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AN ORDER LEVEL INVENTORY MODEL FOR WEIBULL DISTRIBUTED DETERIORATING ITEMS WITH INVENTORY RETURNS AND SPECIAL SALES

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Short Profile :

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1.ABSTRACT :

In this chapter we have considered that the inventory is depleted not only by demand but also by Weibull distribution deterioration. Moreover we find that the feasibility condition for working of this model is proposed. Also the above sensitivity tables shown that the influence of the parameter ‘β’ is more significant than the changes in the other parameter like ‘α’ and C_4 values. It would be interesting to deal with this model in the context of finite Horizon Model. However, one cannot expect a closed form solution for the optimum quantity to be retained. In such situation one can use any search method like Genetical Algorithm.

Keywords:
Level Inventory Model , Inventory Returns and Special Sales , Genetical Algorithm.

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1.INTRODUCTION

In this Chapter We reconsider the order level Inventory model with inventory returns and special sales discussed by Dave [1]. Here the items deteriorate with time. The main stress is on the discussion that the situation where the optimal stock level of an Inventory system is smaller than the amount on hand.

Naddor [2] has considered this problem in case of EOQ Inventory system. Dave [1] has extended this model for the case of order level inventory system. However, this model is presented in a novel manner by considering shortages with prescribed scheduling period for deterministic demand. In these two models the assumption is that the order level inventory is less than the on-hand inventory. This type of situation may arise in any wholesale or retail business. The demand of a particular product decreases due to launching of a new product, which is cheaper and or superior, due to the effects of new budget such as price increase or due to any, other market fluctuations. In any such instances the optimum amount to be retained or sold, if any should be determined by minimizing the losses due to various costs involved in the inventory system.

In all these classical inventory models the depletion of inventory caused by a constant demand rate alone. But in several situations it may be noticed that the depletion of inventory may take place due to deterioration also. This deterioration plays a major role excepting in items like steel, hardware, etc. for these items the rate of deterioration is negligible on the other hand all food items, chemicals after same time will become useless for consumption. This loss must be considered while analyzing the inventory system. In this connection many researchers include Ghare and Schrader [3], Covert and Philip [4], Goyel et al [5] are very important. Misra [6] developed two parameter Weibull distribution deterioration for an inventory model. This investigation was followed by shah and Jaiswal [7], Aggarwal [8], Dave and Patel [9], Datta and Pal [10], Jalan et a1 [11], DiXit and Shah [12] etc. We now develop a single period inventory model with inventory returns and special case for the case of Weibull distributed deteriorates items. The Weibull distribution which is capable of representing constant, increasing and decreasing rates of deterioration is used to represent the distribution of the time to deterioration. The present inventory system is intended to obtain optimal Quantity to be retained for Weibull distributed deteriorate items.

3. ASSUMPTIONS AND NOTATIONS

The models are developed under the following assumptions

1. Demand is deterministic at a constant rate of 'R' units per unit time.

2. Scheduling period is a prescribed constant, T.

3. Replenishment size is constant and its rate is infinite. The fixed lot size 'q_p' raises the inventory level in each scheduling period to the order level 'S'.

4. Shortages are allowed and completed backlogged.

5. The inventory carrying cost C1 per unit time, the shortage cost C2 unit per unit time, the cost of each deteriorated unit C3 and the returning or selling cost Ci; per unit are known and constant during the period under consideration.

6. The system starts with an amount of 'Q' unit's on-hand of which only 'P' units are retained after returning or selling the rest the problem is to determine Optimal value of 'P'.

7. The deterioration rate functions for two parameter Weibull Distribution

$$\theta(t) = \alpha\beta t^{\beta-1}, 0 < \alpha, \beta > 0, t > 0.$$

Where β = l, $\theta(t)$ becomes a constant which is the case of an exponential decay. When β<1, the rate of deterioration is decreasing with time and β >1, is increasing with time't'.

4. MATHEMATICAL FORMULATION

Consider the period't' with the initial inventory level of 'Q' units and final inventory is assumed to be zero. This assumption is meaningful since (Q-P) units are sold with special sales price i.e.
C_4. The retained 'P' units are to be exhausted during the time t_1< T, during the remaining period (T - t_1) the optimal order level system will be operated.

