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FUNCTIONAL MINIMIZATIONS WITH CONJUGATE GRADIENT ALGORITHM FOR SPECTRAL THREE-TERM CONSTRAINED

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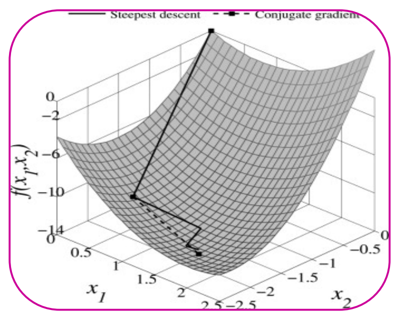
ABSTRACT

The area of limited optimization methods of three-term conjugate gradient (CG) technique which is mainly based on Dai Liao (DL) formula. The new proposed technique satisfies the conjugate properties and the descent conditions of Karush-Kuhn Tucker (K.K.T.). Our planned limited technique uses strong wolf line search conditions with some assumptions. We have a tendency to prove the global convergence of new planned techniques. (30-thirty) Comparison of points for limited optimization problems ensures the effectiveness of the new planned formula.



KEY WORDS: *conjugate gradient (CG), Karush-Kuhn Tucker (K.K.T.), global convergence.*

INTRODUCTION

All policies for limited issues will be classified into (2) basic categories; In particular, direct and indirect ways. Some special optimizations transform uncontrolled optimization type problems into constrained problems by creating unrelated sub-problem paths for the latter type of square measurement for indoor and outdoor penalty function techniques. The main simple and robust technique for the earlier technique known as Sequential Unconstrained Minimization Technique (SUMT)required optimization issues with limited inequality this way…

$$\min f(x) s.t. c_j(x) \geq 0, \qquad i = 1, \ldots m$$

Equation – 1

The problem is to recreate the function of the figure in an arbitrary minimalist technique

$$\varphi(x, u) = f\ (x) +\ \mu B(x)$$

Equation – 2

Where,

$\mu\ \rightarrow 0$and$B(x)$ is defined by equation 1

$$B(x) = \sum_{j=1}^{m} \frac{1}{c_j(x)}$$

Equation – 3

So, we can write the equation – 2 as follows…

$$\varphi(x,u) = f\ (x) + \mu \sum_{j=1}^{m} \frac{1}{c_j(x)}$$

Equation – 4

There are derivatives of these functions$\nabla f(x)$ and $\nabla c_i(x)$, for $i = 1, \dots, n$ are linear independent so that,

$$\nabla\varphi(x,u) = \nabla f\ (x) + \mu \sum_{j=1}^{m} \frac{1}{c_j(x)} \nabla c_j(x)$$

Equation – 5

Now we turn to the second part parallel to the importance of the previous part, which is the arbitrary optimization technique and let us know the problem (2), where $\varphi: R^n \to R$ the real-valuable integral and scalable derivative function. This is repetitive

$$x_{k+1} = x_k + \alpha_k d_k$$

Equation – 6

Where,

α_k = it is step length and the new search direction d_{k+1} is:

$$d_{k+1} = \begin{cases} -\nabla\varphi\left(x_{k+1,}\mu_{k+1}\right)\ for\ k = 0 \\ -\nabla\varphi\left(x_{k+1,}\mu_{k+1}\right) + \beta_k d_k\ \ for\ k \geq 1 \end{cases}$$

Equation – 7

At the current point is the value of the derived function$g(x_{k+1}) = \nabla\varphi(x_{k+1}, \mu_{k+1})$ and $\beta_k$is a positive scalar called the conjugate gradient parameter.

