given.

6)Aryabhatta described the model of the solar system, where the sun and moon are each carried by

epicycles that in turn revolve around the Earth. He also talks about the number of rotations of the earth, describes that the earth rotating on its axis, the order of the planets in terms of distance from earth.
7)Aryabhatta describes the solar and lunar eclipses scientifically.
8)Aryabhatta calculated the sidereal rotation which is the rotation of the earth with respect to the stars as 23 hours, 56 minutes and 4.1 seconds.
9)He calculated the length of the sidereal year as 365 days, 6 hours, 12 minutes and 30 seconds. The actual value shows that his calculations was an error of 3 minutes and 20 seconds over a year.

Aryabhata was the genius who continues to baffle mathematicians even to this day. Several centuries later, the Muslim mathematician Abu Rayhan Biruni described the *Aryabhatiya* as a "mix of common pebbles and costly crystals". India's first satellite Aryabhata and the lunar crater Aryabhata are named in his honour. An Institute for conducting research in astronomy, astrophysics and atmospheric sciences is the Aryabhatta Research Institute of Observational Sciences(ARIES) near Nainital, India. The inter-school Aryabhata Maths Competition is also named after him. as is *Bacillus aryabhata*, a species of bacteria discovered by ISRO scientists in 2009.

Indians were brilliant mathematicians. In particular they made substantial progress in algebra. The decimal system and the invention of "Zero" were other achievements.

Bhāskara II (Bhaskaracharya)
Rasa-Gu?a-Pur?a-Mahisama
Saka-N?pa Samaye 'Bhavat Mamotpatti? /
Rasa-Gu?a-Var?e?a Maya
Siddhanta-Siroma?i Racita? //

Bhaskara II(Bhaskaracharya) was born in 1036 of the Saka era (1114 CE), and that he composed the Siddhanta Siroma?i when he was 36 years old. He also wrote another work called the *Kara?a-kutuhala* in 1183. His works show the influence of Brahmagupta, Sridhara, Mahavira, Padmanabha and other predecessors.

He was born near Vijjadavida (believed to be Bijapur in modern Karnataka). Bhaskara is said to have been the head of anastronomical observatory at Ujjain, the leading mathematical center of medieval India. He lived in the Sahyadri region Patnadevi, in Jalgaon district, Maharashtra.

The Siddhanta-Shiromani : *Siddhanta Shiromani* is divided into four parts called L*ilavati, Bijaga?ita, Grahaga?ita* and *Goladhyaya*

1) Lilavati :

The first section *Lilavati* (also known as *Pa?iga?ita or A?kaga?ita*) consists of 277 verses. It covers calculations, progressions, mensuration, permutations, and other topics.

2) Bijaganita:

The second section *Bijaga?ita* has 213 verses. It discusses zero, infinity, positive and negative numbers, and indeterminate equations including (the now called) Pell's equation, solving it using a *ku??aka* method. In particular, he also solved the $61x^2 + 1 = y^2$ case that was to elude Fermat and his European contemporaries centuries later.

3) Grahaganita :

In the third section Grahaga?ita, while treating the motion of planets, he considered their instantaneous speeds. He arrived at the approximation:

$\sin y' - \sin y \approx (y' - y)\cos y$ for y' close to y, or in modern notation:

$$\frac{d}{dy}\sin y = \cos y$$

In his words -

Bimbardhasya Ko?ijya Gu?astrijyahara? Phala? Dorjyayorantaram

Bhaskara also stated that at its highest point a planet's instantaneous speed is zero.

MATHEMATICS :

Some of Bhaskara's contributions to mathematics include the following:

•A proof of the Pythagorean theorem by calculating the same area in two different ways and then canceling out terms to get $a^2 + b^2 = c^2$.

•In *Lilavati*, solutions of quadratic, cubic and quartic indeterminate equations are explained.

•Solutions of indeterminate quadratic equations (of the type $ax^2 + b = y^2$).

•Integer solutions of linear and quadratic indeterminate equations (*Kuttaka*). The rules he gives are (in effect) the same as those given by the Renaissance European mathematicians of the 17th century

•A cyclic Chakravala method for solving indeterminate equations of the form $ax^2 + bx + c = y$. The solution to this equation was traditionally attributed to William Brouncker in 1657, though his method was more difficult than the chakravalamethod.

•The first general method for finding the solutions of the problem $x^2 - ny^2 = 1$ (so-called "Pell's equation") was given by Bhaskara II.

•Solutions of Diophantine equations of the second order, such as $61x^2 + 1 = y^2$. This very equation was posed as a problem in 1657 by the French mathematician Pierre de Fermat, but its solution was unknown in Europe until the time of Euler in the 18th century.

