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HYPERBOLIC FUNCTIONS

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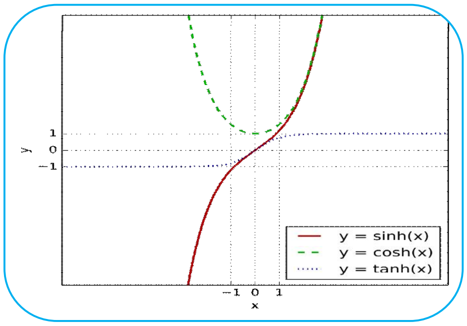
INTRODUCTION: -

The hyperbolic functions in mathematics are comparable to the trigonometric or circular functions. Typically, algebraic formulations using the exponential function are used to define hyperbolic functions (e^x) and its inverse exponential functions (e^{-x}), when the Euler's constant, e, is used. Here, we'll go into great detail about the fundamental hyperbolic functions, their properties, identities, and instances.



HYPERBOLIC FUNCTION DEFINITION

The trigonometric or circular functions have analogues in the hyperbolic functions. The hyperbolic function can be found in Laplace's equations in cartesian coordinates, solutions to linear differential equations, and calculations of distance and angles in hyperbolic geometry. The hyperbolic function, often known as the hyperbolic angle, typically occurs in the real argument. The fundamental hyperbolic operations are:

- Hyperbolic sine (sinh)
- Hyperbolic cosine (cosh)
- Hyperbolic tangent (tanh)

The additional functions, such as the hyperbolic cosecant (cosech), hyperbolic secant (sech), and hyperbolic cotangent (coth) functions, are derived from these three fundamental functions. Let's go into further detail on the fundamental hyperbolic functions, graphs, attributes, and inverse hyperbolic functions.

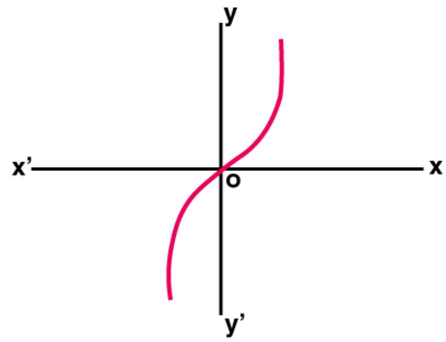
Hyperbolic Functions Formulas

The following lists the fundamental formulas for hyperbolic functions and their graph functions:

Hyperbolic Sine Function

The hyperbolic sine function is a function f: R → R is defined by $f(x) = [e^x - e^{-x}]/2$ and it is denoted by sinh x

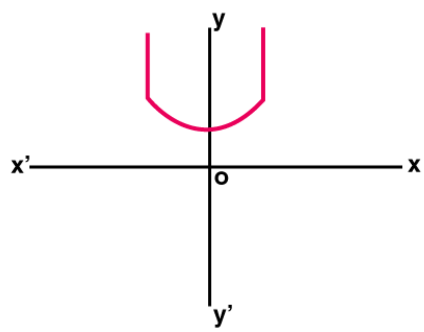
$\text{Sinh } x = [e^x - e^{-x}]/2$
Graph : y = Sinh x



Hyperbolic Cosine Function

The hyperbolic cosine function is a function f: R ⟶ R is defined by $f(x) = [e^x + e^{-x}]/2$ and it is denoted by cosh x

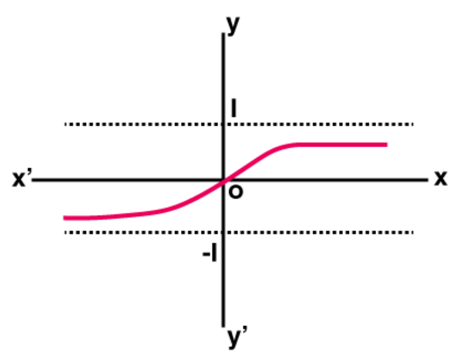
$\cosh x = [e^x + e^{-x}]/2$
Graph : y = cosh x



Hyperbolic Tangent Function

The hyperbolic tangent function is a function f: R ⟶ R is defined by $f(x) = [e^x - e^{-x}] / [e^x + e^{-x}]$ and it is denoted by tanh x

$\tanh x = [e^x - e^{-x}] / [e^x + e^{-x}]$
Graph : y = tanh x



Properties of Hyperbolic Functions

The properties of hyperbolic functions are analogous to the trigonometric functions. Some of them are:

1. Sinh (-x) = -sinh x
2. Cosh (-x) = cosh x
3. Sinh 2x = 2 sinh x cosh x
4. Cosh 2x = $\cosh^2 x + \sinh^2 x$

The derivatives of hyperbolic functions are:

1. d/dx sinh (x) = cosh x
2. d/dx cosh (x) = sinh x

Some relations of hyperbolic function to the trigonometric function are as follows:

1. Sinh x = – i sin(ix)
2. Cosh x = cos (ix)
3. Tanh x = -i tan(ix)

Hyperbolic Function Identities

The hyperbolic function identities are similar to the trigonometric functions. Some identities are:

Pythagorean Trigonometric Identities

- $\cosh^2 (x) - \sinh^2 (x) = 1$
- $\tanh^2 (x) + \text{sech}^2 (x) = 1$
- $\coth^2 (x) - \text{cosech}^2 (x) = 1$

Sum to Product

- sinh x + sinh y = 2 sinh((x+y)/2) cosh((x-y)/2)
- sinh x – sinh y = 2 cosh((x+y)/2) sinh((x-y)/2)
- cosh x + cosh y = 2 cosh((x+y)/2) cosh((x-y)/2)
- cosh x – cosh y = 2 sinh((x+y)/2) sinh((x-y)/2)

Product to Sum

- 2 sinh x cosh y = sinh(x + y) + sinh(x -y)
- 2 cosh x sinh y = sinh(x + y) – sinh(x – y)
- 2 sinh x sinh y = cosh(x + y) – cosh(x – y)
- 2 cosh x cosh y = cosh(x + y) + cosh(x – y).

Sum and Difference Identities

- sinh(x ± y) = sinh x cosh x ± coshx sinh y
- cosh(x ±y) = cosh x cosh y ± sinh x sinh y
- tanh(x ±y) = (tanh x ± tanh y) / (1± tanh x tanh y)
- coth(x ±y) = (coth x coth y ± 1) / (coth y ±coth x)

Inverse Hyperbolic Functions

Inverse hyperbolic functions are the opposite of hyperbolic functions. It also goes by the name of "area hyperbolic function." The hyperbolic angles corresponding to the specified value of the hyperbolic function are provided by the inverse hyperbolic function. The symbol for the operations is $\sinh^{-1}$, $\cosh^{-1}$, $\tanh^{-1}$, csch^{-1}, sech^{-1}, and $\coth^{-1}$. The inverse hyperbolic function in complex plane is defined as follows:

- $\text{Sinh}^{-1} x = \ln(x + \sqrt{[1+x^2]})$
- $\text{Cosh}^{-1} x = \ln(x + \sqrt{[x^2-1]})$
- $\text{Tanh}^{-1} x = (½)[\ln(1+x) - \ln(1-x)$

Hyperbolic Function Example

Example: Solve $\cosh^2 x - \sinh^2 x$

Solution:

Given: $\cosh^2 x - \sinh^2 x$

We know that

Sinh $x = [e^x - e^{-x}]/2$

cosh $x = [e^x + e^{-x}]/2$

$\cosh^2 x - \sinh^2 x = [\,[e^x + e^{-x}]/2\,]^2 - [\,[e^x - e^{-x}]/2\,]^2$

$\cosh^2 x - \sinh^2 x = (4e^{x-x})\,/4$

$\cosh^2 x - \sinh^2 x = (4e^0)\,/4$

$\cosh^2 x - \sinh^2 x = 4(1)\,/4 = 1$

Therefore, $\cosh^2 x - \sinh^2 x = 1$

What are hyperbolic functions used for?

Additionally, we can estimate the angles and distances in hyperbolic geometry by using hyperbolic functions to describe distance in a particular non-Euclidean geometry.

How do you find the hyperbolic function?

We can find the hyperbolic functions using the formulas given below:

sinh x = [e^x– e^-x]/2

cosh x = [e^x + e-^x]/2

tanh x = [e^x – e^-x] / [e^x + e^-x] Using the reciprocal relation of these functions, we can find the other hyperbolic functions.

What is Sinh used for?

The hyperbolic sine function, or Sinh, is the trigonometric equivalent of the Sin circular function. For real numbers, it is defined by making the area twice the axis and a ray intersecting the unit hyperbola through the origin. Additionally, it is used to second-order ordinary differential equations.

What is the difference between trigonometric functions and hyperbolic functions?

Rotations along a circle can be used to produce trigonometric functions, while rotations along a hyperbola can be used to define hyperbolic functions.

Are hyperbolic functions periodic?

It is obvious that the hyperbolic functions in R are not periodic because they are exponential functions. Thus, for the imaginary component, hyperbolic functions are periodic, with period $2\pi i$.

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