



STUDIES ON PLASMA BURN-THROUGH SIMULATION: AN OVERVIEW

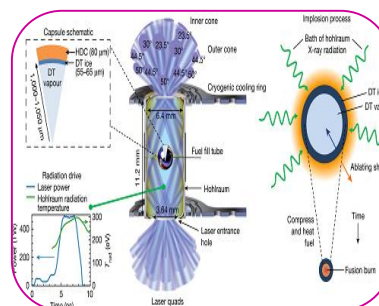
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ABSTRACT

It has been observed that most failures in tokamak start-up occur during the plasma burn through phase. However, our understanding of plasma burn-through is limited, and tokamak start-up has relied on trial and error methods without investigation on the physics of plasma burn-through. The plasma burn-through modelling is a very useful tool to understand the key physics aspects in the burn-through phase, and it will make a contribution to ensuring a reliable tokamak start-up and can be used in reducing flux consumption of the ohmic transformer at the start of the plasma discharge in tokamaks. Plasma burn-through modelling can also be a basis for the research on non-inductive start-up (or solenoid-free start-up), which are of crucial importance for a fusion power plant.



KEYWORDS : ITER, Electric field & Plasma burn.

INTRODUCTION

In ITER, the allowable toroidal electric field for start-up is limited up to 0.35[V/m] due to the engineering issues explained in section 1.2. Since tokamak start-up using such a low E field is only possible for a narrow range of prefill gas pressure, low magnetic error fields, and low impurity content, RF-assisted start-up [1-5] has been proposed in ITER. In order to estimate the required RF power, plasma burn-through in ITER was modelled [6]. However, the previous models were overly simplified, and furthermore, they have never been validated against experimental data. In this section, the basic structure of plasma burn-through modelling and the enhancement in the DYON code are introduced.

Basic structure of plasma burn-through simulator:

In this paper, all physical quantities are expressed in SI units except for the prefill gas pressure. A prefill gas pressure is indicated in [Torr]. Temperatures in the equations are written in [Joules]. Whenever the temperature given is in [eV], this is explicitly stated.

Circuit equation for plasma current:

Assuming a plasma as a circular current loop, the plasma current I_p can be calculated using the circuit equation.

$$U_t = I_p R_p + L_p \frac{dI_p}{dt} \quad (1)$$

where U_t is a toroidal loop voltage, induced by the external coil voltage. In this section, the self-inductance L_p and electric resistance R_p of a plasma are derived, respectively.

Plasma inductance:

Inductance can be calculated from the relation of electric current and the stored magnetic energy in electromagnetics,

$$\frac{1}{2}LI^2 = \int_V \frac{B^2}{2\mu_0} dV \quad (2)$$

Derivation of inductance requires surprisingly lengthy calculations depending on the coil geometry, but it is well known that the electric inductance of a circular current loop is

$$L = \mu_0 R \left(\ln \frac{8R}{a} - 2 + Y \right) \quad (3)$$

where a is a minor radius, R is a major radius, and Y is a function of the current profile in the coil (plasma cross-section)¹. Y represents how much magnetic energy ($= B^2/2\mu$) is stored within the coil. When the electric current is uniformly distributed over the surface of the coil (no magnetic energy within the coil), $Y = 0$.

Equation (3) can be applied to calculate the inductance L_p of the plasma current. L_p can be separated into two parts,

$$L_p = L_e + L_i \quad (4)$$

L_e is external inductance which is related to the magnetic energy stored in the external volume (outside the coil),

$$L_e = \mu_0 R \left(\ln \frac{8R}{a} - 2 \right) \quad (5)$$

The inductance L_i represents the stored magnetic energy within the plasma volume, and is defined as

$$L_i = \mu_0 R Y \quad (6)$$

Y (and in turn L_i) for a plasma current can be derived using Equation (3.2),

$$\frac{1}{2} L_i I_p^2 = \int_{\text{plasmavolume}} \frac{B_\theta(x, y, z)^2}{2\mu_0} dx dy dz, \quad (7)$$