In existing convergence analysis and implementation of CG techniques, weak wolf conditions are defined as:

$$\varphi\left(x_{k+1,}\mu_{k+1}\right) - \varphi\left(x_{k,}\mu_k\right) \leq \delta\alpha_k \nabla\left(x_{k,}\mu_k\right)^T d_k$$

Equation – 8

$$\nabla\left(x_{k+1,}\mu_{k+1}\right)^T d_k \geq \sigma\nabla\varphi\left(x_{k,}\mu_k\right)^T d_k$$

Equation – 9

and $0 < \delta < \sigma < 1$

A strong wolf situation by improving conditions equation [7] consists of equation (8) and

$$\left|\nabla\varphi\left(x_{k+1,}\mu_{k+1}\right)^T\right| \leq -\sigma\nabla\varphi\left(x_{k,}\mu_k\right)^T d_k$$

Equation – 10

Moreover, sufficient descent property i.e.

$$d_{k+1}^T\,\varphi(x_{k+1,}\mu_{k+1}) \le -c\left\|\nabla\varphi(x_{k+1,}\mu_{k+1})\right\|^2$$

Equation – 11

Modified Three Term CG Technique:

Several researchers have provided various updates that are appropriate for the Dai-Liao (DL) CG-method parameters:

$$\beta_k^{DL} = \frac{g_{k+1}^T(y_k - ts_k)}{S_k^T y_k}$$

Equation – 12

Recall the work of Leverage and Pintelus who forwarded new updates to the parameters β_k^{DL} which were based on the revised second equation and which they replaced y_k with this new one. Other researchers, e.g. Babai-Kafaki and Ghanbari derive two modified CG-methods based on Perry's work in their work; They got better numerical results than the original result given by DL. Researchers continued various updates of DL parameters to get some suitable formulas.

$$d_{k+1} = -g_{k+1} + \frac{g_{k+1}^T(y_k - ts_k)}{S_k^T y_k} s_k - \frac{g_{k+1}^T d_k}{S_k^T y_k}(y_k - ts_k), \qquad t > 0$$

Equation – 13

This situation is a satisfactory direction($d_{k+1}^T g_{k+1} \le -c_1\|g_{k+1}\|^2$) for all k. now exploitation of equation -13 in the limited CG-technique mentioned in the yield with equation 1 – equation 4.

$$d_{k+1} = -\nabla\varphi(x_{k+1,}\mu_{k+1}) + \frac{\nabla\varphi(x_{k+1,}\mu_{k+1})^T(y_k - ts_k)}{S_k^T y_k} - \frac{S_k^T\nabla\varphi(x_{k+1,}\mu_{k+1})}{S_k^T y_k}(y_k - ts_k)$$

Equation – 14

By updating this formula using the improved technique of Dai-Liao CG in Equation-14, we obtain:

$$d_{k+1} = \frac{S_k^T(y_k - ts_k)}{S_k^T y_k}\varphi(x_{k+1,}\mu_{k+1}) + \frac{\nabla\varphi(x_{k+1,}\mu_{k+1})^T(y_k - ts_k)}{S_k^T y_k} - \frac{S_k^T\nabla\varphi(x_{k+1,}\mu_{k+1})}{S_k^T y_k}(y_k - ts_k)$$

Equation – 15

This is as follows when rewriting a new search direction

$$d_{k+1} = -Q_{k+1}\nabla\varphi(x_{k+1,}\mu_{k+1})$$

Equation – 16

Where,

$$Q_{k+1} = \frac{1}{S_k^T y_k}\left[S_k^T(y_k - ts_k).I + (y_k S_k^T - S_k y_k^T)\right]$$

Equation – 17

Since $y_k S_k^T > 0$(Due to the strong wolf condition), we get through this inequality and the half Newton condition:

$$Q_{k+1}s_k = y_k \Rightarrow S_k^T Q_k s_k > 0$$

Equation – 18

New Theorem:

The new direction d_{k+1} in Equation-15 satisfies the actual descending position in Equation-11.

