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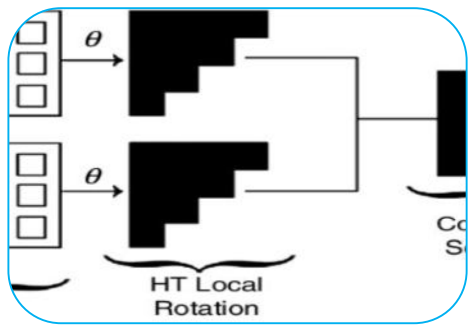
AN APPROACH TO MULTIDIMENSIONAL HERMITE TRANSFORM

S. S. Chauhan
Department Of Mathematics D.V. College, Orai (Jalaun), U.P., India

ABSTRACT

In the present paper making an appeal to generalized multidimensional Lagurre Transform and New multidimensional Integral Transform by Chandel and Chauhan [7, 8] and theory of generalized multiple hypergeometric functions of several variable due to Lauricalla [15], Srivastava-Daoust [17], Exton ([12],[13]),Chandel [2],Chandel, Gupta ([3],[22]), Karlsson [14], Chandel-Vishwakarma ([14],[15]), including multivariable H-function of Srivastava-Panda ([18],[19]), we introduce multidimensional Hermit Transform and we find results involving generalized multiple hypergeometric functions of Srivastava and Daoust with Hermit Transform. Also we find the results for their special cases.



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INTRODUCTION

Chandel and Chauhan |7| introduced multidimensional Laguerre transforms defined by

$$(1.1)\quad L_{\gamma_1,\ldots,\gamma_n}^{(\alpha,\beta,\gamma)}\{\ \} = \frac{(-1)^n m!\,\Gamma(\beta-\alpha-m+\gamma_1+\cdots+\gamma_n)\Gamma(\gamma_1+\cdots+\gamma_n)}{\Gamma(\beta-\alpha+\gamma_1+\cdots+\gamma_n)\Gamma(\beta+\gamma_1+\cdots+\gamma_n)\Gamma(\gamma_1)\ldots\Gamma(\gamma_n)}$$

$$\int_0^\infty \ldots \int_0^\infty e^{-(x_1+\cdots+x_n)}(x_1+\cdots+x_n)^\beta\, x_1^{\gamma_1-1}\ldots . x_n^{\gamma_n-1} L_m^{(\alpha)}(x_1+\cdots+x_n)\{\ \}dx_1 \ldots . dx_m$$

where $\mathrm{Re}(\beta-\alpha-m+\gamma_1+\cdots+\gamma_n)>0$; m, n are arbitrary positive integers; $\mathrm{Re}(\gamma_j)>0$, $j=1,\ldots,n$.

Chandel and Chauhan [7] also introduced another generalized multidimensional Laguerre defined as

$$(1.2)\quad L^{(\beta_1,\ldots,\beta_n;n_1,\ldots n_n;K)}_{\gamma_1,\ldots,\gamma_n\gamma_1,\ldots,\gamma_n}\{\ \} = K\prod_{j=1}^{n}\frac{(-1)^n n_j!\,\Gamma(\gamma_j-\beta_j-\eta_j)}{\Gamma(\gamma_j-\beta_j)\Gamma(\gamma_j)}\int_0^\infty \ldots \int_0^\infty e^{-\sum_{i=1}^n(\alpha_1^i x_1+\cdots+\alpha_n^i x_n)}$$

$$\prod_{j=1}^{n}\left(\alpha_1^j x_1+\cdots+\alpha_n^j x_n\right)^{\gamma_j-1} L_{n_i}^{(\beta_j)}(\alpha_1^j x_1+\cdots+\alpha_n^j x_n)^{\gamma_j-1}\{\ \}dx_1\ldots dx_n$$

where Re(γ_j)>0; Re ($\gamma_j-\beta_j-\eta_j$)>0, $\eta_j\in N; j\ \ 1\ldots.m$

and

$$(1.3)\qquad K=\begin{vmatrix}\alpha_1^1 & \alpha_2^1 & \ldots & \alpha_n^1\\ \alpha_1^2 & \alpha_2^2 & \ldots & \alpha_n^2\\ \ldots & \ldots & \ldots & \ldots\\ \alpha_1^n & \alpha_2^n & \ldots & \alpha_n^n\end{vmatrix}\neq 0.$$

They presented their applications to the theory of generalized multiple hypergeometric functions of several variables due to Lauricella [15], Srivastava-Daoust [17], including multivariable *H*-function of srivastava and Panda ([18],[19]; also see [20].p.251).