where B_θ is the poloidal magnetic field, produced by a plasma current. In order to integrate B_θ over the plasma volume, which is a torus, we should change the rectangular coordinate to be suitable for the integration of the torus volume,

$$\begin{aligned} (x, y, z) &\rightarrow (R_\phi, r, \theta) \\ dx dy dz &\rightarrow R d\phi dr rd\theta \end{aligned} \quad (8)$$

where ϕ and θ are the toroidal and poloidal angle, and R and r are the length in directions of major radius and minor radius, respectively. Then, assuming toroidal symmetry, $dB_\theta(R_\phi, r, \theta)/d\phi = 0$, we can have

$$\begin{aligned} \frac{1}{2} L_i I_p^2 &= \int_0^a \int_0^{2\pi} \int_0^{2\pi} \frac{B_\theta(R\phi, r, \theta)^2}{2\mu_0} R r d\phi dr d\theta \\ &= 2\pi \int_0^a \int_0^{2\pi} \frac{B_\theta(r, \theta)^2}{2\mu_0} R r dr d\theta \end{aligned} \quad (9)$$

We assume a plasma current in a circular cross-section and assume poloidal symmetry i.e. $dB_\theta(r, \theta)/d\theta=0$. Then Equation (9) is

$$\frac{1}{2} L_i I_p^2 = 4\pi^2 \int_0^a \frac{B_\theta(r)^2}{2\mu_0} R r dr \quad (10)$$

Using Ampere's law, i.e. $\nabla \times B\theta = \mu_0 J_p$, we can substitute I_p in Equation (10) with

$$I_p = \frac{2\pi a B_\theta(a)}{\mu_0} \quad (11)$$

then, L_i and Y can be obtained as

$$L_i = \mu_0 R \times \frac{\int_0^a B_\theta(r)^2 r dr}{a^2 B_\theta^2(a)} \quad (12)$$

$$Y = \frac{\int_0^a B_\theta(r)^2 r dr}{a^2 B_\theta^2(a)}$$

In plasma physics, it is conventional to express the plasma inductance, L_p using a dimensionless normalized plasma inductance l_i^2 ,

$$L_p = \mu_0 R \left(\ln \frac{8R}{a} - 2 + \frac{l_i}{2} \right) \quad \text{where } l_i = \frac{2 \int_0^a B_\theta(r)^2 r dr}{a^2 B_\theta^2(a)} \quad (13)$$

In the case of a flat profile of the plasma current, $B_\theta(r)$ can be substituted using the constant plasma current density J_p ,

$$B_\theta(r) = \frac{\mu_0 J_p \pi r^2}{2\pi r} \quad (14)$$

Then, l_i for a uniform plasma current is

$$l_i = \frac{2 \int_0^a \left(\frac{\mu_0 J_p \pi r^2}{2\pi r} \right)^2 dr}{a^2 \left(\frac{\mu_0 J_p \pi a^2}{2\pi a} \right)^2} = \frac{2 \int_0^a r^2 dr}{a^4} = 0.5 \quad (15)$$

Plasma Resistance:

We can derive the plasma resistance R_p assuming that coulomb collisions are dominant compared to electron-neutral collisions. The derivation begins from the fluid equation of motion for electrons,

$$m_e n_e \frac{d\vec{u}_e}{dt} = -en_e (\vec{E} + \vec{u}_e \times \vec{B}) - \nabla p_e + \vec{F}_{e-i} \quad (16)$$

where \vec{F}_{e-i} is the friction force, which results from collisional transfer of momentum between electron and ion, and \vec{u}_e is the fluid velocity of electrons (\vec{u}_i is ion fluid velocity). Defining momentum transfer frequency of an electron v_{e-i}^m , \vec{F}_{e-i} can be expressed as

$$\vec{F}_{e-i} = m_e n_e (\vec{u}_i - \vec{u}_e) \langle v_{e-i}^m \rangle, \quad (17)$$