Now Q_1(t) denotes the inventory position at time t (0 ≤ t ≤ t_1) then the differential equation governing the system for the Weibull distributed deteriorating items is given by

$$\frac{d}{dt}Q(t) + \Theta(t)Q(t) = -R \quad ; \qquad 0 \le t \le t_1 \qquad \ldots (2.1)$$

$$\frac{d}{dt}Q(t) = -R; \qquad t_1 \le t \le T \qquad \ldots (2.2)$$

Where $(t) = \alpha\beta t^{\beta-1}$, 0 < a < l, β > 0, t > 0

The boundary conditions are

Q_1(0) = P and $Q_1(t_1)$ = 0 . . . (2.3)

When 0 < α < 1, we ignore the terms of $0(\alpha^2)$ and use the conditions (2.3), then the solutions of the above equations are here under

$$Q(t) = \left\{-Rt - \frac{R\alpha}{\beta+1}t^{\beta+1}\right\}\left(1+\alpha t^{\beta}\right)^{-1} + P\left(1-\alpha t^{\beta}\right)^{-1} \quad \ldots (2.4)$$

And

$$Q(t) = -R(t - t_1); \qquad t_1 \le t \le T \qquad (2.5)$$

Since Q (t_1) = 0 at t = t_1 we get

$$P = Rt_1 + \frac{R\alpha}{(\beta+1)} t_1^{\beta+1} \qquad (2.6)$$

From (2.4) and (2.6) the total inventory carried during the period 't_1' is

$$Q(t_1) = \int_0^{f_1} Q(t)dt = \frac{Rt_1^2}{2} + \frac{R\alpha\beta}{(\beta+1)(\beta+2)} t_1^{\beta+2} \qquad (2.7)$$

The total cost of the system during the period 'T' is given by

$$K(P) = C_4(Q-P) + C_1Q(t_1) + C_1Q(t_1) + (T-t_1)C(t_1) \qquad (2.8)$$

Where C(t_1) is the average total cost per unit of optimum order level operating system during (T-t_1) and is given by

$$C(t_1) = \frac{C_3R\alpha}{T(\beta+1)} t_1^{\beta+1} + \frac{C_1}{T}\left[\frac{Rt_1^2}{2} + \frac{R\alpha\beta}{(\beta+1)(\beta+2)} t_1^{\beta+2}\right] + \frac{C_2}{T}\frac{R(T-t_1)^2}{2} \qquad (2.9)$$

The t_1^* of $t_1$can be obtained by differentiating the above euation with resdpect to t_1 and equation to zero. However one should ensure that the second derivative must be greater than zero to get optimum value of t_1^* of t_1 i.e.., t_1^* is the solution of the following equation.

$$C_3\alpha t_1^{\beta-1} + C_1\left(1 + \frac{\alpha\beta t_1^{\beta+1}}{\beta+1}\right) - C_2(T-t_1) = 0 \qquad (2.10)$$

Proceeding in similar fashion of equation (2.6), we get the optimum order level s^0 of S as

$$S^0 = R_1t_1^* + \frac{R\alpha}{\beta+1} t_1^{*\beta+1} \qquad (2.11)$$

again the total amount of back order at the end of the cycle is R(T -t_1). Therefore the optimum value of $q_p^* of\ q_p$ is given by

$$q_p^* = S^0 + R(T - t_1^*)$$

$$q_p^* = \frac{R\alpha}{\beta+1} t_1^{*\beta+1} + RT \qquad (2.12)$$

and the minimum value of the average total cost C(t_1) is C(t_1^*).

5. RESULTS IN THE ABSENCE OF DETERIORATIONS:

If the deterioration of the item is switched off (α = O), the equation (2.10) for the optimum value of t_1 reduces to linear equation.

$$C_1t_1 - C_2T + C_2t_1 = 0$$

$\Rightarrow\ t_1 = \frac{C_2 T}{C_1 + C_2}$

And corresponding value of t_1=t_1^*

$t_1^* = \frac{C_2 T}{C_1 + C_2}$ (2.13)

Moreover the expressions for S^0 of q* can be obtained by substituting α = O in the equation (2.11) and (2.12)

$S^0 = Rt_1^*$. (2.14)

And

q_p^*= RT. (2.15)

which agrees with Naddor [2].

Using the equation (2.6), (2.7), (2.9) in (2.8) we get

$$K(P) = C_4\left[Q - \left(Rt_1 + \frac{R\alpha}{\beta+1} t_1^{\beta+1}\right)\right] + C_1\left[\frac{Rt_1^2}{2} + \frac{R\alpha\beta}{(\beta+1)(\beta+2)} t_1^{\beta+2}\right] + (T - t_1)C(S) \ \ldots\ldots.(2.16)$$

Since the above equation is a function of t_1, it is denoted as K (p, t_1) again P is a function of ti as in (2.6), the necessary condition for the minimization of the cost K (t_1) is

$$\frac{d}{dt} k(t_1) = 0$$

After little simplification, the condition can be written as

$$C_1\left(\alpha\beta t_1^{\beta+1} + (\beta+1)t_1\right) - C_4(1 - \alpha t_1^{\beta})(\beta+1) - C(S)R(\beta+1) = 0 \quad (2.17)$$