Further Chandel and Chauhan [8] introduced a new multidimensional integral transform defined by

$$(1.4)\quad R_{\alpha_1,\ldots\alpha_n}^{(a,b)}\{\ \}=\frac{\Gamma(\alpha_1+\cdots+\alpha_n)\Gamma(1/2+a-b+\alpha_1+\cdots+\alpha_n)}{\Gamma(\alpha_1)\ldots\Gamma(\alpha_n)\Gamma(2a+2\alpha_1+\cdots+\alpha_n)}$$

$$\frac{2^{2a+2\alpha_1+\cdots+2\alpha_n-1}}{\Gamma\left(\frac{1}{2}-(\alpha+b+\alpha_1+\cdots+\alpha_n)\right)}\int_0^\infty\ldots\int_0^\infty(x_1+\cdots+x_n)^a(1+x_1+\cdots+x_n)^{-1/2}$$

$$[(x_1+\cdots+x_n)^{1/2}+(1+x_1+\cdots+x_n)^{1/2}]^{2b}x_1^{\alpha_1-1}\ldots x_n^{\alpha_n-1}\{\ \}dx_1\ldots dx_n.$$

where 0<Re(a+$\alpha_1+\cdots+\alpha_n$)<1/2 – Re(b), Re(α_1)>0, *i=1…….n* ;

and presented its certain interesting applications to the theory of generalized multiple hypergeometric function of several variables due to Lauricella [15], Srivastava-Daoust [17], Exton([12],[13]), Chandel [2], Chandel-Gupta [3], Gupta and Chandel [22], Karlsson [14], Chandel-Vishwakarma ([4],[5],[6]) including multivariable *H*-function of Srivastava-Panda ([18],[19]).

We also see Chandel and Kumar [9] and studied some multidimensional integral transforms involving Srivastava and Panda's *H*-function of several variables ([18],[19]).

We also see Chandel and Kumar [10], we introduced and studied generalized multidimensional Laguerre transforms.

Motivated by above work, we introduce multidimensional Hermite transform defined by

$$(1.5)\quad H_{\alpha_1,\ldots,\alpha_n}^{\rho,v}\{\ \}=\frac{\Gamma(\alpha_1+\cdots+\alpha_n)\Gamma(\rho-v+1+\frac{\alpha_1+\cdots+\alpha_n}{2})}{\Gamma(\alpha_1)\ldots\Gamma(\alpha_n)\sqrt{\pi}2^{2v-2\rho+\alpha_1+\cdots+\alpha_n}}$$

$$\frac{1}{\Gamma(2\rho+1+\alpha_1+\cdots+\alpha_n)}\int_{-\infty}^\infty\ldots\int_{-\infty}^\infty(z_1+\cdots+z_n)^{2\rho+1}e^{-(z_1+\cdots+z_n)^2}$$

$$z_1^{\alpha_1-1}\ldots z_n^{\alpha_n-1}H_{2v}(z_1+\cdots+z_n)\{\ \}dz_1\ldots dz_n$$

where ρ = 0, 1, 2,…; Re$\left(\rho-v+1+\frac{\alpha_1+\cdots+\alpha_n}{2}\right)$ >0 and $\alpha_1+\cdots+\alpha_n$ is an even positive integer and present its certain interesting applications to the theory of generalized multiple hypergeometric functions of several variables due to Lauricella [7] and Srivastava-Daoust ([17]; also see [21] p.64) including multivariable *H*-function of Srivastava and Panda ([18],[19]; also see [20], p.251). We also discuss their interesting special cases.

2. A Multidimensional Integral. Chandel [1] gave a generalization of the result ([11], p.117)

(2.1) $$\int_0^\infty \int_0^\infty f(x+y)x^{\alpha-1}y^{\beta-1}dxdy = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \int_0^\infty f(t)t^{\alpha+\beta-1}dt$$

where Re(α) >0, Re(β) >0, in the form

(2.2) $$\int_0^\infty \ldots \int_0^\infty f(x_1+\cdots+x_n)x_1^{\alpha_1-1}\ldots x_n^{\alpha_n-1}dx_1\ldots dx_n$$

$$=\frac{\prod_{i=1}^n \Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^n(\alpha_i))} \int_0^\infty f(t)t^{\alpha_1+\cdots+\alpha_n-1}dt,$$

where $\mathrm{Re}(\alpha_j) > 0, j = 1,\ldots,n$.

