



## A STUDY OF ORDINARY PARTIAL DIFFERENTIAL EQUATIONS AND JACOBIANS

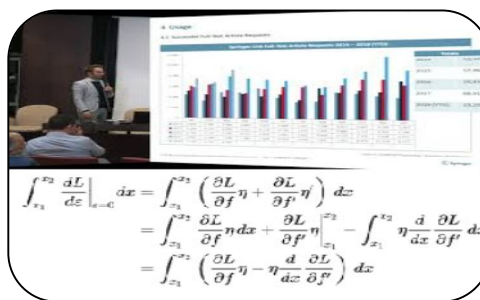
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### ABSTRACT

Partial differential equation is the most useful in the differential calculus we use in the mathematical field partial differential Equations are most probably used in differential calculus which is the one type of differential equation, derivatives w.r.t more than one independent variables are called partial differential equation, partial differential equations, and variance are having variable has been consider more often we will have to consider more than one independent variables suppose It is also called partial derivatives Up to the boundary regularity results for second order linear elliptic systems in divergence form with Dirichlet or partial derivative type boundary condition are well-known. However, a large class of elliptic systems can be written more crisply in the language of differential forms and often,



**KEYWORDS:** Boundary regularity, elliptic system, Campana to method, Hodge Laplacian, Maxwell system, Stokes system, div- curl system, Gaffney-Friedrichs inequality, tangential and normal boundary condition.

### INTRODUCTION

Up to the boundary regularity results for second order linear elliptic systems in divergence form with Dirichlet or conormal derivative type boundary condition are well-known. However, a large class of elliptic systems can be written more crisply in the language of differential forms and often, for such systems, neither of those is the re-example is the Poisson problem of the Hodge Laplacian with prescribed ‘tangential part’ prescribed ‘normal part’ on the boundary levant boundary condition. A respectively, namely the systems,  $\Delta \partial \nu \Omega$ . tepache main result of the present article (Theorem and  $\int_{\partial \Omega} \nu \wedge \omega = \nu \wedge \omega_0$  on  $\partial \Omega, A d \omega) + B^T d \delta (B \omega) = \text{fin } \Omega, \nu \wedge \delta (B \omega) = \nu \wedge \delta (B \omega_0)$  on  $\partial \Omega$ ,

Ex : Let  $f(x,y)$  is a functions of two valuables and  $f(x,y,z)$  is also functions of three valuables. Let  $f(x,y)$  be a continuous funtions of  $x,y,z$  changes when  $x$  changes or  $y$  changes or when both  $x,y$  changes, when  $x$  changes  $y$  remains constant, when  $y$  changes  $x$  remains constant, partial differential denotes as  $\partial f, \partial f, \partial f$  etc. In partial differential equation or partial  $\partial x \partial y \partial z$  derivatives are two, three and  $n$  types (1) first order (2) second order (3)  $n^{\text{th}}$  order.

Ex : -

i)  $\partial f, \partial f, \partial f$  are called first order partial differential equation  $\partial x \partial y \partial z$

ii) are called second order partial differential equation  $\frac{\partial}{\partial r} \left( \frac{\partial f}{\partial n} \right) = \frac{\partial^2 f}{\partial n^2} = f_{x_2}$  In generally  $\frac{\partial^2 f}{\partial v \partial x} = \frac{\partial^2 f}{\partial x \partial v}$  order of partial differentiation is independent.

In partial differential having “Homogeneous Functions” of  $f(\beta x, \beta y) = \beta^n f(x, y)$  then  $f(x, y)$  is called also homogeneous function in degree  $n$ . Till now functions of one variable has considered. We will have to consider functions of more than one independent variable. Partial differential equations having total derivative and total differential, for example let  $u=f(p, q)$  (1) Where  $p=\partial(t)$ ,  $q = \beta(t)$  (2) where  $p$  and  $q$  are functions of single variable. In equation (1) or (2) have single variable  $t$ , is  $\frac{du}{dt}$  is called total derivative with respect to  $t$ . Let  $\partial u, \partial x, \partial y$ , changes  $p$  and  $q$  corresponding to the change  $\partial t$  in

$$t \text{ is } \frac{du}{dt} = \frac{\partial u}{\partial p} \cdot \frac{dp}{dt} + \frac{\partial u}{\partial q} \cdot \frac{dq}{dt}$$

$\cdot f(p, q) = u$ . partial differential equations having many applications which has more than one.