•Solved quadratic equations with more than one unknown, and found negative and irrational solutions.

•Preliminary concept of mathematical analysis.

•Preliminary concept of infinitesimal calculus, along with notable contributions towards integral calculus.

•Conceived differential calculus, after discovering the derivative and differential coefficient.

•Stated Rolle's theorem, a special case of one of the most important theorems in analysis, the mean value theorem. Traces of the general mean value theorem are also found in his works.

•Calculated the derivatives of trigonometric functions and formulae. (See Calculus section below.)

•In *Siddhanta Shiromani*, Bhaskara developed spherical trigonometry along with a number of other trigonometric results. (See Trigonometry section below.)

1) Arithmetic

Bhaskara's arithmetic text *Leelavati* covers the topics of definitions, arithmetical terms, interest computation, arithmetical and geometrical progressions, plane geometry, solid geometry, the shadow of the gnomon, methods to solve indeterminate equations, and combinations.

Lilavati is divided into 13 chapters and covers many branches of mathematics, arithmetic, algebra, geometry, and a little trigonometry and mensuration.

His work is outstanding for its systemisation, improved methods and the new topics that he has introduced. Furthermore the *Lilavati* contained excellent recreative problems and it is thought that Bhaskara's intention may have be.

2)Algebra

His *Bijaganita* ("*Algebra*") was a work in twelve chapters. It was the first text to recognize that a positive number has twos quare roots (a positive and negative square root).[15] His work *Bijaganita* is effectively a treatise on algebra and contains the following main topics:

- Positive and negative numbers.
- Zero.
- The 'unknown' (includes determining unknown quantities).
- Determining unknown quantities.
- Surds (includes evaluating surds).
- *Kuttaka* (solving indeterminate equations &Diophantine equations)

Bhaskara derived a cyclic, *chakravala* method for solving indeterminate quadratic equations of the form $ax^2 + bx + c = y$. Bhaskara's method for finding the solutions of the problem $Nx^2 + 1 = y^2$ (the so-called "Pell's equation") is of considerable importance.

3) Trigonometry

The *Siddhanta Shiromani* (written in 1150) demonstrates Bhaskara's knowledge of trigonometry, including the sine table and relationships between different trigonometric functions. He also discovered spherical trigonometry, along with other interesting trigonometrical results. In particular Bhaskara seemed more interested in trigonometry for its own sake than his predecessors who saw it only as a tool for calculation. Among the many interesting results given by Bhaskara, discoveries first found in his works include computation of sines of angles of 18 and 36 degrees, and the now well known formulae for sin (a+b) and sin(a-b) .

4) Calculus

His work, the *Siddhanta Shiromani*, is an astronomical treatise and contains many theories not found in earlier works. Preliminary concepts of infinitesimal calculus and mathematical analysis, along with a number of results in trigonometry, differential calculus and integral calculus that are found in the work are of particular interest.

Evidence suggests Bhaskara was acquainted with some ideas of differential calculus. There is evidence of an early form of Rolle's theorem in his work.

If $f(a) = f(b) = 0$ then $f'(x) = 0$ for some x with $a < x < b$

He gave the result that if $x \approx y$ then $\sin(y) - \sin(x) \approx (y - x)\cos(y)$, thereby finding the derivative of sine, although he never developed the notion of derivatives.

- Bhaskara uses this result to work out the position angle of the ecliptic, a quantity required for accurately predicting the time of an eclipse.
- In computing the instantaneous motion of a planet, the time interval between successive positions of the planets was no greater than a *truti,* or a $^{1/33750}$ of a second, and his measure of velocity was expressed

in this infinitesimal unit of time.

•He was aware that when a variable attains the maximum value, its differential vanishes.

•He also showed that when a planet is at its farthest from the earth, or at its closest, the equation of the centre (measure of how far a planet is from the position in which it is predicted to be, by assuming it is to move uniformly) vanishes. He therefore concluded that for some intermediate position the differential of the equation of the centre is equal to zero. In this result, there are traces of the general mean value theorem, one of the most important theorems in analysis, which today is usually derived from Rolle's theorem. The mean value theorem was later found by Parameshvara in the 15th century in the *Lilavati Bhasya*, a commentary on Bhaskara's *Lilavati*.

In the 12th century, Bhaskara II lived in southern India and wrote extensively on all then known branches of mathematics. His work contains mathematical objects equivalent or approximately equivalent to infinitesimals, derivatives, the mean value theorem and the derivative of the sine function. To what extent he anticipated the invention of calculus is a controversial subject among historians of mathematics.

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