To introduce multidimensional Hermite transform, here in this section, we prove following extension of (2.2).

(2.3) $$\int_{-\infty}^\infty \ldots \int_{-\infty}^\infty f(x_1+\cdots+x_n)x_1^{\alpha_1-1}\ldots x_n^{\alpha_n-1}dx_1\ldots dx_n$$

$$= \frac{\prod_{i=1}^n \Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^n(\alpha_i))} \int_{-\infty}^\infty f(t)t^{\alpha_1+\cdots+\alpha_n-1}dt,$$

where $\mathrm{Re}(\alpha_j) > 0, j = 1,\ldots,n$.

Proof. It is just the Lioville's extension of Dirichlet's theorem. By Disichlet integral

$$I' = \int \ldots \int x_1^{\alpha_1-1}\ldots x_n^{\alpha_n-1}dx_1\ldots dx_n = \frac{\prod_{i=1}^n \Gamma(\alpha_i)h^{\alpha_1+\cdots+\alpha_n}}{\Gamma(1+\sum_{i=1}^n(\alpha_i))},$$

where $x_1+\cdots+x_n \le h$.

Thus if $x_1+\cdots+x_n \le h+\delta h$,

$$I' = \frac{\prod_{i=1}^n \Gamma(\alpha_i)(h+\delta h)^{\alpha_1+\cdots+\alpha_n}}{\Gamma(1+\sum_{i=1}^n(\alpha_i))}.$$

Thus if

$$h \le x_1+\cdots+x_n \le h+\delta h,$$

the value of the integral

$$= \frac{\sum_{i=1}^n \Gamma(\alpha_i)}{\Gamma(1+\sum_{i=1}^n(\alpha_i))} \{(h+\delta h)^{\alpha_1+\cdots+\alpha_n} - h^{\alpha_1+\cdots+\alpha_n}\}$$

$$= \frac{\prod_{i=1}^n \Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^n(\alpha_i))} h^{\alpha_1+\cdots+\alpha_n-1}\delta h \qquad \text{(to the first order of approximation).}$$

Again taking $x_1+\cdots+x_n = h{+}\in$, we have

$$f(x_1+\cdots+x_n) = f(h{+}\in) = f(h){+}\in f'(h)+\cdots$$

That is

$$\delta h\, f(h{+}\in) = \delta h\, f(h) \qquad \text{(to the first order of approximation).}$$

Hence the value of $f(x_1+\cdots+x_n)$ can be taken to be $f(h)$ through out.

Thus in small interval δh, the integral

$$I = \int \ldots \int f(x_1+\cdots+x_n) x_1^{\alpha_1-1} \ldots x_n^{\alpha_n-1} dx_1 \ldots dx_n$$

$$= \frac{\prod_{i=1}^{n}\Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^{n}(\alpha_i))} f(h) h^{\alpha_1+\cdots+\alpha_n-1}\delta h$$

hence for $h_1 \le x_1+\cdots+x_n \le h_2$

$$I = \frac{\prod_{i=1}^{n}\Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^{n}(\alpha_i))} \int_{h_1}^{h_2} f(h) h^{\alpha_1+\cdots+\alpha_n-1} dh.$$

If $h_1 \to -\infty$ and $h_2 \to \infty$, we obtain (2.3).

3. Multidimensional Hermite Transform. Making an appeal to the result (2.3),we can write

(3.1) $$\int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} (z_1+\cdots+z_n)^{2\rho+1} e^{-(z_1+\cdots+z_n)^2} z_1^{\alpha_n-1} \ldots z_n^{\alpha_n-1}$$

$$H_{2v}(z_1+\cdots+z_n) dz_1 \ldots dz_n$$

$$= \frac{\prod_{i=1}^{n}\Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^{n}(\alpha_i))} \int_{-\infty}^{\infty} t^{2\rho+\alpha_1+\cdots+\alpha_n} e^{-t^2} H_{2v}(t) dt\,,$$

where $H_v(z)$ are Hermite polynomials

Now applying orthogonal property of Hermite polynomials due to Rainville [16]

