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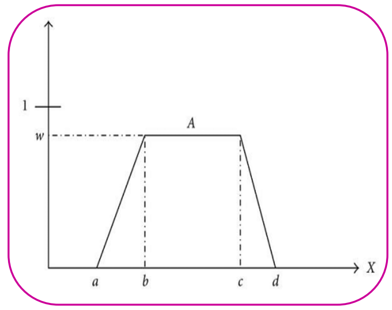
# GENERALIZED TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBER VIA NEW RANKING METHOD

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## ABSTRACT

*In this paper, new ranking method for ordering generalized trapezoidal intuitionistic fuzzy numbers (GTRIFNs) is introduced. Intuitionistic max- min method and generalized intuitionistic modified distribution method is introduced for computing the initial basic feasible solution (IBFS) and optimal solution respectively of transportation problem in which the costs are represented by GTRIFNs.*

**KEY WORDS:** *method and generalized intuitionistic modified distribution method .*

## INTRODUCTION

In 1970, Bellman and Zadeh (1970) introduced the concept of decision making in fuzzy environment. The concept of optimization in intuitionistic fuzzy environment was given by Angelov (1997). One of the important applications of linear programming is in the area of transportation of goods and services from several supply centres to several demand centres. The simplest transportation model was first presented by Hitchcock (1941) in 1941. Several other extensions were successively developed.

In 1984, Chanas. Et. Al (1984) presented a fuzzy approach to the transportation problem. Fuzzy zero point method is introduced by Pandian and Natarajan (2010), which was extended to intuitionistic fuzzy zero point method by Hussain and kumar (2012) to compute optimal solution of transportation problem. To the best of our knowledge, till now no one has used generalized trapezoidal intuitionistic fuzzy numbers for solving transportation problems

## PRELIMINARIES

In this section, some basic results related to intuitionistic fuzzy set theory are reviewed.

Definition 1 (Atanassov, 1999): Let X be a universal set. An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the form $A = \{ < x, \mu_A(x), \upsilon_A(x) > : x \in X \}$ where the functions $\mu_A : X \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ to the set A respectively and for every $x \in X$ in A, $0 \le \mu_A(x) + \upsilon_A(x) \le 1$ holds.

Definition 2 (Atanassov, 1999): For every common intuitionistic fuzzy subset A on X, intuitionistic fuzzy index of x in A is defined as $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$. It is also known as degree of hesitancy or degree of uncertainty of the element x in A.

Obviously, for every $x \in X$, $0 \le \pi_A(x) \le 1$.

Definition 3 (Mahapatra and Mahapatra, 2010): An Intuitionistic Fuzzy Number (IFN) $\tilde{a}^I$ is

a) An intuitionistic fuzzy subset of the real line.
b) Convex for the membership function $\mu_a(x)$, that is, $\mu_a(\lambda x_1 + (1-\lambda)x_2) \ge \min(\mu_a(x_1), \mu_a(x_2))\ \forall\ x_1, x_2 \in R, \lambda \in [0, 1]$.
c) Concave for the non-membership function $\upsilon_a(x)$, that is, $\upsilon_a(\lambda x_1 + (1-\lambda)x_2) \le \max(\upsilon_a(x_1), \upsilon_a(x_2))\ \forall\ x_1, x_2 \in R, \lambda \in [0, 1]$.
d) Normal, that is, there is some $x_0 \in R$ such that $\mu_a(x_0) = 1$, $\upsilon_a(x_0) = 0$.

Definition 4 (Mahapatra and Mahapatra, 2010): An intuitionistic fuzzy number $\tilde{a}^I = \langle (a_1, a_2, a_3, a_4)(\bar{a}_1, a_2, a_3, \bar{a}_4) \rangle$ is said to be trapezoidal intuitionistic fuzzy number (TRIFN) if its membership and non-membership functions are respectively given by

$$\mu_a(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & \text{if } a_1 \le x \le a_2 \\ 1 & \text{if } a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} & \text{if } a_3 \le x \le a_4 \\ 0 & \text{otherwise} \end{cases}$$

$$\upsilon_a(x) = \begin{cases} \dfrac{a_2 - x}{a_2 - \bar{a}_1} & \text{if } \bar{a}_1 \le x \le a_2 \\ 0 & \text{if } a_2 \le x \le a_3 \\ \dfrac{a_3 - x}{a_3 - \bar{a}_4} & \text{if } a_3 \le x \le \bar{a}_4 \\ 1 & \text{otherwise} \end{cases}$$

