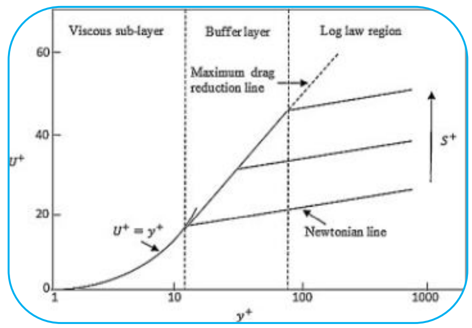
REVIEW OF RESEARCH
ISSN: 2249-894X
IMPACT FACTOR : 5.7631(UIF)
VOLUME - 11 | ISSUE - 5 | FEBRUARY - 2022

ANALATICAL ANALYSSIS ON WALL-ROUGHNESS EFFECTS ON THE FLOW OF POWER-LAW FLUID AS DRAG-REDUCING POLYMER SOLUTIONS

S. S. Shukla
Associate Professor , Department of Mathematics,
D.B.S. (P.G.) College, Kanpur.

ABSTRACT:

In this Paper the effect of wall roughness on the inner surface of circular pipe, the findings of this paper present a comparison of drag reducing properties in rough and smooth popes for different concentrations of the polymers in solutions. In order to observe the effect of roughness on drag reduction, in this paper it is concluded that the drag reducing agents are effective in rough pipes. When the concentration of the polymer in solution is very small, the drag reducing property is more effective in rough pipes than in the smooth pipes. As soon as the concentration increase from a critical value (about 10 PPM), the percentage of drag reduction starts decreasing in rough pipes with the higher rate than that in smooth pipes.



KEY-WORDS: *Dilute Polymer Solution, Concentration, Friction, Poly-acrilamide, Rugosities Project, Buffer layer, Reynolds number, Cartesian Co-Ordinates System, Differential Concentrations, Incompressible, Drag reduction, Consistency indices, Pseudo plastic fluid.*

1. INTRODUCTION:

Hoyt and Fabula (1964) demonstrated that the polymer additives were effective on rough surfaces by using rotating disc apparatus, Shah and Zhou (2003) experimented with a square pipe, two sides of which were smooth and the other two were made rough by cementing Silicon Carbide grains to the surface. Polyethylene Oxide in very small concentration as low as 15 parts per million was found to cause a reduction in friction. Acoording to Ayegba, Abubakar (2019) and Barenblant (1969) ply-ethylene oxide is more effective in rough pipes than poly-acrilamide, where as for both these polymers; the results for the drag reduction in identical smooth pipes with water were similar. Later on, te data obtained by the series of researchers whitw (1967) Mcnally (1968), Spangler (1969), Brandt et.al. (1969) Virk, (1971) all showed the drag reduction in rough pipies.

A pipe may be considered hydrodynamically rough if the hight of the roughness elements is less than the viscous sublayer thickness and it is fully rough if the rugosities project beyond the buffer layer into the turbulent core. Porch (1970) has developed a theoretical model based on the assumption that the effect of the relative roughness size is similar for flow with or without polymers. This model appears to be successful in qualitatively describing the available experimental data. Pilipenko (1978) has shown that the specific simulation of boundary layer characteristics is possible in flow of dilute polymer solutions. He has observed that the flow over a rough surface corresponds to the flow over a smooth surface but at a lower Reynolds number.

Recently, Al Wahaibi (2012) and Thirsness and Hanratty (1979) have considered mass transfer between a fluid and a small amplitude wavy solid surface. The solid wavy surface under consideration was taken in Cartesian coordinate system in the form

$$y = a\cos(\alpha x) \tag{1}$$

2. FORMULATION OF THE PROBLEM:

The rough pipe has no well defined configuration. We have considered the radius of the rough pipe.

$$R(z) = R + aR\cos\frac{2\pi Z}{LoR} \tag{2}$$

Where aR and LoR are the amplitude and wave length of the roughness in the pipe, and these are very small. One dimensional steady flow of polymer solution has been considered for different concentrations. We have taken power law fluid model to represent the flow properties of dilute polymer solution.