Proof:

Now multiply each side of equation-15 by $\nabla\varphi(x_{k+1,}\mu_{k+1})$ which is the tendency to get if it is capable of random optimization

$$\nabla\varphi(x_{k+1,}\mu_{k+1})^T d_{k+1}$$

$$= -\frac{(y_k - ts_k)^T S_k}{S_k^T y_k}\|\nabla\varphi(x_{k+1,}\mu_{k+1})\|^2 + \frac{(y_k - ts_k)^T\nabla\varphi(x_{k+1,}\mu_{k+1})}{S_k^T y_k}\nabla\varphi(x_{k+1,}\mu_{k+1})s_k$$

$$-\frac{S_k^T\nabla\varphi(x_{k+1,}\mu_{k+1})}{S_k^T y_k}\nabla\varphi(x_{k+1,}\mu_{k+1})(y_k - ts_k)$$

$$= -\left[\frac{y_k^T s_k}{y_k^T s_k}\|\nabla\varphi(x_{k+1,}\mu_{k+1})\|^2 - t\frac{\|S_k\|^2}{y_k^T s_k}\|\nabla\varphi(x_{k+1,}\mu_{k+1})\|^2\right]$$

$$= \left[-1 + t\frac{\|S_k\|^2}{y_k^T s_k}\right]\|\nabla\varphi(x_{k+1,}\mu_{k+1})\|^2$$

Equation - 19

Let $s_k = \alpha_k d_k$

$$\nabla\varphi(x_{k+1,}\mu_{k+1})^T d_{k+1} = \left[-1 + t\frac{\|S_k\|^2}{y_k^T s_k}\right]\|\nabla\varphi(x_{k+1,}\mu_{k+1})\|^2 = \left[-1 + t\alpha_k\frac{\|S_k\|^2}{y_k^T s_k}\right]\|\nabla\varphi(x_{k+1,}\mu_{k+1})\|^2$$

Equation – 20

Scalar β_k^{DL} is known, it means $d_k = -g_k$, moreover, when the other end of the direction is multiplied y_k we get:

$$y_k^T d_k = \nabla\varphi(x_{k+1,}\mu_{k+1})^T d_k - \nabla\varphi(x_{k,}\mu_k)^T d_k = \nabla\varphi(x_{k+1,}\mu_{k+1})^T d_k + \|d_k\|^2 \geq \|d_k\|^2$$

Equation – 21

$$-\nabla\varphi(x_{k,}\mu_k)^T d_{k+1} \leq \left[-1 + t\alpha_k\frac{\|d_k\|^2}{\|d_k\|^2}\right]\|\nabla\varphi(x_{k+1,}\mu_{k+1})\|^2 \leq [-1 + t\alpha_k]\nabla\varphi(x_{k+1,}\mu_{k+1})$$

Equation – 22

Where $c = -(1 - t\alpha_k)$ is a positive constant, now we have following equation

$$\nabla\varphi(x_{k+1,}\mu_{k+1})^T d_{k+1} \leq -c\|\nabla\varphi(x_{k+1,}\mu_{k+1})\|^2 \left(\nabla f(x_{k+1}) - \mu_{k+1}\sum_{j=1}^{m}\frac{1}{c_j^2(x_{k+1})}\nabla c_j(x_{k+1})\right)^T_{d_{k+1}}$$

$$\leq -c\left[\nabla f(x_{k+1}) + \mu_{k+1}\sum_{j=1}^{m}\frac{-1}{c_j^2(x_{k+1})}\nabla c_j(x_{k+1})\right]^2 \left(\nabla f(x_{k+1}) - \mu_{k+1}\sum_{j=1}^{m}\frac{1}{c_j^2(x_{k+1})}\nabla c_j(x_{k+1})\right)^T_{d_{k+1}}$$

$$\leq -c\left((\nabla f(x_{k+1}))^T\nabla f(x_{k+1}) - 2\mu_{k+1}\nabla f(x_{k+1})^T\sum_{j=1}^{m}\frac{1}{c_j^2(x_{k+1})}\nabla c_j(x_{k+1})\nabla c_j(x_{k+1}) + \mu_{k+1}^2\left(\sum_{j=1}^{m}\frac{1}{c_j^2(x_{k+1})}\nabla c_j(x_{k+1})^T\sum_{j=1}^{m}\frac{1}{c_j^2(x_{k+1})}\nabla c_j(x_{k+1})\right)\right)$$

