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# GAUSSIAN ELIMINATION OF SYSTEM OF LINEAR EQUATION

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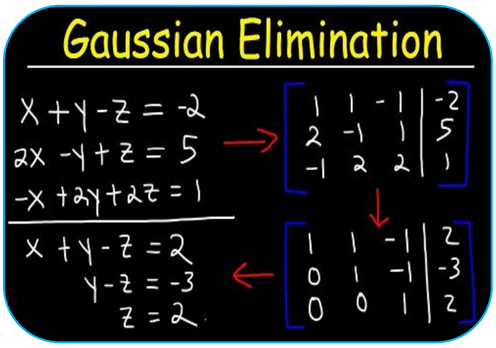
**ABSTRACT:**

*Guassian Elimination and Guass Jordan schemes are run to solve the linear system of equations. This paper introduces matrix and direct methods of linear equations. The aim of this research was to analyse the various elimination techniques of linear equations and to measure the efficiency of Guassian Elimination and Guassian Jordan methods to find their relative importance and advantage in the field of symbolic and numerical calculations. The aim of this research is to refine linear equations, matrix theory and an introductory concept of Guassian elimination types by which the efficiency of Guassian Jordan and Guassian elimination can be measured.*




**KEYWORDS:** *Direct, Indirect, Backward Stage.*

## INTRODUCTION:

An equation system is a set or collection of equations solved together. The collection of linear equations is called the system of linear equations. They are usually based on the same set of variables. Various methods have been developed for solving linear equations but no best method for solving linear equations has been proposed yet. Different mathematicians have suggested different methods based on speed and accuracy. However, velocity is an important factor in solving linear equations where the calculation rate is very large. Linear equation methods are divided into two categories. Direct and indirect. Each category contains several elimination methods used to solve equations. This paper deals with the Guassian elimination method, a direct method of solving linear equations. An introductory part of the Guass Jordan elimination is also performed to analyse the effectiveness of both methods. Indirect methods are basically repetitive methods and one of the advantages of these methods is that they require fewer multiplication steps for larger calculations. Repeat methods can be applied to smaller programs and they are fast enough. Along with the study of linear equation systems, one must be familiar with the matrix theory to see how many different operations are performed on the desired matrix to calculate the result.

## HISTORY OF GUASSIAN ELIMINATION:

Guassian Method of Elimination - Without Evidence - Chapter Eight in Chinese Mathematical Text: A rectangular array of nine chapters on mathematical art appears. Its use is explained in eighteen problems with two to five equations. The first reference to the book under this title dates back to 179 AD, but parts of it were written around 150 BC. It was commented on by Liu Hui in the third century. The method in Europe originated from the notes of Isaac Newton. In 1670, he wrote that not all algebra books he knew had a lesson in solving equations at once, which Newton later provided. After Newton left academic life in 1707, the University of Cambridge finally published notes as Arithmetica Universalis. Notes were heavily imitated, making Guassian abolition a standard lesson in algebra textbooks in the late 18th century. Carl Friedrich Gauss created a notation for symmetric elimination in 1810 that was adopted in the 19th century by commercial hand computers to solve common equations of minimal-square problems. The algorithms taught in high school were only named for Gauss in the 1950s because of the confusion of history in the subject. Some authors use the term Guassian Elimination only to refer to the process until the matrix comes in the form of Akelon, and use the term Guassian-Jordan Elimination, which ends in the process of less Akelon. The name is used because it is a form of Guassian extermination, as described by Wilhelm Jordan in 1888. However, Klaassen's article, published the same year, also shows this method. Jordan and Klaassen may have independently explored the Gauss-Jordan abolition.

## GUASSIAN ELIMINATION:

Guassian elimination is the standard method for solving linear equations. Because it is a ubiquitous algorithm and plays a fundamental role in scientific calculations. Guassian Elimination is a tool for solving equations, for calculating determinants, for drawing a range of coefficient matrix. However, the Guassian Elimination Matrix relies more on analysis and calculation. It emphasizes block pivoting, repetition methods, and tools for improving the quality of calculated solutions. It consists of two stages, the forward and the backward stages.

The objectives of Guassian elimination are to make the element in the upper-left corner 1, to use the primary row operations to get 0s in all the positions below the first 1, to get 1s for the leading coefficients in each row slanting from top-left to bottom. Right angle, and get 0s below all leading coefficients. Basically, you subtract all the variables except the one in the last row, all the variables except the two in the equation above that one, and so on until the equation above, which contains all the variables. Then you can use back substitution to solve one variable at a time by adding the values you know from bottom to top in the equations. You complete this elimination by removing the x from all the equations except the first one. Then subtract the second variable from all the equations except the first two. This process continues until only one variable remains in the last line, removing one more variable in each line. Then fix that variable.

## STEPS TO SOLVE GUASSIAN ELIMINATION:

Guassian elimination is the systematic use of primary row operations in a system of equations. It converts the linear system of equations into an upper triangular shape, from which the solution of the equation is determined. The steps mentioned above summarize the Guassian elimination.

- For a system of linear equations, it is necessary to write augmented matrix.
- Convert A to the top triangle using the row operations on {A / b. Diagonal elements cannot be zero.
- Use back replacement to find a solution to the problem.