where $\langle v_{e-i}^m \rangle$ is an average value of v_{e-i}^m in the fluid. Assuming uniform electron pressure ($\nabla p_e = 0$), parallel fluid velocity to the electric field and the magnetic field ($\vec{u}_e \parallel \vec{u}_i \parallel \vec{E}, \vec{u}_e \times \vec{B} = 0$), and small electron inertia term ($m_e n_e \frac{d\vec{u}_e}{dt} = 0$), Equation (16) can be reduced to

$$en_e E = m_e n_e (u_i - u_e) \langle v_{e-i}^m \rangle \quad (18)$$

The electric resistivity is a function of the electric current and the electric field, $\eta = \frac{E}{J}$. Substituting J and E with $en_e(u_i - u_e)$ and Equation (18), the electric resistivity η_{e-i} due to the e-i collisions can be obtained,

$$\eta_{e-i} = \frac{E}{J} = \frac{m_e \langle v_{e-i}^m \rangle}{e^2 n_e} \quad (19)$$

For a deuterium plasma, v_{e-i}^m is a product of n_i (ion density), σ_{e-i}^m (cross-section for collisional momentum transfer), and v_e (velocity of an electron).

$$v_{e-i}^m = n_i \sigma_{e-i}^m v_e. \quad (20)$$

σ_{e-i}^m can be calculated with the impact parameter b_0 which is defined as a distance between an electron and an ion when electrons have a 90 scattering coulomb collision.

$$b_0 = \frac{e^2}{4\pi \epsilon_0 m_e (v_e)^2} \quad (21)$$

Then, σ_{e-i}^m equals $\pi b_0^2 \times (4 \ln \Delta)$,

$$\sigma_{e-i}^m = \pi \left(\frac{e^2}{4\pi \epsilon_0 m_e (v_e)^2} \right)^2 \times 4 \ln \Delta \quad (22)$$

where $(4 \ln \Delta)$ is a correction factor to include the cumulative effect of many small-angle deflections. The coulomb logarithm $\ln \Delta$ is insensitive to plasma parameters ($\ln \Delta \approx 10$), and is defined by $\ln (\lambda_D/b_0)$.⁴

Substituting σ_{e-i}^m in Equation (20) with Equation (22), we can have

$$\begin{aligned} v_{e-i}^m &= n_i \times \pi \left(\frac{e^2}{4\pi \epsilon_0 m_e (v_e)^2} \right)^2 \times 4 \ln \Delta \times v_e \\ &= \frac{n_i e^4 \ln \Delta}{4\pi \epsilon_0^2 m_e^2 v_e^3}. \end{aligned} \quad (23)$$

Since v_{e-i}^m is a function of v_e^{-3} , $\langle v_{e-i}^m \rangle$ must be calculated using a distribution function. The detailed derivation of $\langle v_{e-i}^m \rangle$ using Equation (23) for a Maxwellian distribution is well described in [4]. Defining $v_e^{th} = \sqrt{T_e / m_e}$, the average value of the momentum collision frequency $\langle v_{e-i}^m \rangle$ is

$$\langle v_{e-i}^m \rangle = \frac{n_i e^4 \ln \Delta}{6\sqrt{2}\pi^{3/2} \epsilon_0^2 m_e^{1/2} (T_e)^3} \quad (24)$$

The plasma resistivity η_{e-i} can be calculated by substituting the momentum transfer frequency v_{e-i}^m in Equation (19) with Equation (24),

$$\begin{aligned} \eta_{e-i} &= \frac{m_e \langle v_{e-i}^m \rangle}{e^e n_e} = \frac{m_e}{e^e n_e} = \frac{n_i e^4 \ln \Delta}{6\sqrt{2}\pi^{3/2} \epsilon_0^2 m_e^{1/2} (T_e)^3} \\ &= \frac{m_e^{1/2} e^e \ln \Delta}{6\sqrt{2}\pi^{3/2} \epsilon_0^2 (T_e)^{3/2}} \end{aligned} \quad (25)$$