Equation – 23

Which we can written in differently

$$\left(\nabla f(x_{k+1}) - \mu_{k+1}\sum_{j=1}^{m}\frac{1}{c_j^2(x_{k+1})}\nabla c_j(x_{k+1})\right)^T_{d_{k+1}} \leq -c[\nabla f(x_{k+1}) - \mu_{k+1}\nabla B(x_{k+1})]^2$$

Equation – 24

and $B(x_{k+1})$ is the Barrier function at point $k + 1$

$$\mu_{k+1} = \frac{\mu_k}{10}, \mu_0 > 0$$

Equation – 25

So, when μ_{k+1} and to get a minute of the function $f(x)$ we take the function limit $\varphi(x, u)$ when $\mu \to 0$ in the form:

$$\nabla f(x_{k+1})^T d_{k+1} \leq -c\|\nabla f(x_{k+1})\|^2$$

Equation – 26

We get the direction of our new algorithm needed, the right slope.

The new direction of d_{k+1} is defined in equation – 15 is satisfying the conjugacy condition.

Proof:

Let,

$$y = (y_k - ts_k)$$

$$\tilde{y}_k^T d_{k+1} = \frac{\tilde{y}_k^T S_k}{y_k^T s_k} \tilde{y}_k^T \nabla\varphi(x_{k+1,}\mu_{k+1}) + \frac{\tilde{y}_k^T \nabla\varphi(x_{k+1,}\mu_{k+1})}{y_k^T s_k} \tilde{y}_k^T S_k - \frac{s_k^T \nabla\varphi(x_{k+1,}\mu_{k+1})}{y_k^T s_k} \tilde{y}_k^T y_k$$

Equation – 27

$$\tilde{y}_k^T d_{k+1} = \frac{\|\tilde{y}\|^2}{y_k^T s_k} S_K^T \nabla\varphi(x_{k+1,}\mu_{k+1}) = -\emptyset S_K^T \nabla\varphi(x_{k+1,}\mu_{k+1})$$

Equation – 28

Where, $\emptyset > 0$ and condition is = $y_k^T d_{k+1} = tS_k^T g_{k+1}$

Where,

$$g_{k+1} = \nabla\varphi(x_{k+1,}\mu_{k+1})$$

Equation – 30

New Theorem Convergence at Global:

Consider the new three-term CG-technique equation 15 which is a satisfactory equation -13 and assume that the size of the step completes -equation -8 and equation -10.

$$\lim_{k\to\infty} \|\nabla\varphi(x_{k+1,}\mu_{k+1})\| = 0$$

Equation – 33

Proof:

The Direction of New Search is:

$$d_{k+1} \le \frac{|y_k - ts_k||\|S_k\|}{\|y_k\|\|S_k\|} \|\nabla\varphi(x_{k+1,}\mu_{k+1})\| + \frac{|y_k - ts_k||\|S_k\|}{\|y_k\|\|S_k\|} \|\nabla\varphi(x_{k+1,}\mu_{k+1})\|$$

$$+ \frac{\|\nabla\varphi(x_{k+1,}\mu_{k+1})\|\|S_k\||y_k - ts_k|}{\|y_k\|\|S_k\|} \le (\|y_k\|t\|s_k\|)\left(\frac{3\|\nabla\varphi(x_{k+1,}\mu_{k+1})\|}{\|y_k\|}\right)$$

Equation – 34

From Lipschitz condition and:

$$\mu\|S_k\|^2 \le y_k^T S_k \le L\|S_k\| \le (L\|S_k\| + t\|S_k\|)\left(\frac{3y}{\mu\|S_k\|}\right) \le (L+t)\left(\frac{3y}{\mu}\right) = r$$

Equation – 35

Due to that by taking summation of each direction of search we get following:

$$\sum_{k\ge1} \frac{1}{\|d_{k+1}\|^2} \ge \frac{1}{r}\sum_{k\ge1} 1 = \infty$$

This means that the equation 33 is proved.

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