Consider a system of linear equations consisting of n variables

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}xn = a_1,n+1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}xn = a_2,n+1$$
$$a_{31}x_1 + a_{32}x_2 + \dots + a_{3n}xn = a_3,n+1$$
$$\dots\dots\dots\dots\dots\dots\dots\dots$$
$$A_{n1}x_1 + a_{n2}x_2 + \dots + a_{3n}xn = a_n,n+1$$

Where $a_{ij}$, and $a_{i,j+1}$ are constants, xi's are variables. The system become equals to:

$AX = B$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & a_{3n} & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \dots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{1,n+1} \\ a_{2,n+2} \\ a_{3,n+3} \\ \dots \\ a_{n,n+1} \end{bmatrix}$$

### First Step:

Store the coefficients in an augmented matrix. The superscript on $a_{ij}$ means that this is the first time to store numbers in location (i, j).

$$\left[\begin{array}{cccc|c} a_{11} & a_{12}a_{13} & \dots & a_{1n} & a_{1,n+1} \\ a_{21} & a_{22}a_{23} & \dots & a_{2n} & a_{2,n+2} \\ a_{31} & a_{32}a_{33} & \dots & a_{3n} & a_{3,n+3} \\ \dots & \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n}a_{3n} & \dots & a_{nn} & a_{n,n+1} \end{array}\right]$$

### Second Step:

If necessary, shift the row so that a ≠ 1 10, then remove x1 n from 2. The new elements are written aij to indicate that this is the second time the number is stored in the matrix allocation (i, j).

$$\left[\begin{array}{cccc|c} a_{11} & a_{12}a_{13} & \dots & a_{1n} & a_{1,n+1} \\ 0 & a_{22}a_{23} & \dots & a_{2n} & a_{2,n+2} \\ 0 & a_{32}a_{33} & \dots & a_{3n} & a_{3,n+3} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & a_{2n}a_{3n} & \dots & a_{nn} & a_{n,n+1} \end{array}\right]$$

### Third Step:

Newly written $a_{ij}$ indicates that this is the third time to store numbers in the matrix at location (i, j).

$$\left[\begin{array}{cccc|c} a_{11} & a_{12}a_{13} & \dots & a_{1n} & a_{1,n+1} \\ 0 & 0\ a_{23} & \dots & a_{2n} & a_{2,n+2} \\ 0 & 0\ a_{33} & \dots & a_{3n} & a_{3,n+3} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0\ a_{3n} & \dots & a_{nn} & a_{n,n+1} \end{array}\right]$$

The final result after the row operation can come in the form above:

$$\left[\begin{array}{cccc|c} a_{11} & a_{12}a_{13} & \dots & a_{1n} & a_{1,n+1} \\ 0 & 0\ a_{23} & \dots & a_{2n} & a_{2,n+2} \\ 0 & 0\ a_{33} & \dots & a_{3n} & a_{3,n+3} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0\ 0 & \dots & a_{nn} & a_{n,n+1} \end{array}\right]$$

### LU Factorization Guassian Elimination:

LU factorization is one of the most important mathematical concepts in Guassian Elimination. It plays an important role in the implementation of GE in modern computers and, ultimately, facilitates spherical error analysis of algorithms. The LU factorization method is done in three steps
Following is the Matrix to consider….
Ax=b:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{22} & a_{23} & a_{24} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

## Computational Efficiency:

The number of arithmetic operations required to reduce rows is one way of measuring the computer performance of an algorithm. For example, a system of *n* equations for *n* unknowns requires $n(n + 1) / 2$ divisions, $(2n^3 + 3n^2 - 5n) / 6$ multiplication, and $(2n^3 + 3n^2 - 5n) / 6$ subtraction to solve it in a single form by performing row operations on the matrix, and then in reverse order for each unknown. For a total of approximately $2n^3 / 3$ operations. Thus its arithmetic complexity O $(n^3)$;.

This arithmetic complex is a good measure of the time required for a complete calculation when the time of each arithmetic operation is approximately constant. This happens when the coefficients are represented by floating-point numbers or when they belong to a finite area. If coefficient integers or rational numbers are shown precisely, the intermediate entries can be exponentially large, so the bit complexity is exponential. However, there is a type of Guassian elimination, called the Bareis algorithm, which avoids this exponential growth of intermediate entries and has a slight complexity of O $(n^5)$, with the same arithmetic complexity of O $(n^3)$.

This algorithm can be used on computers for thousands of equations and systems that are unknown. However, the price becomes restrictive for systems with millions of equations. These large systems are usually solved using iterative methods. Specific methods exist for systems whose coefficients follow a regular pattern.

To keep n × n the matrix in a singleon form reduced by row operations, one needs $n^3$ arithmetic operations, which are approximately 50% more calculation steps. Numerical instability is a potential problem, caused by the possibility of dividing into very small numbers. If, for example, the leading coefficient of a row is very close to zero, then to reduce the matrix row, one must divide by that number. This means that any existing error for a number close to zero will be increased. Guassian elimination is numerically stable for oblique or positive-fixed matrix. For the general matrix, Guassian elimination is generally considered static, when using partial pivoting, although there are instances of static matrix for which it is unstable.

## CONCLUSION:

There are different direct and indirect methods for calculating the linear system of equations. Guassian elimination is a type of direct method used to calculate unknown variables. Many scientific and engineering fields of computation can take the form of linear equations. Equations in this field can have a large number of variables and that is why it is important to solve these equations efficiently. The efficient method of solving these equations involves the Guassian elimination method in this study.

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