3. MATHEMATICAL ANALYSIS:

As proposed in the introduction above the one dimensional axisymmetric flow of power law fluid flowing in a pipe of geometry given in equation (2) has been considered. The equation of motion, of power law fluid in cylindrical polar coordinates, assuming that the motion is steady and axisymmetric and the fluid is incompressible is:

$$r\frac{\partial p}{\partial z} = -\frac{\partial}{\partial r}(r\,T_{rz}) \tag{3}$$

Where T_{rz} is given by-

$$T_{rz} = m\left(-\frac{\partial u}{\partial r}\right)^n \tag{4}$$

After introducing equation (4) in equation (3) and solving with the boundary conditions:

T_{rz} Is finite at r=0 (5)

And u=o at r=R (z)

We get

$$U = \frac{n}{n+1}\left\{\left(\frac{1}{2m}\right)\left(-\frac{\partial p}{\partial z}\right)\right\}^{\frac{1}{n}} \times$$

$$R(z)^{\frac{n+1}{n}}\left\{1-\left(\frac{r}{R(z)}\right)^{\frac{n+1}{n}}\right\} \tag{6}$$

The flow rate is given by

$$Q = \int_0^{R(z)} 2\pi\; r\,u\,dr \tag{7}$$

Introducing (6) in (7) we get

$$Q = \frac{n\pi}{3n+1}\left\{\frac{1}{2m}\left(-\frac{\partial p}{\partial z}\right)\right\}^{\frac{1}{n}} R(z)^{\frac{3n+1}{n}} \tag{8}$$

Therefore, for the constant flow rate

$$\frac{\left(-\frac{\partial p}{\partial z}\right)}{Q^n} = \left(\frac{3n+1}{n\pi}\right)^n 2m\, R(z)^{-(3n+1)} \tag{9}$$

Integrating equation (9) with respect to z from O to L, we get,

$$\frac{1}{Q^n}\left(\frac{P_o - P_L}{L}\right) = \left(\frac{3n+1}{n\pi}\right)^n 2m \left(\frac{1}{R}\right)^{(3n+1)} \beta_n \tag{10}$$

Where

$$\beta_n = 1 + \frac{(3n+1)(3n+2)}{4} a^2 \tag{11}$$

The resistance to the flow for the constant flow rate is defined as:

$$F = \left(\frac{P_o - P_L}{L\, Q}\right) \tag{12}$$

Hence

$$F = \left(\frac{P_o - P_L}{L}\right)^{1-\frac{1}{n}} \frac{3n+1}{n\pi} \times$$

$$(2m\beta_n)^{\frac{1}{n}} \left(\frac{1}{R}\right)^{\frac{3n+1}{n}} \tag{13}$$

4. CALCULATION OF DRAG REDUCTION IN SMOOTH PIPES:

The percentage of drag reduction in smooth pipe may be given as:

$$DR\% = 100 \times \frac{F_1 - F_2}{F_1} \tag{14}$$

Where $F_1 = \frac{8\mu}{\pi R^4}$ (15)

$$F_2 = \left(\frac{P_o - P_L}{L}\right)^{1-\frac{1}{n}} \frac{3n+1}{n\pi} (2m)^{\frac{1}{n}} \left(\frac{1}{R}\right)^{\frac{3n+1}{n}} \tag{16}$$

5. CALCULATION OF DRAG REDUCTION IN ROUGH PIPES:

The percentage of drag reduction in tough pipe may be given as:

$$DR_r\% = 100 \times \frac{F_3 - F_4}{F_3} \tag{17}$$

Where $F_3 = \frac{8\mu\beta_1}{\pi R^4}$ (18)

And F_4 is given by

$$F_4 = \left(\frac{P_o - P_L}{L}\right)^{1-\frac{1}{n}} \frac{3n+1}{n\pi} \times$$

$$(2m\beta_n)^{\frac{1}{n}} \left(\frac{1}{R}\right)^{\frac{3n+1}{n}} \qquad (19)$$

6. CALCULATION OF EXCESS DRAG REDUCTION PERCENTAGE:

The excess drag reduction percentage in rough pipe is defined as

$$\text{DR \%(Excess in rough pipe)} = \frac{DR_r\% - DR_s\%}{DR_s\%} \times 100 \qquad (20)$$

7. RESULTS AND DISCUSSIONS:

In order to observe the effects of roughness on drag reduction, we have introduced the same values of flow behavior and consistency indices, calculated these values from the data given by Mc B Comb (1974) for different concentrations of the polymer solutions so that we may relate the effectiveness of the drag reduction phenomenon with the concentration of the polymers as drag reducing agents in the rough pipes.