**Definition 5:** An intuitionistic fuzzy number $\tilde{a}^I = \langle (a_1, a_2, a_3, a_4; \omega_a)(\bar{a}_1, a_2, a_3, \bar{a}_4; \sigma_a) \rangle$ is said to be a generalized intuitionistic fuzzy number (GTRIFN) if its membership and non-membership function are respectively given by

$$\mu_a(x) = \begin{cases} \dfrac{(x - a_1)\omega_a}{a_2 - a_1} & \text{if } a_1 \le x \le a_2 \\ \omega_a & \text{if } a_2 \le x \le a_3 \\ \dfrac{(a_4 - x)\omega_a}{a_4 - a_3} & \text{if } a_3 \le x \le a_4 \\ 0 & \text{otherwise} \end{cases}$$

$$\upsilon_a(x) = \begin{cases} \dfrac{a_2 - x + \sigma_a(x - \bar{a}_1)}{a_2 - \bar{a}_1} & \text{if } \bar{a}_1 \le x \le a_2 \\ \sigma_a & \text{if } a_2 \le x \le a_3 \\ \dfrac{x - a_3 + \sigma_a(\bar{a}_4 - x)}{a_3 - \bar{a}_4} & \text{if } a_3 \le x \le \bar{a}_4 \\ 1 & \text{otherwise} \end{cases}$$

Where $\omega_a$ and $\sigma_a$ represent the maximum degree of membership and minimum degree of non-membership respectively, satisfying $0 \le \omega_a \le 1$, $0 \le \sigma_a \le 1$, $0 \le \omega_a + \sigma_a \le 1$.

**Observation:** GTRIFN defined in definition 5 is different from the TRIFNs considered in (De and Das 2012), since in (De and Das, 2012) $\bar{a}_1 = a_1$ and $\bar{a}_4 = a_4$ but in definition 5, $\bar{a}_1$ and $\bar{a}_4$ may not necessarily be equal to $a_1$ and $a_4$ respectively. Also, in Wan (2013); Wu and Cao (2013); Shen. et. al (2011), $\upsilon_a(x) = 0$ for $x < \bar{a}_1$ and $x > \bar{a}_4$ but in definition 5, $\upsilon_a(x) = 1$ for $x < \bar{a}_1$ and $x > \bar{a}_4$. Graphical representation of GTRIFN is illustrated in Figure 1.

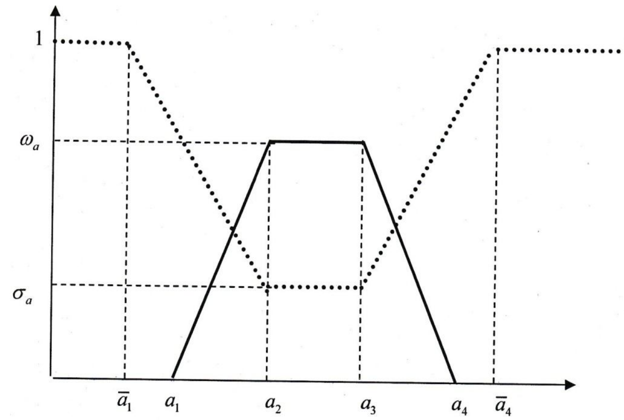


**Fig1: Generalized Trapezoidal Intuitionistic Fuzzy Number (GTRIFN)**

## Arithmetic Operations

In a similar way to the arithmetic operations of TRIFNs (De and Das, 2012) and triangular IFNs (Li, 2008), arithmetic operations over GTRIFNs are defined as follows.

Let $\tilde{a}^I = < (a_1, a_2, a_3, a_4; \omega_a)(\bar{a}_1, a_2, a_3, \bar{a}_4; \sigma_a) >$ and $b^I = < (b_1, b_2, b_3, b_4; \omega_b)(b_1, b_2, b_3, b_4; \sigma_b) >$ be two GTRIFNs, then

1. $\tilde{a}^I + b^I = < (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; \min(\omega_a , \omega_b)) (\bar{a}_1 + b_1, a_2 + b_2, a_3 + b_3, \bar{a}_4 + b_4; \max(\sigma_a , \sigma_b)) >$
2. $\tilde{a}^I - b^I = < (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4; \min(\omega_a , \omega_b)) (\bar{a}_1 - b_1, a_2 - b_2, a_3 - b_3, \bar{a}_4 - b_4; \max(\sigma_a , \sigma_b)) >$
3. $\lambda\tilde{a}^I = < (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4; \lambda\omega_a)( \lambda\bar{a}_1, \lambda a_2, \lambda a_3, \lambda\bar{a}_4; \lambda\sigma_a) >$ if $\lambda > 0$.
4. $\lambda\tilde{a}^I = < (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1; \lambda\omega_a)( \lambda\bar{a}_4, \lambda a_3, \lambda a_2, \lambda\bar{a}_1; \lambda\sigma_a) >$ if $\lambda < 0$.