(3.2) $$\int_{-\infty}^{\infty} z^{2\rho} e^{-z^2} H_{2v}(z) dz = \frac{\sqrt{\pi}\, 2^{2(v-\rho)}\Gamma(2\rho+1)}{(\rho-v+1)}\,, \qquad \rho = 0,1,2,\ldots$$

we get

(3.3) $$\int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} (z_1+\cdots+z_n)^{2\rho+1} e^{-(z_1+\cdots+z_n)^2} z_1^{\alpha_n-1} \ldots z_n^{\alpha_n-1}$$

$$H_{2v}(z_1+\cdots+z_n) dz_1 \ldots dz_n$$

$$= \frac{\prod_{i=1}^{n}\Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^{n}\alpha_i)} \frac{2^{2v-2\rho+\alpha_1+\cdots+\alpha_n}\Gamma(2\rho+1+\alpha_1+\cdots+\alpha_n)}{\Gamma\left(\rho-v+1+\frac{\alpha_1+\cdots+\alpha_n}{2}\right)}\,,$$

where $\rho = 0,1,2,\ldots$; $\mathrm{Re}\left(\rho - v + 1 + \frac{\alpha_1+\cdots+\alpha_n}{2}\right) > 0$; $\alpha_1+\cdots+\alpha_n$ is an even positive integer, which for brevity suggests to introduce multidimensional Hermite operator $H^{\rho,v}_{\alpha_1+\cdots+\alpha_n}\{\ \}$ defined in (1.5) to derive multidimensional Hermite transforms involving different multiple hypergeometric functions of several variables.

It is quite clear that

(3.4) $H^{\rho,v}_{\alpha_1,\ldots,\alpha_n}\{1\}=1,$

(3.5) $H^{\rho,v}_{\alpha_1,\ldots,\alpha_n}\{z_1^{m_1}\ldots z_n^{m_n}(z_1+\cdots+z_n)^{m_1+\cdots+m_n}\}$

$$=\frac{(\alpha_1)_{m_1}\ldots(\alpha_n)_{m_n}(2\rho+\alpha_1+\cdots+\alpha_n+1)_{2(m_1+\cdots+m_n)}}{(\alpha_1+\cdots+\alpha_n)_{m_1+\cdots+m_n}\left(\rho-v+1+\frac{\alpha_1+\cdots+\alpha_n}{2}\right)_{m_1+\cdots+m_n}}$$

and

(3.6) $H^{\rho,v}_{\alpha_1,\ldots,\alpha_n}\{z_1^{\sigma_1 m_1}\ldots z_n^{\sigma_n m_n}(z_1+\cdots+z_n)^{\eta_1 m_1+\cdots+\eta_n m_n}\}$

$$=\frac{2^{(\sigma_1-\eta_1)m_1+\cdots+(\sigma_n-\eta_n)m_n}(\alpha_1)_{\sigma_1 m_1}\cdots(\alpha_n)_{\sigma_n m_n}}{(\alpha_1+\cdots+\alpha_n)_{\sigma_1 m_1+\cdots+\sigma_n m_n}}$$

$$\frac{(2\rho+\alpha_1+\cdots+\alpha_n+1)_{(\sigma_1+\eta_1)m_1+\cdots+(\sigma_n+\eta_n)m_n}}{\left(\rho-v+1+\frac{\alpha_1+\cdots+\alpha_n}{2}\right)_{((\eta_1+\sigma_1)/2)m_1+\cdots+((\eta_n+\sigma_n)/2)m_n}}$$

provide that $\rho=0,1,2,\ldots$; $\mathrm{Re}\left(\rho-v+1+\frac{\alpha_1+\cdots+\alpha_n}{2}\right)>0$ and $\alpha_1+\cdots+\alpha_n$ is an even positive integer.

4. Result Involving Most Generalized Multiple Hypergeometric Function of Srivastava and Daoust. In this section, making an appeal to (3.6), we derive

(4.1) $$H^{\rho,v}_{\alpha_1,\ldots,\alpha_n}\left\{F^{A:B';\ldots;B^{(n)}}_{C:D';\ldots;D^{(n)}}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}];\\ [(c):\Psi',\ldots,\Psi^{(n)}]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\ y_1 z_1^{\sigma_1}(z_1+\ldots+z_n)^{\eta_1},\ldots,y_n z_n^{\sigma_n}(z_1+z_n)^{\eta_n}\right)\right\}$$

$$=F^{A+1:B'+1;\ldots;B^{(n)}+1}_{C+2:D';\ldots;D^{(n)}}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}],\\ [(c):\Psi',\ldots,\Psi^{(n)}],[\alpha_1+\cdots+\alpha_n:\sigma_1,\ldots,\sigma_n],\end{matrix}\right.$$