Independent variable (1) equations of conic in polar form (2) Jacobians (3) Polar co-ordinates etc. for such systems, neither of those is the relevant boundary condition. A typical example is the Poisson problem for the Hodge Laplacian with prescribed ‘tangential part’ or prescribed ‘normal part’ on the boundary respectively, namely the systems,

$v \wedge \omega = 0$  on  $\partial \Omega$ . or  $v \gamma \omega = 0$  on  $\partial \Omega$ .  $\delta d\omega + d\delta\omega = f$  in  $\Omega$  We prove existence and up to the boundary regularity estimates in  $L^p$  and  $H^1$   $s\delta(Ad\omega) + B^T d\delta(B\omega) = \lambda B\omega + f$  in  $\Omega$ , with either  $v \wedge \omega$  and  $v \wedge \delta(B\omega)$  or  $v \gamma B\omega$  and  $v \gamma (Ad\omega)$  prescribed on  $\partial \Omega$ . The proofs are in the spirit of ‘Campanato method’ and thus avoid potential theory and do not require a verification of Agmon-Douglis- Nirenbeor Lopatinskii-Shapiro type conditions. Applications to a number of related problems, such as general versions of the time-harmonic Maxwell system, stationary Stokes problem and the ‘div-curl’ systems, are included Partial differential Equations (PDEs) are accomplished the application of the variance in PDE.

**OBJECTIVES:**

- To Understand the steps of partial differential equation in differential Calculus and its variance
- To know about notation’s of partial differential equations and variance.
- To know different Mathematician, solve the various problems related to variance
- To form the different types of partial differential equations.
- To understand the solving partial differential equations related to variance field.

**RESEARCH METHODOLOGY**

According to William green wald: variance mean of squared deviation about the mean of a sequence, boundary regularity, elliptic system, Campana to method, Hodge Laplacian, Maxwell system, Stokes system, div- curl system, Gaffney-Friedrichs inequality, tangential and normal boundary

$$v_i(t) = \text{sgn}(r(t) - s(t)), \quad A \frac{dx(t)}{dt} + Bx(t) = c(t), t \in (t_0, T],$$

with given initial value  $x(t_0) = x_0$ , where  $x(t) \in \mathbb{R}^n$  is the unknown solution vector consisting for example of currents and voltages,  $A, B \in \mathbb{R}^n \times \mathbb{R}^n$  are matrices, and  $c(t) \in \mathbb{R}^n$  is the right-hand side containing current and voltage sources, e.g., the pulsed voltage  $v_i(t)$ . The system may be assembled from lumped element descriptions based on loop or (modified) nodal analysis as described in Please note, that we focus on the linear case but the approach can be straightforwardly generalized, e.g., considering  $B = B(x)$ . Furthermore, a global strong solution  $x(t)$  may not exist due to the switching of the right-hand side solution.

The large end time point  $T$  while the fast dynamics due to the switching enforce small time steps. This is the motivation to turn to efficient methods that can handle the two ingredients of Parareal: the fine and the coarse propagators. We denote by  $F(t, t_0, x_0)$  and  $G(t, t_0, x_0)$  the solutions of the IVP (2) at  $t \in$

$(t_0, T ]$  obtained with sequential time stepping using fine and coarse time steps, respectively. Partitioning the time interval  $t_0 = T_0 < T_1 < \dots < T_N = T$  we write the Parareal iteration: for  $k = 0, 1, \dots$  and  $n = 1, \dots, N$  solve  $X^{(k+1)} = x_0 X^{(k+1)} = F.T_n, T_{n-1}, X^{(k)} \Sigma + G.T_n, T_{n-1}, X^{(k+1)} \Sigma - G.T_n, T_{n-1}, X^{(k)n-1}$ . The solution operator  $F$  is assumed to deliver a very accurate solution (e.g., using a numerical time-integration method with small time steps  $\delta T$ ) and can be executed in parallel, while  $G$  gives rough information about the solution using a cheap method (e.g., using a numerical method with large time steps  $\Delta T_i = T_{i+1} - T_i$ ) and has to be calculated sequentially. Captures the high frequency behavior was introduced. The idea is to separate the high frequency (pulsed) components from the low frequency components, i.e.,

$$A \frac{dx}{dx(t)} + B x(t) = \bar{c}(t) + \tilde{c}(t), = c(t) - A(1)$$

where  $\bar{c}$  can be given as a few low-frequency sinusoids from a (fast) Fourier transform and  $\tilde{c}(t) := c(t) - \bar{c}(t)$  is the remainder. This allows to define a reduced coarse propagator  $G$  which solves

$$\frac{dx}{dt} Ax(t) + B x(t) = \bar{c}(t) \quad \text{---}$$

Two particular differential operators on differential forms have a special significance for us. A differential  $(k + 1)$ -form  $\phi \in L^1(\Omega; \Lambda^{k+1})$  is differential forms will have an exterior derivative of  $d\phi \in L^1(\Omega; \Lambda^k)$ ; for all  $\eta \in C^\infty \eta \wedge \phi = (-1)^{n-k} d\eta \wedge \omega$ ,  $\omega \in L^1(\Omega; \Lambda^{n-k-1})$ . The Hodge codifferential of  $\omega \in L^1(\Omega; \Lambda^k)$  is  $\delta \omega \in L^1(\Omega; \Lambda^{k-1})$ . Multirate PDEs. The MPDE approach, which is used for obtaining the coarse solution in Parareal uses the MPDE. Its application to yields

$$A \frac{\partial x}{\partial t}(t_1, t_2) + \frac{\partial x}{\partial t^2}(t_1, t_2) + \frac{\partial x}{\partial t} B x(t_1, t_2) = Cx(t_1, t_2), \text{---}(2)$$

where the relation between the original and the MPDE solution and righthand side are given by  $x(t, t) = x(t)$ ,  $c(t, t) = c(t)$ . This implies that if a solution (1) is found, the solution of (2) can be extracted from it. The MPDEs allow us to split the power converter solution which consists of a slowly varying envelope and fast periodically varying ripples explicitly using a solution expansion by variance method.

