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VARIOUS EXPANSION METHODS FOR FEW FRACTIONAL DIFFERENTIALS AS SOLUTION

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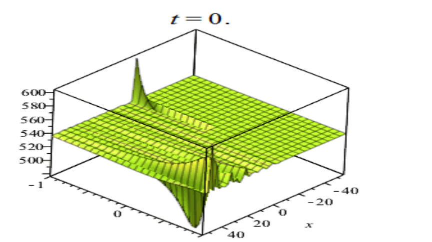
ABSTRACT :
In this paper, we formulate exact solutions of some nonlinear space time fractional differential equations generated by mathematical physics and modified Reimann Liouville in applied mathematics; that is, the fractional modified Benjamin Bona Mahony (MBBM) and the Kawahara equation.

KEYWORDS : *Fractional calculus, Kawahara equation.*



Kawahara equation traveling wave plot
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INTRODUCTION :

Fractional calculus has been used for the physical and engineering science models. Fractional differential equations (FDEs) are considered as models of physical systems. We introduce the space-time fractional MBBM equation (Equation 1, 2, 3).

$$D_t^{\alpha}u + D_t^{\alpha}u - vu^2 D_x^{\alpha}u + D_x^{3\alpha}u = 0 \qquad 1$$

Where u is a nonzero positive constant, we also consider the time fraction mode nonlinear shear defined in equation 4

$$D_t^{\alpha} + u^2 ux + pu_{xx} + qu_{xx} = 0 \qquad 2$$

Where α is the parameter for the order of the fractional time derivative, and 0 <$\alpha \leq$ 1: the modified Riemann-Liouville order α derivative of the Jumaries defined in equation 5

The $\frac{G'}{G}$ expansion method for FDE's

We are considering the following types of common nonlinear fractional differential equations (FDEs)…

$$\left(u, D_t^{\alpha}u, D_t^{\beta}u, D_t^{\alpha}D_t^{\alpha}u, D_t^{\alpha}D_x^{\beta}u, D_x^{\beta}D_x^{\beta}u, \ldots \ldots ..\right) = 0, \quad 0 < \alpha, \beta < 1 \qquad 3$$

Where $u = u\ (x,t)$ is an undefined function, following is the traveling wave variable…

$$u(x,t) = U(\zeta), \zeta = \frac{Tx^{\beta}}{\Gamma(1+\beta)} + \frac{ct^{\alpha}}{\Gamma(1+\alpha)} \quad 4$$

Where Γ is an nonzero arbitrary constants, with using chain rule….

$$D_t^{\alpha} u = \sigma_u' \frac{dU}{d\zeta} D_t^{\alpha}\zeta, D_x^{\alpha} u = \sigma_x' \frac{dU}{d\zeta} D_x^{\sigma}\zeta \quad 5$$

Where,

$\sigma_u'$and σ_x' are denote as sigma tables following equation 7, we can take it deprived of defeat of overview,

$$Q(U, U', U'', U''', \ldots..) = 0 \quad 6$$

Where the principal denotes this derivation of ζ Suppose that the solution of equation 9can be expressed by most $\frac{G'}{G}$ as follows in equation 8,9.

$$u(\zeta) = \sum_{i=0}^{m} a_i \left(\frac{G'}{G}\right)^i, a_m \neq 0 \quad 7$$

Where,

a_i $(i = 0, 1 \ldots \ldots, m)$ are constants, while G(ζ) satisfies following second order

$$G''(\xi) + \lambda G'(\xi) + \mu G(\xi) = 0 \quad 8$$

With λ and μ are stable. The positive integer m can be determined by the uniform equilibrium principle in equation 6 by substituting equation 7 into equation 6, and equation 8 forcing all the orders together. Subsequently, if every sub-angle of the polynomial is reduced to zero, we get a set of algebraic equations for $a_i (i = 0, 1, \ldots m)$, and λ, μ, Γ solving these equations; you can find various exact solutions of equation (3).

Algorithm expansion method of $(G' / G, 1 / G)$

We use linear ordinary differential equation for the second order….

$$G''(\xi) + \lambda G(\xi) = \mu \quad 9$$

We choose

$$\emptyset = G' / G, \vartheta = 1 / G \quad 10$$

For simplicity of after equation of 9 and 10 we gives…..

The details of ordinary differential equation 9 we conclude the following three cases

Case 1 if λ < 0, the simple solution of ordinary differential equation 9 will be read

$$G(\xi) = A_1 sinh(\sqrt{-\lambda\xi}) + A_2 \, cosh(\sqrt{-\lambda\xi}) + \frac{\mu}{\lambda} \quad 11$$

$$\psi^2 = \frac{-\lambda}{\lambda^2\sigma}(\quad - 2\mu\psi + \lambda), \sigma = A_1^2 - A_2^2 \quad 12$$

Case 2 if λ > 0 the simple solution of ordinary differential equation 9 will be read

$$G(\xi) = A_1 \sin(\sqrt{\lambda\xi}) + A_2 cos(\sqrt{\lambda\xi}) + \frac{\mu}{\lambda} \quad 13$$

And corresponding relation will be…

.

$$\psi^2 = \frac{\lambda}{\lambda^2\sigma-\mu^2}(\phi^2 - 2\mu\varphi + \lambda), \sigma = A_1^2 + A_2^2 \quad 14$$

Case 3 if λ = 0, the simple solution of ordinary differential equation 9 will be read

$$G(\xi) = \frac{u}{2}\xi^2 + A_1\xi + A_2 \quad 15$$

And we have

$$\psi^2 = \frac{1}{A_1^2-2\mu A_2}(\phi^1 - 2\mu\psi) \quad 16$$

The arbitrary constant A_1 and A_2, Suppose the solution of ordinary differential equation is polynomial and can be expressed $\phi \; and \; \psi$ in that form

$$u(\xi) = \sum_{i=0}^{N} a_i\phi^i + \sum_{i=1}^{N} b_0\phi^{i-1}\psi \quad 17$$

Where G = $G(\xi)$ is the second solution linear ordinary differential equation 9, $a_i, b_i(i = 1, \dots . N)$, λ and μ are constant and positive integers N can be balanced by the principle in ordinary differential equation 6. Employed in equation 18, using equation 6, using 11, and 13 the left of equation 6 can be expressed as a polynomial, and where the degree is not greater than;, the system of algebraic equations Solve algebraic equations and substitute the values of i; we can obtain the travel wave solution expressed by the hyperbolic functions of Eq. (6). (18) Substitution in equation 6 with using equation 11 and equation 14 or equation 11 and 16 we obtain the travel wave solution of equation 9 expressed by trigonometric and rational functions 10 and 11.

CONCLUSION:

The methods $(G' / G) \; and$ Extension Methods for Solving Nonlinear Fractional Partial Directional Equations These methods have their own advantages for nonlinear FDEs with fractional complex transforms: direct, concise, basic; And so it can also be applied to other FDEs where the uniformly balanced principle is Saint Ed.

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