$$\begin{matrix}[1+2\rho+\alpha_1+\cdots+\alpha_n:\sigma_1+\eta_1,\ldots,\sigma_n+\eta_n]:[(b'):\phi'],[\alpha_1,\sigma_1]\\ \left[\rho-v+1+\dfrac{\alpha_1+\cdots+\alpha_n}{2}:\dfrac{\eta_1+\sigma_1}{2}\ldots\dfrac{\eta_n+\sigma_n}{2}\right]:[(d'):\delta']\end{matrix}$$

$$\left.\begin{matrix},\ldots,[(b^{(n)}):\phi^{(n)}],[\alpha_n:\sigma_n];\\ ;\ldots;[(d^{(n)}):\delta^{(n)}];\end{matrix}\ 2^{\sigma_1-\eta_1}y_1,\ldots,2^{\sigma_n-\eta_n}y_n\right)$$

provided that $\rho=0,1,2,\ldots$; $\mathrm{Re}\left(\rho-v+1+\frac{\alpha_1+\cdots+\alpha_n}{2}\right)>0$ *and* $\alpha_1+\cdots+\alpha_n$ is an even positive integer and

$$1-\left(\frac{\sigma_1+\eta_1}{2}\right)+\sum_{j=1}^{c}\Psi_j^{(i)}+\sum_{j=1}^{D^{(i)}}\delta_j^{(i)}-\sum_{j=1}^{A}\delta_j^{(i)}-\sum_{j=1}^{B'}\phi_j^{(i)}>0.$$

5. Result Involving Other Multiple Hypergeometric Functions.

Making an appeal to (3.5), we derive

(5.1) $H^{\rho,v}_{\alpha_1,\ldots,\alpha_n}\{{}_1F_1\left[\rho-v+1+\frac{\alpha_1+\cdots+\alpha_n}{2};\ \rho+\frac{\alpha_1+\cdots+\alpha_n+1}{2};(u_1z_1+\cdots+u_nz_n)(z_1+\ldots+z_n)\right]\}$

$$=F_D^{(n)}\left(\rho+1+\frac{\alpha_1+\cdots+\alpha_n}{2},\alpha_1,\ldots,\alpha_n;\alpha_1+\cdots+\alpha_n;4u_1,\ldots,4u_n\right)$$

where $\rho=0,1,2,\ldots;$ $\mathrm{Re}\left(\rho-v+1+\frac{\alpha_1+\cdots+\alpha_n}{2}\right)>0$ *and* $\alpha_1+\cdots+\alpha_n$ is an even positive integers; $|u_i|<\frac{1}{4}, i=1,\ldots,n$ and $F_D^{(n)}$ is Lauricella's multiple hypergeometric function [15].

(5.2) $H^{\rho,v}_{\alpha_1,\ldots,\alpha_n}\{{}_1F_1\left[\rho-v+1+\frac{\alpha_1+\cdots+\alpha_n}{2};\ \rho+1+\frac{\alpha_1+\cdots+\alpha_n}{2};(u_1z_1+\cdots+u_nz_n)(z_1+\ldots+z_n)\right]\}$

$$=F_D^{(n)}\left(\rho+\frac{\alpha_1+\cdots+\alpha_n+1}{2},\alpha_1,\ldots,\alpha_n;\alpha_1+\cdots+\alpha_n;4u_1,\ldots,4u_n\right)$$

valid if all conditions of (5.1) are satisfied

(5.3) $H^{\rho,v}_{\alpha_1,\ldots,\alpha_n}\{{}_1F_1\left(\alpha_1+\cdots+\alpha_n;\ \rho+\frac{\alpha_1+\cdots+\alpha_n+1}{2};(u_1z_1+\cdots+u_nz_n)(z_1+\cdots+z_n)\right)\}$

$$=F_D^{(n)}\left(\rho+1+\frac{\alpha_1+\cdots+\alpha_n}{2},\alpha_1,\ldots,\alpha_n;\rho-v+1+\frac{\alpha_1+\cdots+\alpha_n}{2};4u_1,\ldots,4u_n\right)$$

provided that all conditions of (5.1) hold true.