In order to calculate an accurate plasma resistivity, e-e collisions should also be considered. Spitzer and co-workers have found the correction factor for this as 0.51. Hence, accurate resistivity in a deuterium plasma i.e. *Spitzer resistivity* η_s is

$$\eta_s = 0.51 \frac{m_e^{1/2} e^e \ln \Delta}{6\sqrt{2}\pi^{3/2} \epsilon_0^2 (T_e)^{3/2}}$$

$$\approx 5 \times 10^{-5} \times \ln\Lambda \times T_e^{-3/2} [eV]. \tag{26}$$

In the case of impurities in a plasma, η_s should be modified to $Z_f\eta_s$ where Z_f is an effective charge, defined as [1]

$$Z_f = \frac{\sum_A \sum_Z n_A^{z+} z^2}{\sum_A \sum_{Z \geq 1} n_A^{z+} z} \tag{27}$$

where subscript A represents deuterium or an impurity. z means an ionic charge state. Accordingly, n_A^{z+} indicates deuterium ion density n_D^{1+} or impurity ion densities n_I^{z+} with charge state z .

Plasma resistance R_p is a function of plasma resistivity η_s and the size of the current loop (cross-section A and length l),

$$\begin{aligned} R_p &= Z_f \eta_s \frac{1}{A} = Z_f \eta_s \frac{2\pi R}{\pi a^2} \\ &= 5 \times 10^{-5} \times \ln\Lambda \times Z_f \times \frac{2R}{a^2} \times T_e^{-3/2} [eV] \end{aligned} \tag{28}$$

where R and a are the major and minor radius of the plasma.

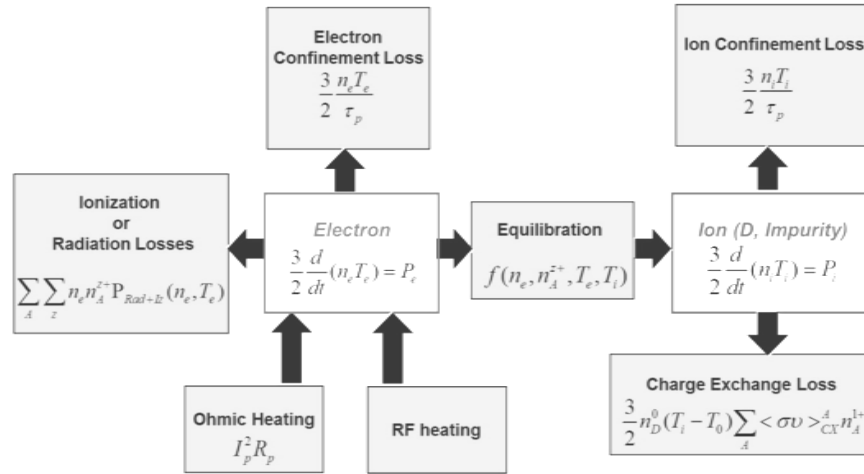


Figure- 1: Energy flow in plasma

Electron energy balance

It should be noted that plasma resistance R_p is a function of electron temperature T_e and effective charge Z_f . In order to calculate I_p with (1), T_e and Z_f should be obtained by solving the energy and particle balance equations.

Figure 1 shows how energy flows in a plasma through various heating and loss channels. The net heating power i.e. $P_{\text{heating}} - P_{\text{loss}}$ remains as the internal energy of the plasma, $\frac{3}{2} n_e T_e$ and $\frac{3}{2} n_i T_i$. Assuming a uniform plasma density and temperature, we can calculate the electron energy balance equation,

$$\frac{3}{2} \frac{d(n_e T_e)}{dt} = P_{oh} + P_{aux} - (P_{iz} + P_{rad}) - P_{equi} - P_{conv}^e \quad (29)$$

where P_{oh} and P_{aux} are ohmic heating and auxiliary heating such as ECH or ICH. Electron power losses in Equation (29) consist of collisional ionization power loss P_{iz} , radiation power loss P_{rad} , equilibration power loss P_{equi} , and convective transport power loss P_{conv}^e .

Electron heating power

Ohmic heating P_{oh} is the main electron heating source, and all ohmic heating power is assumed to be absorbed by electrons without ion heating. Ohmic heating power per unit volume is

$$P_{oh} = \frac{I_p^2 R_p}{V_p} \quad (30)$$

where V_p is the plasma volume [6].

