



SOME CONTRIBUTION TO THE SCOPE AND DEVELOPMENT OF HINDU MATHEMATICS

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ABSTRACT

It is generally admitted that the decimal place value system of numeration notation was invented and first used by Hindus but it is not yet fully realised to what extent we are indebted to them for our elementary mathematics. This is due to the lack of a reliable and authentic history of Hindu Mathematics. It is said that in ancient India no science did ever attain an independent existence what-so-ever of any science is found in vedic India is supposed to have been originated and grown as the Hindi man of one of the other of the six members of vedas and helping the vedic rituals.



KEY WORD: *reliable and authentic history , ancient India.*

INTRODUCTION

It is said that once upon a time Narada approached the sage Sanat Kumar and begged of him the Brahmagvidya i.e. the supreme knowledge. There-upon Narada enumerated the various science and arts studied by him. This included astronomy and arithmetic also. So the culture of the science of mathematics or of any branches of secular knowledge was in advance stage. Importance to the culture of ganita or mathematics is also given by the Jainas. Their religious like value is generally classified into four branches known as exposition of Principles one of them is the exposition of the principles of mathematics i.e. the science of number meaning thereof arithmetic and astronomy is stated to be one of the principal accomplishments of the Jain Priest. In Buddhist literature too, arithmetic is regarded as the first and the noblest of the arts. All these give us a fair idea of the importance and value set-upon the culture of ganita in ancient India. The following appreciation of mathematics is found to be very interesting specially as it comes from the pen of Mahavir : one of the best mathematicians of his time.

In the early 6th century AD, the Indian mathematician Aryabhata incorporated an existing version of this system in his work, and experimented with different notations. In the 7th century, Brahmagupta established the use of 0 as a separate number and determined the results for multiplication, division, addition and subtraction of zero and all other numbers, except for the result of division by 0. His contemporary, the Syriac bishop Severus Sebokht described the excellence of this system as "... valuable methods of calculation which surpass description". The Arabs also learned this new method and called it *hesab*. Although the Codex Vigilanus described an early form of Arabic numerals (omitting 0) by 976 AD, Fibonacci was primarily responsible for spreading their use throughout Europe after the publication of his book *Liber Abaci* in 1202. He considered the significance of this "new" representation of numbers, which he styled the "Method of the Indians" (Latin *Modus Indorum*), so fundamental that all related mathematical foundations, including the results of Pythagoras and the algorism describing the methods for performing actual calculations, were "almost a mistake" in comparison. In the Middle Ages, arithmetic was

one of the seven liberal arts taught in universities. The flourishing of algebra in the medieval Islamic world and in Renaissance Europe was an outgrowth of the enormous simplification of computation through decimal notation. Various types of tools exist to assist in numeric calculations. Examples include slide rules (for multiplication, division, and trigonometry) and nomographs in addition to the electrical calculator

1.1 Prime numbers

If $a, b \in \mathbb{Z}$ we say that a divides b (or is a divisor of b) and we write $a \mid b$, if

$$b = ac$$

for some $c \in \mathbb{Z}$.

Thus $-2 \mid 0$ but $0 \nmid 2$.

Definition 1.1.1 The number $p \in \mathbb{N}$ is said to be prime if p has just 2 divisors in \mathbb{N} , namely 1 and itself.

Note that our definition excludes 0 (which has an infinity of divisors in \mathbb{N}) and 1 (which has just one).

Writing out the prime numbers in increasing order, we obtain the *sequence of primes*

2, 3, 5, 7, 11, 13, 17, 19,

which has fascinated mathematicians since the ancient Greeks, and which is the main object of our study.

Definition 1.2 We denote the n th prime by p_n .

Thus $p_5 = 11$; $p_{100} = 541$.

It is convenient to introduce a kind of inverse function to p_n .

Definition 1.3 If $x \in \mathbb{R}$ we denote by $\pi(x)$ the number of primes $< x$:

$$\pi(x) = |\{p < x : p \text{ prime}\}|$$

Thus

$$\pi(1:3) = 0; \pi(3:7) = 2:$$

Evidently $\pi(x)$ is monotone increasing, but discontinuous with jumps at each prime $x = p$.

Theorem 1.4 (Euclid's First Theorem) The number of primes is infinite.

Proof ▶ Suppose there were only a finite number of primes, say

p_1, p_2, \dots, p_n :

Let

$$N = p_1 p_2 \dots p_n + 1:$$

Evidently none of the primes p_1, \dots, p_n divides N .

Lemma 1.1 Every natural number $n > 1$ has at least one prime divisor.

Proof of Lemma ▶ The smallest divisor $d > 1$ of n must be prime. For otherwise d would have a divisor e with $1 < e < d$; and e would be a divisor of n smaller than d . ◀

By the lemma, N has a prime factor p , which differs from p_1, \dots, p_n . ◀

Our argument not only shows that there are an infinity of primes; it shows that

$$p_n < 2^{2^n}$$

a very feeble bound, but our own. To see this, we argue by induction. Our proof shows that

$$p_{n+1} < p_1 p_2 \dots p_n + 1:$$

But now, by our inductive hypothesis,

$$p_1 < 2^{2^1}, p_2 < 2^{2^2}, \dots, p_n < 2^{2^n}$$

It follows that

$$p_{n+1} < 2^{2^1+2^2+\dots+2^n}$$

But

$$2^1 + 2^2 + \dots + 2^n = 2^{n+1} - 1 < 2^{n+1};$$

Hence

$$p_{n+1} < 2^{2^{n+1}}$$

It follows by induction that

$$p_n < 2^{2^n}$$

for all $n > 1$, the result being trivial for $n = 1$.

This is not a very strong result, as we said. It shows, for example, that the 5th prime, in fact 11, is $< 2^{2^5} = 2^{32} = 4294967296$:

In general, any bound for p_n gives a bound for $\pi(x)$ in the opposite direction, and vice versa; for

$$p_n < x \Leftrightarrow \pi(x) > n:$$

In the present case, for example, we deduce that

$$\pi(2^{2^y}) > [y] > y - 1$$

and so, setting $x = 2^{2^y}$,

$$\pi(x) > \log_2 \log_2 x > 1 > \log \log x - 1:$$

for $x > 1$. (We follow the usual convention that if no base is given then $\log x$ denotes the logarithm of x to base e .)

The *Prime Number Theorem* (which we shall make no attempt to prove) asserts that

$$p_n \sim n \log n;$$

or, equivalently,

$$\pi(x) \sim \frac{x}{\log x}$$

This states, roughly speaking, that the probability of n being prime is about $1/\log n$. Note that this includes even numbers; the probability of an *odd* number n being prime is about $2/\log n$. Thus roughly 1 in 6 odd numbers around 10^6 are prime; while roughly 1 in 12 around 10^{12} are prime.

CONCLUSION

There are several alternative proofs of Euclid's Theorem. We shall give one below. But first we must establish the Fundamental Theorem of Arithmetic (the Unique Factorisation Theorem) which gives prime numbers their central role in number theory; and for that we need Euclid's Algorithm

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