(5.4) $H^{\rho,v}_{\alpha_1,\ldots,\alpha_n}\{{}_1F_1\left(\alpha_1+\cdots+\alpha_n;\ \rho+1+\frac{\alpha_1+\cdots+\alpha_n}{2};(u_1z_1+\cdots+u_nz_n)(z_1+\cdots+z_n)\right)\}$

$$=F_D^{(n)}\left(\rho+\frac{\alpha_1+\cdots+\alpha_n+1}{2},\alpha_1,\ldots,\alpha_n;\rho-v+1+\frac{\alpha_1+\cdots+\alpha_n}{2};4u_1,\ldots,4u_n\right),$$

where all conditions of (5.1) are satisfied.

(5.5) $H^{\rho,v}_{\alpha_1,\ldots,\alpha_n}\{{}_1F_2\left(\alpha_1+\cdots+\alpha_n;\ \rho+\frac{\alpha_1+\cdots+\alpha_n+1}{2},\rho+1+\frac{\alpha_1+\cdots+\alpha_n}{2};(u_1z_1+\cdots+u_nz_n)(z_1+\cdots+z_n)\right)\}$

$$=\phi_2^{(n)}\left(\alpha_1,\ldots,\alpha_n;\rho-v+1+\frac{\alpha_1+\cdots+\alpha_n}{2};4u_1,\ldots,4u_n\right),$$

provided that $\rho=0,1,2,\ldots;$ $\mathrm{Re}\left(\rho-v+1+\frac{1}{2}(\alpha_1+\cdots+\alpha_n)\right)>0,\alpha_1+\cdots+\alpha_n$ is an even positive integers.

(5.6) $H^{\rho,v}_{\alpha_1,\dots,\alpha_n}\{ {}_1F_2 \left(\rho - v + 1 + \frac{\alpha_1+\cdots+\alpha_n}{2};\ \rho + \frac{\alpha_1+\cdots+\alpha_n+1}{2}; \rho + 1 + \frac{\alpha_1+\cdots+\alpha_n}{2}; (u_1z_1 + \dots + u_nz_n)(z_1+\cdots+z_n)\right)\}$

$$= \phi_2^{(n)}(\alpha_1,\dots,\alpha_n; \alpha_1+\cdots+\alpha_n; 4u_1,\dots,4u_n),$$

where all conditions of (5.5) are satisfied.

(5.7) $H^{\rho,v}_{\alpha_1,\dots,\alpha_n}\{ {}_2F_2 \left(\alpha_1+\cdots+\alpha_n;\ \rho - v + 1 + \frac{\alpha_1+\cdots+\alpha_n}{2}; \rho + \frac{\alpha_1+\cdots+\alpha_n+1}{2};\ \rho + 1 + \frac{\alpha_1+\cdots+\alpha_n}{2}; (u_1z_1 + \cdots + u_nz_n)(z_1+\cdots+z_n)\right)\}$

$$= (1-4u_1)^{-\alpha_1} \dots (1-4u_n)^{-\alpha_n}$$

provided that $\rho = 0,1,2,\dots;$ $\mathrm{Re}(\rho - v + 1 + (\alpha_1+\cdots+\alpha_n)/2) > 0, \alpha_1+\cdots+\alpha_n$ is an even positive integers while $|u_1|$<1/4,..., $|u_n|$<1/4.

(5.8)
$H^{\rho,v}_{\alpha_1,\dots,\alpha_n}\{ {}_2F_3 \left(\alpha_1+\cdots+\alpha_n;\ \rho - v + 1 + \frac{\alpha_1+\cdots+\alpha_n}{2}; a,\ \rho + \frac{\alpha_1+\cdots+\alpha_n+1}{2};\ \rho + 1 + \frac{\alpha_1+\cdots+\alpha_n}{2}; (u_1z_1 + \cdots + u_nz_n)(z_1+\cdots+z_n)\right)\}$

$$= \phi_2^{(n)}(\alpha_1,\dots,\alpha_n; a; 4u_1,\dots,4u_n),$$

where $\rho = 0,1,2,\dots;$ $\mathrm{Re}(\rho - v + 1 + (\alpha_1+\cdots+\alpha_n)/2) > 0, \alpha_1+\cdots+\alpha_n$ is an even positive integer and $\phi_2^{(n)}$ is Lauricell'a confluent hypergeometric function [15].