The solution of the above equation gives the optimal value of t_1 say t_1^* . The above equation in t1 can be solved by using Newton Rapshson method or any other search method substituting t_1^* in (2.6) we get the optimum value of P^0 of P, the sufficient condition of minimum total cost is

$$\frac{d^2}{dt^2} K(t_1) \text{ at } t_1 = t_1^*$$

$$= -C_4\alpha\beta t_1^{*\beta-1} + C_1 + C_1\alpha\beta t_1^{*\beta} > 0 \text{ Should be satisfied}$$

Note that the maximum Quantity that can be returned or sold if ever is Q i.e. the optimum value of P must be less than or equal to Q. However P depends on $t_1$and therefore the optimum solution of the present inventory system should be represented as follows.

$$P^0 = Rt_1^* + \frac{R\alpha}{\beta+1} t_1^{*\beta+1} \qquad ; if\ 0 \le C_4 < \frac{C_1\left(1+\alpha\beta t_1^{*\beta}\right)}{\alpha\beta t_1^{*\beta-1}}$$

$= Q$; Otherwise ……………………………….. (2.18)

The above equation (2.18) gives the optimal value of P i.e. the optimal Quantity to be retained.

6. NUMERICAL ILLUSTRATIONS

Let the hypothetical values of parameters of the inventory models be $C_1 = 3$; $C_2 = 15$; $C_3 = 5$; $C_4 = 4$; $R = 100$; $T = 1$. All the parameters are expressed in consistent units per month. For different values of 'α' and 'β', we have determined the optimal Quantity to be retained and the associated costs are portrayed in the following table. To do this, at first we solved the (2.17) by using Newton Raphson method and t_1^* is substituted in equations (2.18), (2.16) to get optimum quantity to retained and associated minimum cost respectively.

TABLE 2.1: SENSITIVITY OF THE MODEL WITH RESPECT TO DETERIORATION RATES I.E.., 'A' AND 'B'

α values	β values									
	1	2	3	4	5	6	7	8	9	10
0.01	112.85	115.86	117.54	118.32	118.46	118.16	117.56	116.78	115.89	114.65
	173.24	174.21	174.93	175.41	175.64	175.61	175.36	175.02	174.75	174.74
	97.31	145.36	245.23	448.81	831.48	1491.79	2533.97	4043.46	6133.17	8878.12
0.02	121.38	127.28	130.58	132.09	132.37	131.79	130.63	129.10	127.36	125.51
	178.47	180.41	181.83	182.69	183.00	182.82	182.64	182.55	182.86	183.78
	116.92	224.53	454.03	915.70	1737.41	3038.41	4913.25	7445.32	10780.62	15071.62
0.03	129.76	138.45	143.28	145.51	145.92	145.08	143.38	141.15	138.59	135.88
	183.74	186.64	188.70	189.82	190.13	189.99	189.87	190.16	191.12	192.99
	139.45	318.39	705.59	1465.15	2743.78	4643.79	7248.10	10662.16	15085.84	20756.44
0.04	137.99	149.36	155.68	158.59	159.13	158.04	155.84	152.93	149.59	146.04
	189.06	192.90	195.53	196.81	197.07	196.95	197.04	197.77	199.42	202.21
	164.98	427.01	996.81	2079.23	3811.83	6272.72	9548.55	13780.37	19227.04	26210.19
0.05	146.07	160.03	167.77	171.33	172.01	170.68	168.00	164.45	160.37	156.02

	194.41	199.17	202.28	203.64	203.85	203.79	204.15	205.65	207.69	211.41
	193.57	550.19	1323.75	2741.73	4916.49	7908.01	11819.39	16833.07	23267.16	31527.32
0.06	154.01	170.47	179.57	183.76	184.57	183.03	179.90	175.73	170.93	165.82
	199.79	205.44	208.95	210.32	210.49	210.54	211.19	212.89	215.92	220.57
	25.25	687.53	1680.78	3439.37	6041.68	9540.58	14063.33	19835.87	27235.88	36751.10
0.07	161.80	180.67	191.09	195.89	196.83	195.08	191.52	186.77	181.29	175.43
	205.21	211.70	215.25	216.86	217.01	217.20	218.18	220.38	224.11	229.68
	260.02	838.39	2064.10	4161.89	7177.35	11165.52	16282.96	22798.04	31148.66	41904.16
0.08	169.46	190.65	202.33	207.73	208.79	206.86	202.89	197.58	191.45	184.88
	210.66	217.94	221.99	223.27	223.42	223.78	225.10	227.82	232.26	238.76
	297.99	1002.05	2468.99	4901.55	8316.97	12779.97	18478.06	25725.00	35015.51	47000.77
0.09	176.97	200.42	213.32	219.28	220.48	218.38	214.02	208.18	201.42	194.17
	216.14	224.15	228.37	229.57	229.74	230.29	231.97	235.21	240.36	247.79
	339.07	1177.64	2891.48	5652.59	9456.12	14382.37	20651.45	28620.70	38842.49	52049.63
0.10	184.36	209.97	224.05	230.57	231.90	229.64	224.92	218.57	211.21	203.29
	221.63	230.33	234.65	235.77	235.97	236.72	238.78	242.56	248.42	256.78
	383.28	1364.27	3328.15	6410.67	10591.85	15971.78	22803.57	31631.77	42633.91	57056.73