Since most simulations in this thesis are for ohmic start-up cases (no RF assist), we set $P_{aux} = 0$. In order to provide more input power to ensure robust tokamak start-up, auxiliary heating such as Electron cyclotron Heating (ECH) or Ion Cyclotron Heating (ICH) is planned for ITER. The auxiliary heating power P_{aux} to the plasma is a complicated function of various plasma parameters. For RF-assisted start-up for ITER, we will assume P_{aux} as a constant (in time and over the plasma volume) absorbed heating power.

Electron power losses

The collisional ionization process is a power loss mechanism from an electron point of view since a free electron loses their kinetic energy as much as the binding energy of an electron in an atom [6]. Therefore, collisional ionization power loss P_{iz} is

$$P_{iz} = \frac{V_n^A}{V_p} \sum_A \langle \sigma v \rangle_{A,iz}^{0 \rightarrow 1+} W_A^{0 \rightarrow 1+} n_e n_A^0 + \sum_A \sum_{z \geq 1} \langle \sigma v \rangle_{A,iz}^{z+ \rightarrow (z+1)+} W_A^{z+ \rightarrow (z+1)+} n_e n_A^{z+} \quad (31)$$

where $W_A^{z+ \rightarrow (z+1)+}$ is the ionization energy required to ionize an atom or a non-fully ionized ion from $z+$ to an $(z+1)+$. Here V_n^A represents a neutral volume of species A within a plasma volume. Since the ionization reaction of neutrals can occur only in the neutral volume within a plasma volume, the different volume occupied by neutrals or ions must be taken into account. The first term on the right-hand-side in Equation (31) is the electron power loss required to ionize neutrals. The second term is for further ionization of non-fully ionized ions to higher charge states.

$\langle \sigma v \rangle_{A,iz}^{0 \rightarrow 1+}$ and $\langle \sigma v \rangle_{A,iz}^{z+ \rightarrow (z+1)+}$ are ionization rate coefficients. In this thesis, the reaction rate coefficients and power coefficients are expressed as $\langle \sigma v \rangle$. Their superscript indicates the change of the ion charge in the atomic reaction, the subscripts represent the species of the reaction particle and the kind of the reaction. For example, $\langle \sigma v \rangle_{A,rec}^{z+ \rightarrow (z-1)+}$ indicates a recombination rate coefficient of species A of which the ionic charge transits to $(z-1)+$ from $z+$ through a recombination reaction. In the case of charge exchange reaction, the subscript is cx . $\langle \sigma v \rangle_{A,line}^{z+}$ is a power coefficient for line radiation and $\langle \sigma v \rangle_{A,RB}^{Z+ \rightarrow (z-1)+}$ is a power coefficient for Recombination and Bremsstrahlung radiation. The rate coefficients and power coefficients used in the burn-through simulation are obtained from Atomic Data and Analysis Structure (ADAS) package.

DISCUSSION & CONCLUSION

The ADAS atomic rate coefficients are based on the generalized collisional-radiative theory, and the data can cover various range of plasmas e.g. space plasma, industrial plasma, and the thermonuclear fusion plasma in current devices. It is assumed in the ADAS data that the free electrons have a Maxwellian velocity distribution and the dominant populations of impurities are in the quasi-equilibrium *i.e.* the ground and metastable states. If there is a collisional excitation of an atom or an ion, a free electron also loses its kinetic energy. In the case of optically thin plasma (no reabsorption of photons in the plasma), which is assumed in this thesis, the amount of the electron power loss for collisional excitations is equal to the subsequent line radiation power. The electron power loss resulting from the electron deceleration due to the background ions is also equal to the Bremsstrahlung radiation power loss. However, in the case of recombination, the radiation power loss is greater than the electron power loss for the recombination reactions since the potential energy in an atom or an ion is included in the total recombination radiation power. Therefore, this amount must be subtracted from the total recombination radiation power in order to calculate the electron power loss.

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