6. Special Case. When $\rho = \frac{\alpha_1+\cdots+\alpha_n}{2} - 1.$

It is clear

(6.1) $H^{\left(\frac{\alpha_1+\cdots+\alpha_n}{2}-1,v\right)}_{\alpha_1,\dots,\alpha_n}\{(u_1z_1)^{m_1}\dots(u_nz_n)^{m_n}(z_1+\cdots+z_n)^{m_1+\cdots+m_n}\}$

$$= \frac{\left(\frac{\alpha_1+\cdots+\alpha_n-1}{2}\right)_{m_1+\cdots+m_n}(\alpha_1)_{m_1}\dots(\alpha_n)_{m_n}(4u_1)^{m_1}\dots(4u_n)^{m_n}}{(\alpha_1+\cdots+\alpha_n-v)_{m_1+\cdots+m_n}}.$$

which further suggests that

(6.2) $H^{\left(\frac{\alpha_1+\cdots+\alpha_n}{2}-1,v\right)}_{\alpha_1,\dots,\alpha_n}\{\exp[(u_1z_1+\cdots+u_nz_n)(z_1+\cdots+z_n)]\}$

$$= F_D^{(n)}\left(\frac{\alpha_1+\cdots+\alpha_n-1}{2}, \alpha_1,\dots,\alpha_n; \alpha_1+\cdots+\alpha_n; 4u_1,\dots,4u_n\right)$$

provided that $\alpha_1+\cdots+\alpha_n$is an even positive integer and $|u_i| < \frac{1}{4};$ $i = 1,\dots,n.$

(6.3) $H_{\alpha_1,\ldots,\alpha_n}^{\left(\frac{\alpha_1+\cdots+\alpha_n}{2}-1,v\right)}\left\{\Psi_2^{(n)}(\alpha_1+\cdots+\alpha_n-v;c_1,\ldots,c_n;\ u_1z_1(z_1+\cdots+z_n),\ldots,u_nz_n(z_1+\cdots+z_n))\right\}$

$$=F_A^{(n)}\left(\frac{\alpha_1+\cdots+\alpha_n-1}{2},\alpha_1,\ldots,\alpha_n;c_1,\ldots,c_n;4u_1,\ldots,4u_n\right)$$

where $\alpha_1+\cdots+\alpha_n$ is an every positive integer and $|u_1|+\ldots+|u_n|<1/4$.

(6.4) $H_{\alpha_1,\ldots,\alpha_n}^{\left(\frac{\alpha_1+\cdots+\alpha_n}{2}-1,v\right)}\left\{\Psi_2^{(n)}(a;\alpha_1,\ldots,\alpha_n;u_1z_1(z_1+\cdots+z_n),\ldots,u_nz_n(z_1+\cdots+z_n))\right\}$

$$=\sum_{m_1,\ldots,m_n=0}^{\infty}\frac{(a)_{m_1+\cdots+m_n}\left(\frac{\alpha_1+\cdots+\alpha_n-1}{2}\right)_{m_1+\cdots+m_n}}{(\alpha_1+\cdots+\alpha_n-v)_{m_1+\cdots+m_n}}(4u_1)^{m_1}\ldots(4u_n)^{m_n}$$

$$={}_2F_1\left(a,\frac{\alpha_1+\cdots+\alpha_n-1}{2},\alpha_1+\cdots+\alpha_n-v;4(u_1+\cdots+u_n)\right),$$

provided that $\alpha_1+\cdots+\alpha_n$ is every positive integer and $|u_1+\ldots+u_n|<1/4$.

(6.5) $H_{\alpha_1,\ldots,\alpha_n}^{\left(\frac{\alpha_1+\cdots+\alpha_n}{2}-1,v\right)}\left\{\Psi_2^{(n)}(\alpha_1+\cdots+\alpha_n-v;\alpha_1,\ldots,\alpha_n;u_1z_1(z_1+\cdots+z_n),\ldots,\ u_nz_n(z_1+\cdots+z_n))\right\}$

$$=[1-4(u_1+\cdots+u_n)]^{\frac{1-(\alpha_1+\cdots+\alpha_n)}{2}},$$

valid if $\alpha_1+\cdots+\alpha_n$ is an even positive integer and $|u_1+\ldots+u_n|<1/4$.

(6.6) $H_{\alpha_1,\ldots,\alpha_n}^{\left(\frac{\alpha_1+\cdots+\alpha_n}{2}-1,v\right)}\left\{\phi_2^{(n)}\left(b_1,\ldots,b_n;\frac{\alpha_1+\cdots+\alpha_n-1}{2};u_1z_1(z_1+\cdots+z_n),\ldots,u_nz_n(z_1+\ldots+z_n)\right)\right\}$

$$=F_B^{(n)}(b_1,\ldots,b_n,\alpha_1,\ldots,\alpha_n;\alpha_1+\cdots+\alpha_n-v;4u_1,\ldots,4u_n)$$

where $\alpha_1+\cdots+\alpha_n$ is an even positive integer and $|u_i|<1/4$. $i=1,\ldots,n$.