## Ranking Index of GTRIFN

In literature there are various algorithms for ranking IFNs, but most of the algorithms are used to rank triangular IFNs or TRIFNs with $\bar{a}_1 = a_1$ and $\bar{a}_4 = a_4$ (De and Das, 2012; Das and Duha, 2013). So in order to rank GTRIFN, firstly we define a new single function $\rho_a$ involving both membership and non-membership function of GTRIFN $\tilde{a}^I$ as follows:

Define $\rho_a : R \rightarrow [\,0, \omega_a]$ such that

$$\rho_a = \frac{(\mu_a(x) - \upsilon_a(x) + 1)\,\omega_a}{\omega_a - \sigma_a + 1} \qquad \forall\, x \in R$$

Here, $\mu_a(x)$ and $\upsilon_a(x)$ are membership and non-membership of GTRIFN $\tilde{a}^I$.
Lemma: $\rho a = <(x, \rho a(x)): x\in R>$ is trapezoidal non-normal fuzzy number.
Proof: Let $x \in R$ be arbitrary. Then,

$$
\rho_a(x) = \begin{cases}
0 & \text{if } x \leq \bar{a}_1,\ x \geq \bar{a}_4 \\
\dfrac{\omega_a}{\omega_a - \sigma_a + 1}\left\{\dfrac{-a_2 + x - \sigma_a(x - \bar{a}_1)}{a_2 - \bar{a}_1} + 1\right\} & \text{if } \bar{a}_1 \leq x \leq a_1 \\
\dfrac{\omega_a}{\omega_a - \sigma_a + 1}\left\{\dfrac{x - a_1}{a_2 - a_1}\ \omega_a - \dfrac{a_2 - x + \sigma_a(x - \bar{a}_1)}{a_2 - \bar{a}_1} + 1\right\} & \text{if } a_1 \leq x \leq a_2 \\
\omega_a & \text{if } a_2 \leq x \leq a_3 \\
\dfrac{\omega_a}{\omega_a - \sigma_a + 1}\left\{\dfrac{(a_4 - x)\omega_a}{a_4 - a_3} - \dfrac{x - a_3 + \sigma_a(\bar{a}_4 - x)}{a_3 - \bar{a}_4} + 1\right\} & \text{if } x_3 \leq x \leq a_4 \\
\dfrac{\omega_a}{\omega_a - \sigma_a + 1}\left\{\dfrac{-x + a_3 + \sigma_a(\bar{a}_4 - x)}{\bar{a}_4 - a_3} + 1\right\} & \text{if } a_4 \leq x \leq \bar{a}_4
\end{cases}
$$

Therefore, $\rho_a(x)$ can be written as

$$
\rho_a(x) = \begin{cases}
q(x) & \text{if } \bar{a}1 \leq x \leq a2 \\
\omega a & \text{if } a_2 \leq x \leq a_3 \\
r(x) & \text{if } a_3 \leq x \leq \bar{a}_4 \\
0 & \text{otherwise}
\end{cases}
$$

Where q(x) is defined as q(x): $[\bar{a}_1, a_2] \rightarrow [0, \omega_a]$ such that

$$
r(x) = \begin{cases}
\dfrac{\omega_a}{\omega_a - \sigma_a + 1}\left\{\dfrac{(a_4 - x)\omega_a}{a_4 - a_3} - \dfrac{x - a_3 + \sigma_a(\bar{a}_4 - x)}{a_3 - \bar{a}_4} + 1\right\} & \text{if } x_3 \leq x \leq a_4 \\
\dfrac{\omega_a}{\omega_a - \sigma_a + 1}\left\{\dfrac{-x + a_3 + \sigma_a(\bar{a}_4 - x)}{\bar{a}_4 - a_3} + 1\right\} & \text{if } a_4 \leq x \leq \bar{a}_4
\end{cases}
$$

Here, q(x) is continuous and monotonically increasing function and r(x) is continuous and monotonically decreasing function. Also range of $\rho_a(x)$ lies in $[0, \omega_a]$.

Therefore, $\rho_a = < (x, \rho_a(x); x \in R>$ is non-normal trapezoidal fuzzy number.

To rank GTRIFNs, firstly we will find the centroid of fuzzy number $\rho_a$. Functions q(x) and r(x) defined in the lemma are both strictly monotone.