**LITERATURE REVIEW;**

We prove existence and up to the boundary regularity estimates in  $L^p$  and Hölder spaces for weak solutions of the linear  $s\delta (A d\omega) + B^T d\delta (B\omega) = \lambda B\omega + f$  in  $\Omega$ , with either  $v \wedge \omega$  and  $v \wedge \delta (B\omega)$  or  $v \wedge B\omega$  and  $v \wedge (A d\omega)$  prescribed on  $\partial\Omega$ . The proofs are in the spirit of ‘Campana to method’ and thus avoid potential theory and do not require a verification of Agmon-Douglis-Nirenbeor Lopatinski-Shapiro type conditions. Applications to a number of related problems, such as general versions of the time-harmonic Maxwell system, stationary Stokes problem and the ‘div-curl’ systems, are included.

The regularity results for these two systems are known since. However, regularity results for more general linear elliptic systems with these type of boundary conditions do not exist in the literature. The reason for this surprising absence probably lies in the available proofs of these results. The original proof of Morrey relied on potential theory and used the fact that  $\delta d + d\delta$ , i.e the Hodge Laplacian is precisely the component wise scalar Laplacian. Also, after flattening the boundary, the condition  $v \wedge \omega = 0$  and also its counterpart with the ‘normal condition’  $\delta(A d\omega) + B^T d\delta (B\omega) = f$  in  $\Omega$ ,  $v \wedge (v \wedge \delta\omega = 0$  imply that as far as the principal order terms are concerned, the whole system decouples and gets reduced to  $n_k$  number of scalar Poisson problems with Laski-Shapiro, henceforth LS, (Other available proofs verify either the Lopatin- or the Agmon- Douglis-Nirenberg complementing condition, henceforth ADN, and these verifications too rely on the fact that the principal symbol of the operator is rather ‘simple’. Deriving the regularity results for the system  $\delta (A d\omega) + B^T d\delta (B\omega) = f$  or

even the simpler system  $\delta (Ad\omega) + d\delta\omega = f$  with these type of boundary conditions calls for different methods, as the verification of the ADN or LS conditions for these systems looks algebraically tedious. On the other hand, the boundary conditions simply arise out of an integration by parts formula and in principle, verifying ADN or LS should be an avoidable overkill.

For linear elliptic systems with Dirichlet boundary condition the classical Campana to method of deriving estimates in Morrey and Campana to spaces, avoids potential theory and yields at the same time both the Schauder estimates and via an interpolation theorem of also the  $L^p$  estimates. This approach has also been adapted to elliptic systems with conormal derivative type condition, for example in Giaquinta-Modica The crux of the present article is to adapt this approach to these kind of boundary conditions. This, as a particular case yields a new proof for the regularity of the  $\omega$  is a  $k$ -form. Note that unlike the case of the Hodge Laplacian, where solving the tangential boundary value problem for  $k$ -forms is equivalent to solving the normal boundary value problem  $(B, \omega_0) = \nu \partial \gamma \Omega$  on  $\partial \Omega$ . equation in a bounded domain in  $\mathbb{R}^3$  is where  $A$  and  $B$  are matrix fields and  $\omega$  is a  $(n - k)$ -form by Hodge duality, these two problems are not dual to each other in general. Instead, each has their respective dual versions. The results for (1) and (2) yield also the existence and regularity results for a number of related problems.

### CONCLUSIONS:

The importance of partial differential equations and variance due to their recent occurrence in the study of many processes in science and engineering, statistics and also the various limitations posed by the integer order derivative for making the study put up in the thesis entitled Partial Differential Equations The study of symmetries and exact solutions of nonlinear partial differential equations has great theoretical and practical importance. These exact solutions for nonlinear systems are used as models for physical or numerical investigations and often replicate qualitatively on the behavior of more complicated solutions. More specifically, the thesis deals with nonlinear partial differential equations of fractional order representing some interesting physical systems which are the space time fractional Burgers Poisson equation, time fractional potential Burgers' equation, variable coefficient space-time fractional potential Burgers' equation, time fractional Gardner and space time from the view point of their underlying Lie symmetries of infinitesimal transformations. the solutions obtained are of very specific nature and further application of Lie group method on the reduced system led only to trivial symmetries. Thus, the general solution of reduced ODEs, their physical interpretation and the study of higher order symmetries of fractional order differential equations bring forth tremendous scope for future work.

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