Here $F_A^{(n)},F_B^{(n)},F_D^{(n)}$ are Lauricella's multiple hypergeometric function [15] and $\phi_2^{(n)},\Psi_2^{(n)}$ are their confluent forms.

REFERENCES

[1] Chandel, R.C.S., the products of certain classical polynomials and the generalized Laplacian operator, *Ganita* **20** (2) (1969), 79-87.

[2] Chandel, R.C.S., On some multiple hypergeometric functions related to Lauricella function, *Jñānābha Sect. A,* **3** (1973), 119-136.

[3] Chandel, R.C.S. and Gupta, A.K., Multiple hypergeometric functions related Lauricella function, *Jñānābha,* **16** (1986), 195-209.

[4] Chandel, R.C.S. and Vishwakarma, P.K., Karlson's multiple hypergeometric function and its confluent forms, *Jñānābha,* **19** (1989),173-185.

[5] Chandel, R.C.S. and Vishwakarma, P.K., Fractional integration and integral representations of Karlsson's multiple hypergeometric function and its confluent form, *Jñānābha,* **20** (1990), 101-110.

[6] Chandel, R.C.S., and Vishwakarma, P.K., Fractional derivatives of confluent hypergeometric forms of Karlsson's multiple hypergeometric functions ${}^{(k)}F_{CD}^{(n)}$, *Pure Appl. Math. Sci.,* **35** (1992), 31-39.

[7] Chandel, R.C.S. and Chauhan, S., Multidimensional Laguerre Transforms, *Jour. Pure Math. Calcutta,* **20** (2003), 59-72.

[8] Chandel, R.C.S. and Chauhan, S., A new multidimensional integral transform, *Jour Pure Math., Calcutta,* **23** (2006), 101-119.

[9] Chandel, R.C.S. and Kumar, K., On some multidimensional integral transform of Srivastava and Panda's *H*-function of several variables, *Jñānābha,* **30** (2000), 125-130.

[10] Chandel, R.C.S. and Kumar, K., Generalized multidimensional Laguerre transform, *Jñānābha,* **35** (2005), 17-27.

[11] Edwards J., *A Treatise on the Integral Calculus, H,* Chelsea, New York, 1954.

[12] Exton, H., On two multiple hypergeometric functions related to Lauricella's $F_D^{(n)}$, *Jñānābha Sect A,* **2** (1972), 59-73.

[13] Exton, H., *Multiple Hypergeometric Function and Applications,* John Wiley and Sons, New York 1976.

[14] Karlsson, P.W., on intermediate Lauricellas functions, *Jñānābha,* **16** (1986), 211-222.

[15] Lauricella, G., Sulle funzioni ipergeometriche a più variabili, *Rend. Circ. Mat. Palermo,* (1893), 111-158.

[16] Rainville, E.D., *Special Functions,* MacMillan Co., New York 1960; Reprinted Chelsea Publ. C. Bornx. New York, 1971.

[17] Srivastava, H.M. and Daoust, M.C., Certain generalized Neumann expanssions associated with the Kampé de Feriet function. *Nederd. Acad. Wetensh. Proc. Ser. A* **72** = Indag Math. **31** (1969), 449-457.

[18] Srivastava, H.M. and Panda, R., Some bilateral generating functions for a class of generalized hypergeometric polynomials, *J. Reine Angew, Math.,* **283/284** (1976), 265-274.

[19] Srivastava, H.M. and Panda, R., Extenssion Theorems, for the *H*-function of several complex variables, *J. Reine Angew Math.,* **288** (1976), 129-145.

[20] Srivastava, H.M., Gupta, K.C. and Goyal, S.P., *The H- functions, of One and Two variables with applications,* South Asian Publications, New Delhi and Madras, 1982.

[21] Srivastava, H.M. and Manocha, H.L., *A Treatire on Generting Functions,* Halsted Press (Ellis Horwood Limited, Chichester) John Wiley and Sons, New York, Chichester, Brisbane and Toranto, 1984.

[22] Vandna Gupta and R.C. Singh Chandel, Applications of Difference operators in transformations of certain multiple hypergeometric functions of several variables, *Jñānābha, Vol.* **45** (2015), 153-164.