Let $q^{-1}(y)$: $[0, \omega_a] \rightarrow [\bar{a}_1, a_2]$ and $r^{-1}(y)$: $[0, \omega_a] \rightarrow [a_3, \bar{a}_4]$ be the inverse functions of q(x) and r(x) respectively. Then,

$$q^{-1}(y) = \begin{cases} \dfrac{y(a_2-\bar{a}_1)(\omega_a-\sigma_a+1)+\bar{a}_1(1-\sigma_a)\omega_a}{(1-\sigma_a)\omega_a} & \text{if } 0 \le y \le t \\ \dfrac{y(\omega_a-\sigma_a+1)(a_2-a_1)(a_2-\bar{a}_1)-\omega_a(a_4\bar{a}_4\omega_a-a_4a_3\omega_a-a_4\bar{a}_4\sigma_a+a_3\bar{a}_4\sigma_a+a_4\bar{a}_4-a_3\bar{a}_4}{(-\bar{a}_4\omega_a+a_3\omega_a-a_4+a_3+a_4\sigma_a-a_3\sigma_a)\omega_a} & \text{if } t \le y \le \omega_a \end{cases}$$

$$\text{where } t = \frac{(a_1-\bar{a}_1)(1-\sigma_a)\omega_a}{(\omega_a-\sigma_a+1)(a_2-\bar{a}_1)} \qquad \text{and}$$

$$r^{-1}(y) = \begin{cases} \dfrac{y(\bar{a}_4-a_3)(\omega_a-\sigma_a+1)-\bar{a}_4\omega_a(1-\sigma_a)}{(1-\sigma_a)\omega_a} & \text{if } 0 \le y \le s \\ \dfrac{y(a_4-a_3)(\bar{a}_4-a_3)(\omega_a-\sigma_a+1)-\omega_a(a_4\bar{a}_4\omega_a-a_4a_3\omega_a-a_4\bar{a}_4\sigma_a+a_3\bar{a}_4\sigma_a+a_4\bar{a}_4-a_3\bar{a}_4}{(-\bar{a}_4\omega_a+a_3\omega_a-a_4+a_3+a_4\sigma_a-a_3\sigma_a)\omega_a} & \text{if } s \le y \le \omega_a \end{cases}$$

$$\text{where } s = \frac{y(a_2-\bar{a}_1)(\omega_a-\sigma_a+1)+\bar{a}_1(1-\sigma_a)\omega_a}{(1-\sigma_a)\omega_a}$$

Since $\rho_a$ is non-normal trapezoidal fuzzy number, so centroid point $(x_0, y_0)$ of a fuzzy number ρa (based on formula of Wang .et .al, 2006) is given by

$$x_0(\rho_a) = \int_{-\infty}^{\infty} x\rho_a(x)dx \Big/ \int_{-\infty}^{\infty} \rho_a(x)dx$$

$$= \int_{\bar{a}_1}^{a_2} xq(x)dx + \int_{a_2}^{a_3} x\omega a dx + \int_{a_3}^{\bar{a}_4} xr(x)dx \Big/ \int_{\bar{a}_1}^{a_2} q(x)\,dx + \int_{a_2}^{a_3} \omega a dx + \int_{a_3}^{\bar{a}_4} r(x)dx$$

$$= \frac{(1-\sigma_a)(-\bar{a}^2{}_1-a^2{}_2-\bar{a}_1a_2+a^2{}_3+a_3\bar{a}_4+\bar{a}^2{}_4)+\omega_a(-a^2{}_1-a^2{}_2-a_1a_2+a^2{}_3+a_3\bar{a}_4+\bar{a}^2{}_4}{3\{(1-\sigma a)(-\bar{a}_1-a_2+a_3+\bar{a}_4)+\omega_a(-\bar{a}_1-a_2+a_3+\bar{a}_4)\}} \qquad (1)$$

$$y_0(\rho_a) = \int_0^{\omega_a} y(r^{-I}(y) - q^{-I}(y))dy \,/ \int_0^{\omega_a} (r^{-I}(y)-q^{-I}(y))dy \qquad (2)$$

**Remark 1:** If $\mu_a(x) = 1-\upsilon_a(x)$, then $\bar{a}_1=a_1$, $\bar{a}_4=a_4$, $\omega_a=1-\sigma_a$

Also, $\rho_a = <x\ \mu_a(x) : x\in R) >$. Thus, $\rho_a$ reduces to a non-normal trapezoidal fuzzy number with membership function $\mu_a(x)$ (as defined in definition 5). By substituting the values in the above centroid formula, we get

$$x_0(\rho_a) = 1/3\ [\ a_1 + a_2 + a_3 + a_4 - \frac{a_4a_3 - a_1a_2}{(a_4+a_3)-(a_1+a_2)}\ ]$$

$$y_0(\rho_a) = \omega_a/3\ [\ 1 + \frac{a_3 - a_2}{(a_4+a_3)-(a_1+a_2)}\ ],$$

which is exactly the same centroid formula of a trapezoidal non-normal fuzzy number with membership function μa(x), as derived by Wang.et.al (2006).

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