Fig. A describes the percentage of drag reduction with pressure drop for different concentrations of the dag reducing agents in a rough pipe of roughness a=0.1 and of radius 2 cm. As the concentration increases the percentage of drag reduction increases up to some critical value of the concentration (about 10 PPM) and later on it starts decreasing with increasing value of the concentration.

Fig. B describe the percentage of drag reduction with concentration for different pressure drops in rough pipes of roughness (a=0.1) and radius (R=2 cm.).From this figure we see that as the pressure drop increase the percentage of the drag reduction increases.

Table A gives the comparison of drag reduction percentage in rough and smooth pipes for different polymeric solutions and on different pressure drops. From this table we observe, a very important result. As the concentrartion of the polymer increases upto some critical concentration. The drag reduction percentage in rough pipe is higher that that is n smooth pipes. This result is similar to the result of Pillipenko (1978).

Table B gives the theoretical comparison of drag reduction percentage in smooth and rough pipes. For different pseudoplastic fluids (m=.012, R=2 cm). From this table, we also observe that as the roughness in the pipe increase the percentage of drag reduction increase.

From our calculations it is very clear that although the drag in rough pipes is higher than that in smooth pipes, but the percentage of drag reduction is higher in rough pipes than smooth pipes and pseudoplastic fluid such as vary dilute polymers solutions are very much effective in rough pipes.

TABLE-A

Comparison of DR% in rough and smooth pipes for various concentrations of polymers in solution
Where $\delta = .1$, $R = 2\ cm$

S.No.	Concentration	m	n	$\frac{P_0 - P_L}{L}$	DR% in smooth pipe	DR% in rough pipe
1.	8 PPM	.031	.812	10	13.906	13.9428
2.				20	26.672	26.8666
3.				30	32.42	33.425
4.				40	37.544	37.716
5.	10 PPM	.43	.697	10	53.89	55.56952
6.				20	66.016	67.12
7.				30	73.326	72.4419
8.				40	75.58	75.6819
9.	25 PPM	.056	.731	10	13.59	9.66095
10.				20	33.04	30.00
11.				30	42.32	39.70285
12.				40	48.12	45.7600

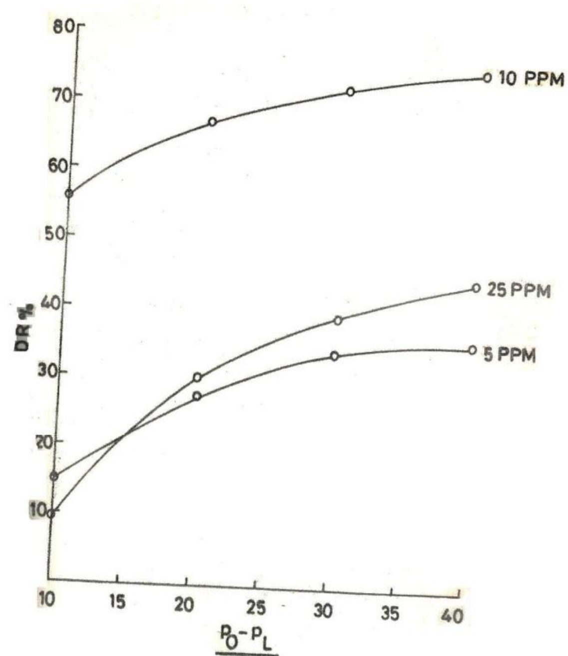
TABLE-B
Comparison of DR% in rough and smooth pipes for various pseudo plastic fluids
Where m=0.012, R=2 cm

S.No.	n	$\frac{P_0 - P_L}{L}$	DR% smooth pipe	DR% Rough Pipe (a=0.1)
1.	.9	10	72.582	72.62476
2.		20	74.614	74.65333
3.		30	75.732	75.76952
4.		40	76.496	76.53142
5.	.8	10	90.499	90.5291
6.		20	92.01142	92.0427
7.		30	92.78095	92.8209
8.		40	93.2819	93.3118
9.	.7	10	97.236	97.24761
10.		20	97.948	97.9619
11.		30	98.274	98.28
12.		40	98.474	98.4819

REFERENCE:

1. Hoyt,J.W.& Febula,A.C.(1964) “The effect of additives on fluid fricition”, Proceadings fifth symposium on Noval Hydrod namics,Bergen,Norwey,Office of Naval Research ACR-112,P-947
2. Shah SN, Zhou Y.(2003) “ An exprimantal study of drag reduction of polymer solutions in coiled tubing” SEP J., P-7-8
3. Barenblant,A.C.,Gareatev,V.A.& Kala,Shikvr,V.N.(1969) “Turbulence of Anomalous Fluids”, Heat Transfer-Soviet Research, Vol.1,P- 102
4. White,A.(1967) “Turbulence and Drag reduction with Polymer additives”,Henden College of Tech.Research Bulletin No.4,P-75
5. Ayegba PO, Edomwonyi-Out LC, Yusuf N, Abubakar A (2019) “ Experimental and natural network modelling of polymer drag reduction in 180 bends” Results Mater, P- 1-11
6. Mc.Nally,W.A.(1968) “Heat and Momentum transport in dilute polu(Ethulene oxide) Solutions”, Naval Under Water Weapons Research in Engg.Station,T.R. 44

7. Spangler,J.G.(1969) "Studies of viscous drag reduction with Polymers including Turbulence measurements and roughness effects",C.S.Wells(Ed) Plenum Press,New York, Viscous Drag reduction, P- 131-155
8. Brandt,H,Mc.Donald,A.T.&Beyl,F.W.(1969) "Turbulent skin friction of Dilute polymer solution in rough pipes", Viscous drag reauction Ed.C.S.Wells,Plenum Press,New York,P-159
9. Virk,P.S.(1971) "An elastic sublayer model for drag reduction by dilute solutions of linear macromolecules", J. Fluid Mech. Vol.45,P-417
10. Al-Wahaibi.T et al (2012) " Experimental study on the effect of drag reducing polymer on flow patterns and drag reduction in a Fluid Flow, P 74-80
11. Porch,M.(1970) "Flow of dilute polymer solutions in rough pipes" ,J. of horizontal oil- water flow" Int J Heat Hydronautics , vol. 6,P-151
12. Piliponke,V.N.(1978) "Simulation of turbulent flows in dilute Polymer solutions", Fluid Mechanics Soviet Research, Vol. 7,No.-2,P-31
13. Thorsness,C.B.& Hanralty,T.J.(1979) "Mass transfer between a flowing fluid and a solid wavy surface", A I Ch E,Vol.25,No.4,P-686
14. Mc.Comb,W.D.(1974) "Drag reduction lpolymers and Liquid column oscillations", Nature 251,P-598



[redacted] A Variation of percentage Drag reduction with change in pressure drop for different concention of polymer in solution in rough pipe(a = 0.1, R = 2 cm).

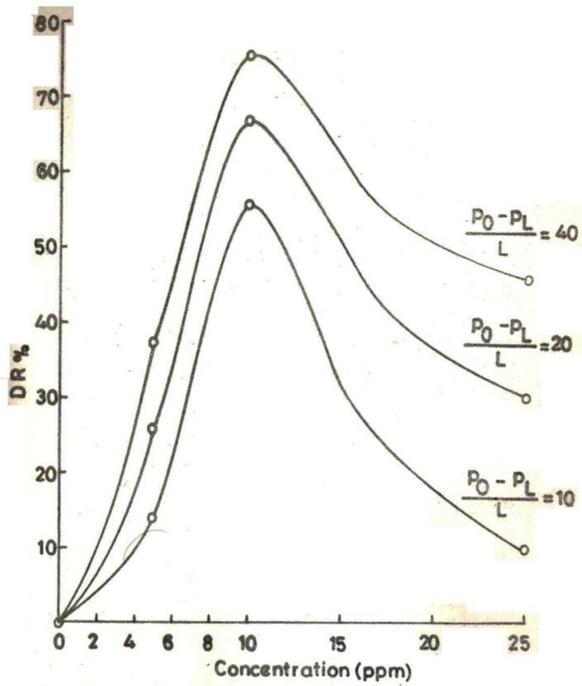


Fig. B Variation of percentage drag reduction with the concentration of polymer in solution for different pressure drops ($a = 0.1$, $R = 2$ cm)

S. S. Shukla
Associate Professor , Department of Mathematics,
D.B.S. (P.G.) College, Kanpur.