REVIEW OF RESEARCH

EULER-LAGRANGE EQUATIONS WITH APPLICATION TO MATTER FIELDS

Dr. Vishwanath Kumar Rajak

Lecturer (Mathematics) ,
Govt. Polytechnic , Darbhanga.

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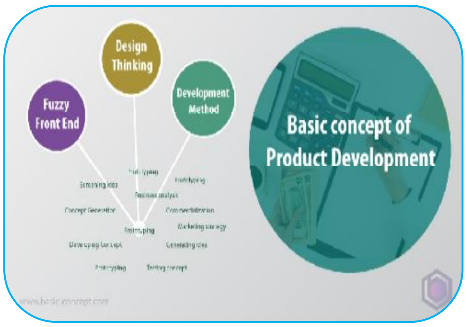

ABSTRACT:

This paper introduces the basic concept of Euker Lagrange equation. Our problem is $\nabla_\nu T^{\mu\nu} = 0$ .for matter fields.
In this paper, step has been taken to proove it numerically.

KEYWORDS: *collection of matter , basic concept.*

INTRODUCTION:-

Here Φ is the collection of matter fields and we consider smooth vector field X with compact support in U. This generates a 1-parameter-group $\{f_t\}$ of diffeomorphisms of U onto itself. [1-5]

PROOF

If A[g, Φ, U]. replacing g by gt = f^*_t g and Φ by $\Phi_t = f^*_t$ Then it is

$$\mathcal{A}(t) := \mathcal{A}[g_t, \Phi_t; \mathcal{U}] = \mathcal{A}[g, \Phi; \mathcal{U}]$$

since the action is an invariant. So A(t) is independent of t. We have

$$0 = \frac{d\mathcal{A}}{dt}|_{t=0} = \int_{\mathcal{U}} \{ -\frac{1}{2} T^{\mu\nu} \dot{g}_{\mu\nu} + E \dot{\Phi} \} d\mu_g = 0$$

E is the variation of A with respect to Φ, which

$$\dot{g} = \mathcal{L}_X g \ , \quad \dot{g}_{\mu\nu} = \nabla_\mu X_\nu + \nabla_\nu X_\mu$$

Hence

$$\begin{aligned} 0 &= -\int_{\mathcal{U}} \frac{1}{2} T^{\mu\nu} (\nabla_\mu X_\nu + \nabla_\nu X_\mu) d\mu_g \\ &= -\int_{\mathcal{U}} T^{\mu\nu} \nabla_\nu X_\mu d\mu_g \\ &= \int_{\mathcal{U}} (\nabla_\nu T^{\mu\nu}) X_\mu d\mu_g . \end{aligned}$$

X is arbitrary smooth of compact

So we write the gravitational stress in the following way.

$$\frac{\partial\sigma}{\partial g^{\mu\nu}} = \partial_\mu \Phi \partial_\nu \Phi$$

$$T^{\mu\nu} = 2 \frac{dL^*}{d\sigma} \partial^\mu \Phi \partial^\nu \Phi - L^* g^{\mu\nu}$$

Electromagnetic field.

First

$$\frac{\partial\alpha}{\partial g^{\mu\nu}} = 2 F_{\mu\kappa} F^\kappa_\nu$$

and

$$\frac{\partial\beta}{\partial g^{\mu\nu}} = \frac{1}{2} \beta g_{\mu\nu} , \quad \text{because of the identity: } F_{\mu\kappa} {}^*F^\kappa_\nu = \frac{1}{4} \beta g_{\mu\nu}$$

Finally, the gravitational stress may be written as

$$T^{\mu\nu} = 4 \frac{\partial L^*}{\partial\alpha} F^\mu_\kappa F^{\nu\kappa} + (\beta \frac{\partial L^*}{\partial\beta} - L^*) g^{\mu\nu}$$

We get

$$L^* = \pi^{\mu\nu} \pi_{\mu\nu}$$

When g vary , we can write

$$\dot{\pi} = \mathcal{L}_U \dot{g}$$

A[U] = $\int_U L^* d\mu_g$, then it is

$$\begin{aligned} \dot{\mathcal{A}} &= \int_{\mathcal{U}} \{ (-2\, \pi^{\mu}_{\kappa}\, \pi^{\nu\kappa} + \frac{1}{2}\, \pi^{\kappa\lambda}\, \pi_{\kappa\lambda}\, g^{\mu\nu})\, \dot{g}_{\mu\nu} + 2\, \pi^{\mu\nu}\, \dot{\pi}_{\mu\nu} \}\, d\mu_g\,, \\ \dot{\pi}_{\mu\nu} &= \mathcal{L}_U \dot{g}_{\mu\nu} = U^{\kappa}\, \nabla_{\kappa}\, \dot{g}_{\mu\nu} + \dot{g}_{\kappa\nu}\, \nabla_{\mu}\, U^{\kappa} + \dot{g}_{\kappa\mu}\, \nabla_{\nu}\, U^{\kappa}\,, \\ \int_{\mathcal{U}} \pi^{\mu\nu}\, \dot{\pi}_{\mu\nu}\, d\mu_g &= \int_{\mathcal{U}} \{ -\nabla_{\kappa}\, (U^{\kappa}\, \pi^{\mu\nu}) + \pi^{\kappa\nu}\, \nabla_{\kappa}\, U^{\mu} + \pi^{\kappa\mu}\, \nabla_{\kappa}\, U^{\nu} \}\, \dot{g}_{\mu\nu}\, d\mu_g\,. \end{aligned}$$

So, we get

$$T^{\mu\nu} = 4\, \pi^{\mu}_{\kappa}\, \pi^{\nu\kappa} - \pi^{\kappa\lambda}\, \pi_{\kappa\lambda}\, g^{\mu\nu} + 4\, \{ \nabla_{\kappa}\, (U^{\kappa}\, \pi^{\mu\nu}) - \pi^{\kappa\nu}\, \nabla_{\kappa}\, U^{\mu} - \pi^{\kappa\mu}\, \nabla_{\kappa}\, U^{\nu} \}$$

Now, we assume the case of conformal properties.

Let

$$g \mapsto \tilde{g} = \Omega^2\, g\,, \quad \mathcal{A}\, [\tilde{g}] = \mathcal{A}\, [g]$$

Hence

$$\dot{g} = \lambda\, g\,, \quad \lambda = 2\, \Omega\, \dot{\Omega}$$

And

$$\dot{\mathcal{A}} = 0$$

We know

$$\dot{\mathcal{A}} = -\frac{1}{2} \int_{\mathcal{U}} T^{\mu\nu}\, \dot{g}_{\mu\nu}\, d\mu_g$$

we get.

$$0 = -\frac{1}{2} \int_{\mathcal{U}} \lambda\, tr\, T\, d\mu_g$$

For arbitrary function λ, it follows that trT = 0.

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