The first row values are corresponding 'α' are obtained using equation (2.9) i.e. the optimal cost during the period (t - t_1) and the second row values for corresponding 'α' are the optimum quantities to be retained i.e. P^0 of P using equation (2.6). The third row values for corresponding 'α' are obtained from equation (2.16).

From the above table we observe that as 'α' increases optimum quantity to be retained will increase and there is a marginal Change in the cost even though 'P' increases. Whereas in case of 'β' the associated cost will increase drastically. This can be noted from the third row values of the table 2.1 When α = 0.01. Similar observation can be made for different values of 'β' and 'α'. Hence the model is very sensitive for changes in 'β' rather than changes in the 'α' values. However, it would be interesting to check this sensitivity of the model with respect to the changes in the special sales i.e. C_4. To do so, the pertinent computations are summarized in table 2.2.

TABLE 2.2: SENSITIVITY OF THE MODEL TO CHANGES IN THE 'C_4' AND 'A'

C_4 values α values	0.20	0.24	0.28	0.32	0.36	0.40	0.44	0.48	0.52
0.01	44.313	45.653	46.994	48.335	49.676	51.018	52.703	53.703	55.045
	283.805	322.037	360.219	398.348	436.426	474.453	512.428	573.239	607.123
0.02	47.189	48.537	49.886	51.235	52.586	53.936	55.288	56.640	57.993
	288.955	327.113	365.222	403.280	441.289	479.249	517.159	577.547	611.672
0.03	50.019	51.375	52.732	54.091	56.131	57.492	58.173	59.536	60.900
	293.958	332.052	370.098	408.097	446.129	484.037	521.809	581.801	616.165
0.04	52.802	54.167	55.534	56.902	58.272	59.643	61.016	62.390	63.766
	298.849	336.889	374.884	412.834	450.739	488.599	526.415	581.801	616.165
0.05	55.539	56.914	58.291	59.670	61.050	62.433	63.817	65.203	66.591
	303.659	341.657	379.612	417.525	455.395	493.223	531.009	590.293	625.125
0.06	58.231	59.617	61.004	62.394	63.786	65.180	66.577	67.975	69.376
	308.420	346.387	384.313	422.200	460.047	497.855	535.624	594.595	629.659
0.07	60.879	62.275	63.674	65.076	66.480	67.886	69.296	70.707	72.121
	313.161	351.107	389.016	426.888	464.724	502.524	540.288	598.973	634.269
0.08	63.487	64.891	66.302	67.715	69.132	70.551	71.974	73.399	74.827
	317.907	322.844	393.747	431.617	469.454	507.257	545.029	603.453	628.980
0.09	63.526	67.463	68.887	70.313	71.743	73.176	74.612	76.052	77.494
	321.750	360.622	398.531	436.409	474.258	512.078	549.869	608.059	643.818
0.10	68.562	69.994	71.430	72.870	74.313	75.760	77.211	78.665	80.123
	327.510	403.389	403.389	441.288	479.161	517.009	554.831	566.429	648.802

From the above table we note that as C_4 values increase there is a marginal change in the values of 'P' and K(P) i.e. the optimum quantity to be retained and the associated costs.

7. DISCUSSION:

In this chapter we have considered that the inventory is depleted not only by demand but also by Weibull distribution deterioration. Moreover we find that the feasibility condition for working of this model is proposed in equation 2.18. Also the above sensitivity tables show that the influence of the parameter 'β' is more significant than the changes in the other parameter like 'α' and C_4 values. lt would be interesting to deal with this model in the context of finite Horizon Model. However, in the subsequent chapter we reconsidered the aspect of inventory returns and special sales in the case of Power pattern demand. This gives a general solution i.e., the general in the sense that this model deals with several patterns of the demand occurs during the planning horizon. It also very interesting to deal with this situation in probabilistic demand. However, one cannot expect a closed form solution for the optimum quantity to be retained. In such situation one can use any search method like Genetical Algorithm (see Manjusri Basu and Sudipta Sinha [13]).

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